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Finite Element Modelling of Viscoelastic Rotors: An Operator Based Approach

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ABSTRACT

Damping exists in every material in varying degrees. So materials in general are viscoelastic in nature. Modelling of viscoelastic materials is always difficult since such materials store energy as well as dissipate it to the thermal domain. This paper presents a theoretical study of the dynamics of a viscoelastic rotor-shaft system, where the internal material damping in the rotor shaft introduces a rotary force well known to cause instability of the rotor-shaft system. For this the material constitutive relationship has been represented by a differential time operator. Use of operators enables to consider general linear viscoelastic behaviours, represented in the time domain, for which, in general, instantaneous stress and its derivatives are proportional to instantaneous strain and also its derivatives. The operator may be suitably chosen according to the material model. An efficient modelling technique for viscoelastic material augmenting thermodynamic field (ATF) has been found in literature. The constitutive relationships for ATF approach is represented in differential time operator to obtain the equations of motion of a rotor-shaft system after discretizing the system using finite element method. The equations thus developed may easily be used to find the stability limit speed of a rotor-shaft system as well as the time response when the rotor-shaft-system is subjected to any kind of dynamic forcing function.

Keywords: Viscoelastic rotor, Internal damping, Augmenting thermodynamic field, Stability limit of spin speed, Unbalance response.

1. INTRODUCTION

Viscoelasticity, as the name implies, is a property that combines elasticity and viscosity. In other words viscoelastic materials store energy and dissipate it as well. For this reason, it is extensively used in various engineering applications for controlling the amplitude of resonant vibrations and modifying wave attenuation and sound transmission properties, increasing structural life through reduction in structural fatigue. Nakra (1998) has reported many such applications.

In viscoelastic materials stress and strain are not in phase under dynamic deformation,, the frequency of which, in the case of cyclic deformation, has considerable influence on energy

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storage and dissipation. For this reason, in linear viscoelastic solids, the instantaneous stress is obtained by operating the instantaneous strain by a linear differential time operator, which is a constant (the Young's modulus) for the special case of linear elastic behaviour. Different multi-element spring-damper models like the 2, 3, 4 element models [Bland (1960)] as well as internal variable models e.g. Augmenting Thermodynamic Field (ATF) [Lesieutre (1989), Lesieutre and Mingori (1990)] and Anelastic Displacement Field (ADF) approaches [Lesieutre and Bianchini (1995), Lesieutre et. al. (1996)] are used to represent the operator as also the viscoelastic material behaviour physically.

This paper attempts to study the dynamics of a viscoelastic rotor shaft system considering the effect of internal material damping in the rotor. Unlike in structures, rotation of rotors introduces a rotary damping force due to internal material damping and acts tangential to the rotor orbit, which is well known to cause instability in rotor-shaft systems after certain spin speed. Thus, a reliable model is indeed necessary to represent the rotor internal damping for correct prediction of stability limit of spin speed of a rotor-shaft system. Modelling the rotor internal damping using viscous and hysteretic model has been attempted by many researchers [Dimentberg (1961), Tondl (1965), Genta (2005)]. Most of the authors have considered, in general, viscous form of internal damping and used 2-element Voigt model for representing the material behaviour to study the dynamics of rotor-shaft systems. Again Zorzi and Nelson (1977), Ozguven and Ozkan (1984), Ku (1998) developed a finite element model of the rotor material damping by representing its constitutive relationship with a Voigt model (2-element model) where internal material damping force was considered as a superposition of viscous and hysteretic damping forces to take into account the frequency dependent and frequency independent components of energy dissipation per cycle for properly representing the properties of structural materials like steel. In this regard Genta (2004) pointed out the correct interpretation and use of the hysteretic damping model.

However viscous and hysteretic damping models are unsuitable for proper representation of viscoelastic material behaviour, which shows considerable dependence on wide range of excitation frequencies. Not many papers are found to report dynamic simulation of viscoelastic rotors. Grybos (1991) used 3-element material model and studied the dynamics of a viscoelastic rotor. Roy et al. (2008) reported a finite-element approach, where viscoelastic behaviour of a rotor-continuum was represented by ATF (Augmenting Thermodynamic Field). Recently Dutt and Roy (2010) obtained the equation of motion of viscoelastic rotor-shaft system after discretizing the continuum by finite beam element method. The rotor-shaft material was assumed to behave as a linear viscoelastic solid for which the instantaneous stress was obtained by operating the instantaneous strain by a generic linear differential time operator. The advantage of using a generic operator approach is that it may be suitably tailored according to the material constitutive relationship to obtain the equations of motion for a particular material model.

In this present reporting, an attempt has been made to study theoretically the stability limit of the spin speed,unbalance vibration response and subsequently time response within the stable zone of operation of a simply supported aluminium rotor-shaft having a central disc made of aluminium. The rotor-shaft material is assumed to behave as a linear viscoelastic solid for which the instantaneous stress is obtained by operating the instantaneous strain by a linear differential time operator. The internal variable approach i.e. ATF is used for modelling the viscoelastic material. The constitutive relationships for ATF approach is represented in differential time operator, where the coefficients of the operator are formed by ATF parameters. The equations of motion of a rotor-shaft system are obtained after discretizing the continuum using finite beam element. So this work is useful for dynamic analysis of viscoelastic rotors under any type of dynamic forcing function.

2. CONSTITUTIVE RELATIONS AND EQUATIONS OF MOTION

The constitutive relationships are obtained from the Helmholtz free energy density function, $\mathcal{H}$ representing a thermodynamic potential, where strain (ε) is an independent variable. The function, $\mathcal{H}$ is defined as [Lesieutre and Mingori (1990)]

$$\mathcal{H} = \frac{1}{2} E\varepsilon^2 - \delta\varepsilon\xi + \frac{1}{2}\alpha\xi^2 \tag{1}$$

The constitutive equations are obtained as:

$$\sigma = \frac{\partial\mathcal{H}}{\partial\varepsilon} = E\varepsilon - \delta\xi,\ \mathfrak{A} = -\frac{\partial\mathcal{H}}{\partial\xi} = \delta\varepsilon - \alpha\xi \tag{2}$$

In the above equation E is the un-relaxed modulus, σ is the mechanical stress, ξ is the augmenting thermodynamic field (ATF), $\mathfrak{A}$ is the affinity, α is a material property relating the changes in $\mathfrak{A}$ to ξ and δ is the strength of coupling between the mechanical displacement field and the thermodynamic field.

Following [Lesieutre and Mingori (1990)] the relaxation equation is given as:

$$\dot{\xi} + B\xi = \frac{B\delta}{\alpha}\varepsilon \quad \text{Or,} \quad \xi = \frac{B\delta}{\alpha}\frac{1}{B+D}\varepsilon \tag{3}$$

Putting values of ξ from equation (3) in equation (2), the constitutive relationship is rewritten as:

$$\sigma = E\left(1-\tfrac{B\delta^2}{\alpha}\tfrac{1}{B+D}\right)\varepsilon = \tfrac{a_0+a_1\mathrm{D}}{b_0+b_1\mathrm{D}}\,\varepsilon \tag{4}$$

where, $a_0 = E-\tfrac{\delta^2}{\alpha},\ a_1=\dfrac{E}{B},\ b_0=1,\ b_1=\dfrac{1}{B},\ \mathrm{D}=\dfrac{d}{dt}$.

The instantaneous normal stress σ_x is obtained from the equation (4) i.e. by operating the expression of ε_x by the operator *E* ( ), the generic form of which is expressed in equation (5) below, where *Nu*(D) and *Dn*(D) are the numerator and denominator polynomials of differential time operator, D ≡ d/dt. For the special case of linear elastic behaviour, *E* is a constant called the Young's modulus.

$$\mathrm{E}(\ ) = \frac{\mathrm{Nu(D)}}{\mathrm{Dn(D)}} \tag{5}$$

Mechanical models for physical representation of viscoelastic behaviour are given by Bland (1960) among others. Expressions of *E* ( ), for 2, 3, and 4-element models for example, as shown in figure 1, are given bellow.

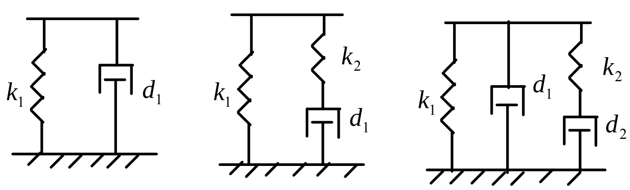

Figure 1: Different Viscoelastic Models

$$E_2() = a_0 + a_1\mathrm{D}, a_0 = k_1, a_1 = d_1 \tag{6a}$$

$$E_3() = \frac{a_0 + a_1\mathrm{D}}{(b_0 + b_1\mathrm{D})}, a_0 = k_1, a_1 = d_1 + \frac{k_1 d_1}{k_2}, b_0 = 1, b_1 = \frac{d_1}{k_2} \tag{6b}$$

$$E_4() = \frac{a_0 + a_1\mathrm{D} + a_2 D^2}{(b_0 + b_2\mathrm{D})}, a_0 = k_1, a_1 = d_1 + d_2 + \frac{k_1 d_2}{k_2}, a_2 = \frac{d_1 d_2}{k_2}, b_0 = 1, b_1 = \frac{d_1}{k_2} \tag{6c}$$

For the nomenclature, all springs and dampers directly connected to the ground are called 'primary' and those connected in series are called 'secondary'. Following this, all springs and dashpots with subscript '1' are primary and the ones with subscript '2' are secondary.

Let ε_x denotes the mechanical strain induced in the element at an instant of time. Zorzi and Nelson (1977) expressed the mechanical strain in the 'x' direction as

$$\varepsilon_x = -r\cos[(\Omega-\omega)t]\frac{\partial^2 R(x,t)}{\partial x^2} \tag{7}$$

Zorzi and Nelson (1977) obtained the bending moment expressions after considering a 2-element material model (Voigt model) to represent the constitutive-relationship of the material. The bending moments expression at any instant of time about the y and z-axes, M_{yy} and M_{zz} respectively, are expressed as given below.

$$
\begin{aligned}
M_{zz} &= \int_0^{2\pi}\int_0^{r_0} -\left(v + r\cos\left(\Omega t\right)\right)\sigma_x r dr d(\Omega t) \\
M_{yy} &= \int_0^{2\pi}\int_0^{r_0} \left(w + r\sin\left(\Omega t\right)\right)\sigma_x r dr d(\Omega t)
\end{aligned} \tag{8}
$$

The displaced cross-section of the rotor-shaft is shown in figure 2, the outer radius r_0 is measured at any distance 'x' from the left end along the length of the shaft, called the 'x' direction. Coordinates of the shaft centre at any instant of time,'t', are given as ($v(x,t)$, $w(x,t)$) along 'y' and 'z', the transverse directions respectively, where (x,t) are the spatial and temporal variables. Angle 'Ωt' in the figure, denotes the instantaneous orientation of the radius vector showing the instantaneous deflection $R(x,t)$, shown in the figure as 'R' for convenience. An infinitesimal element of thickness 'dr', subtending an angle '$d(\Omega t)$' at the centre is chosen at a radius 'r' and angular location 'Ωt', is shown in the figure, where 'Ω' and 'ω' denote the spin and whirl frequency of the rotor in radians per second.

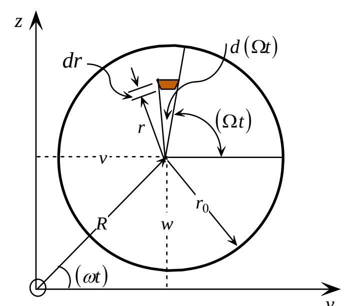

Figure 2: Displaced position of the shaft cross-sedtion

The instantaneous bending moments are written next by extending the work by Zorzi and Nelson (1977). Substituting σ_x from equation (4) in the bending moment expressions (equations (8)) and utilizing the expressions of ε_x given in equation (7), the bending moment expressions are rewritten as

$$
\begin{aligned}
M_{zz} &= \int_0^{2\pi}\int_0^{r_0} -\left(v + r\cos\left(\Omega t\right)\right)\frac{a_0+a_1\mathrm{D}}{b_0+b_1\mathrm{D}}\left[-r\cos\left(\Omega t-\omega t\right)\frac{\partial^2 R(x,t)}{\partial x^2}\right] r dr d(\Omega t) \\
M_{yy} &= \int_0^{2\pi}\int_0^{r_0} \left(w + r\sin\left(\Omega t\right)\right)\frac{a_0+a_1\mathrm{D}}{b_0+b_1\mathrm{D}}\left[-r\cos\left(\Omega t-\omega t\right)\frac{\partial^2 R(x,t)}{\partial x^2}\right] r dr d(\Omega t)
\end{aligned} \tag{9}
$$

It may be noted in equation (9) that the operator E ( ) is operated exclusively on the terms inside the bracket [ ] containing the expression of strain to give the stress. The operator does not work on other terms, ($v + r\cos(\Omega t)$ and $w + r\sin(\Omega t)$) forming the momentums in the respective planes at any instant of time,'t'. Hence the expressions of momentums are perceived as constants as far as the operator E ( ) is concerned. Following this logic the equation (9) may be rewritten as

$$M_{zz} = \frac{1}{b_0 + b_1 \mathrm{D}} \int_0^{2\pi} \int_0^{r_0} r^2 \left(v + r\cos(\Omega t)\right) [a_0 \cos(\Omega t - \omega t) \frac{\partial^2 R}{\partial x^2} + a_1 \cos(\Omega t - \omega t) \frac{\partial^3 R}{\partial x^2 \partial t}$$
$$- a_1 (\Omega - \omega) \sin(\Omega t - \omega t) \frac{\partial^2 R}{\partial x^2}] dr d(\Omega t)$$

$$M_{yy} = -\frac{1}{b_0 + b_1 \mathrm{D}} \int_0^{2\pi} \int_0^{r_0} r^2 \left(w + r\sin(\Omega t)\right) [a_0 \cos(\Omega t - \omega t) \frac{\partial^2 R}{\partial x^2} + a_1 \cos(\Omega t - \omega t) \frac{\partial^3 R}{\partial x^2 \partial t}$$
$$- a_1 (\Omega - \omega) \sin(\Omega t - \omega t) \frac{\partial^2 R}{\partial x^2}] dr d(\Omega t)$$

After performing the integration

$$M_{zz} = \frac{I}{b_0 + b_1 \mathrm{D}} [a_0 v'' + a_1 \dot{v}'' + \Omega a_1 w'']$$

$$M_{yy} = -\frac{I}{b_0 + b_1 \mathrm{D}} [a_0 w'' + a_1 \dot{w}'' - \Omega a_1 v'']$$

Or,

$$\begin{Bmatrix} M_{zz} \\ M_{yy} \end{Bmatrix} = \frac{I}{b_0 + b_1 \mathrm{D}} \left[\begin{bmatrix} a_0 & a_1 \Omega \\ a_1 \Omega & -a_0 \end{bmatrix} \begin{Bmatrix} v'' \\ w'' \end{Bmatrix} + \begin{bmatrix} a_1 & 0 \\ 0 & -a_1 \end{bmatrix} \begin{Bmatrix} \dot{v}'' \\ \dot{w}'' \end{Bmatrix} \right] \tag{10}$$

For developing the equations of motion the rotor-shaft continuum is discretized into finite beam elements having two nodes at the ends and 4 degrees of freedom, which are the displacements and slopes in x-y and z-x planes at each node. The expression of translations and relations with rotations are given as

$$\begin{Bmatrix} v(x,t) \\ w(x,t) \end{Bmatrix} = \left[\phi(x)\right]^T \left\{q(t)\right\}; \qquad \Phi = -\frac{\partial w}{\partial x}; \qquad \Gamma = \frac{\partial v}{\partial x} \tag{11}$$

where v, w denote the deformations along and Φ, Γ are the rotations about the y and z axes respectively.

The equations of motion may easily be written using complex coordinates. Expressions of the stiffness, damping and circulatory matrices due to bending are obtained from the strain energy and dissipation function calculated from the expression of bending moments given in the equation (10). Diagonal elements of each coefficient matrix in this equation give rise to a direct matrix (e.g. direct stiffness, direct damping matrix) whereas the cross-diagonal elements give rise cross coupled matrices; reference (Ozguven and Ozkan (1984)) may be seen for details. The expressions of generalized force vectors comprising of forces and moments acting in the 'x-y' and 'z-x' planes i.e. $\{[\overline{F}_{xy}]_{(1x4)}\ [\overline{F}_{zx}]_{(1x4)}\}^T$. Composition of stiffness, circulatory as well as damping matrices are also shown below.

$$\begin{Bmatrix} \{\overline{F}_{xy}\}_{(1x4)} \\ \{\overline{F}_{zx}\}_{(1x4)} \end{Bmatrix} = \frac{I}{(b_0+b_1D)}\left[a_0[K_b]_{(8x8)}\{q\}_{(1x8)} + a_1\Omega[K_c]_{(8x8)}\{q\}_{(1x8)} + a_1[K_b]_{(8x8)}\{\dot{q}\}_{(1x8)}\right] \tag{12}$$

The expression of $[K_b]$ and $[K_c]$ are given as

$[K_b] = \int_0^l I[\phi''(x)][\phi''(x)]^T\, dx$, $[K_c] = \int_0^l I[\phi''(x)]\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}[\phi''(x)]^T\, dx$ where the Hermite shape function matrix $[\phi(x)]$, (refer to Rao (1996)) is given by $[\phi(x)] = \begin{bmatrix} \{\phi_{xy}(x)\} & \{0\} \\ \{0\} & \{\phi_{zx}(x)\} \end{bmatrix}$, with subscripts in the elements showing the respective planes.

Operating by the operator Dn(D)=$(b_0 + b_1D)$ throughout and arranging terms with same orders of differentiation together, the equations of motion of one shaft element are given below. Assuming the rotor is rotating at a uniform speed (Ω).

$$\begin{aligned} b_1[M]\{\dddot{q}\} + (b_0[M] + b_1[G])\{\ddot{q}\} + (b_0[G] + a_1[K_b])\{\dot{q}\} \\ + (a_0[K_b] + a_1\Omega[K_c])\{q\} = (b_0+b_1D)\{P\} \end{aligned} \tag{13}$$

Where $[M]_{(8x8)} = [M_T]_{(8x8)} + [M_R]_{(8x8)}$, $[M_T]_{(8x8)}$ is the translational mass matrix, $[M_R]_{(8x8)}$ is the rotary inertia matrix, $[G_T]_{(8x8)}$ is the gyroscopic matrix. Effects due to simultaneous action of spin and vibratory motion the rotary inertia and gyroscopic matrix are taken into account. The expressions of translational mass matrix, rotary inertia matrix and gyroscopic matrix are given below after following Rao (1996).

$$[M_T] = \int_0^l \rho A\phi(x)\phi(x)^T\, dx,\ , \quad [M_R] = \int_0^l \rho I\phi'(x)\phi'(x)^T\, dx\, , \quad [G] = \int_0^l 2\rho I\phi'(x)\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\phi'(x)^T\, dx$$

It may be noted that the 3rd order differential equation result in this process. Dutt and Nakra (1992) also obtained a third order differential equation as they modelled the rotor supports of an elastic rotor-shaft using a 4-element spring-dashpot model.

Again equation (13) may be further combined and rewritten as

$$[\mathcal{A}]\{\dot{\mathcal{X}}\} + [\mathcal{B}]\{\mathcal{X}\} = \{\mathcal{P}\} \tag{14}$$

$$[\mathcal{A}] = \begin{bmatrix} [A_3] & [0] & [0] \\ [0] & [A_3] & [0] \\ [0] & [0] & [A_3] \end{bmatrix},\ [\mathcal{B}] = \begin{bmatrix} [0] & -[A_3] & [0] \\ [0] & [0] & -[A_3] \\ [A_0] & [A_1] & [A_2] \end{bmatrix},\ \{\mathcal{X}\} = \begin{Bmatrix} \{q\} \\ \{\dot{q}\} \\ \{\ddot{q}\} \end{Bmatrix},\ \{\mathcal{P}\} = (b_0+b_1D)\begin{Bmatrix} \{0\} \\ \{0\} \\ \{P\} \end{Bmatrix}$$

3. STABILITY LIMIT OF THE SPIN SPEED AND SYNCHRONOUS UNBALANCE RESPONSE OF THE DISC

Stability of the system is determined from the eigenvalues estimated from the equation of motions for free vibration obtained by putting a zero vector on the right hand side of equation (14). A code written in MATLAB (version 7.1) has been used to find out the eigenvalues at a spin speed of the rotor. The eigenvalue to be in general complex, the system is stable only if the maximum real part of the eigenvalues is < 0. Stability limit of the spin speed SLS is the maximum spin-speed till which all the eigen values have negative real parts.

Synchronous unbalance response (UBR) at the disc location for speeds < SLS is obtained from the equation of motion with force vector $\{\mathcal{P}\} = \{\bar{\mathcal{P}}\} e^{i\Omega t}$ where the angle Ωt is measured from the *y*-axis in the direction of rotation of the rotor.

Using the trial solution $\{X\} = \{\bar{X}\} e^{i\Omega t}$ in the equation of motion (14)

$$\{\bar{X}\} = \left[[\mathcal{B}] + i\Omega[\mathcal{A}]\right]^{-1} \{\bar{\mathcal{P}}\}$$

The disc response amplitude is given by $R_{disc} = \max\left|\mathrm{Real}\left(\bar{X}_{y\text{-}disc} e^{i\Omega t}\right) + i\mathrm{Real}\left(\bar{X}_{z\text{-}disc} e^{i\Omega t}\right)\right|$

where $\bar{X}_{y\text{-}disc}$ and $\bar{X}_{z\text{-}disc}$ are the complex elements in $\{\bar{X}\}$ at the disc node along y and z directions.

4. NON-DIMENSIONALISATION

To ensure the applicability of the results to rotor shaft systems and for ease of presentation a few non-dimensional parameters are defined.

Non-dimensional Stability Limit Speed, $\mathrm{SLS} = \dfrac{\Omega_{\lim}}{\omega_n}$

where $\omega_n = \sqrt{k/M_{eff}}$ is the natural frequency of a uniform simply supported shaft of mass M having an central disc of mass m, $k = \dfrac{48EI}{L^3}$ is the stiffness of the simply supported shaft and M_{eff} is its effective mass. Raleigh's principle is used to find out M_{eff} from the expression of maximum kinetic energy calculated by assuming the static deflection function of the elastic curve. For a simply supported beam with a central load as in this situation the maximum kinetic energy $T_{\max}$ is given by

$$T_{\max} = \frac{1}{2} M_{eff} \dot{y}_{\max}^2 = \frac{1}{2} m\dot{y}_{\max}^2 + \frac{1}{2}\left(\frac{M}{L}\right)\dot{y}_{\max}^2 \, 2\int_0^{1/2} \left\{\left(\frac{3x}{L}\right) - 4\left(\frac{x}{L}\right)^3\right\}^2 dx$$

where $\dot{y}_{\max}$ is the maximum velocity at the mid span. From the above expression it is seen that $M_{eff} = m + 0.4853M$.

Non-dimensional Unbalance Response Amplitude, $\text{UBR} = \frac{R_{disc}}{e}$, where, $e = \frac{U}{m}$ and U is the unbalance in the disc.

Non-dimensional spin speed is defined as $\Omega^* = \frac{\Omega}{\omega_n}$

5. RESULTS AND DISCUSSION

5.1 The rotor shaft system

A rotor shaft system as shown in figure 3, is made of aluminium (E = 7.13e10 Pa, ρ = 2750 Kg/m^3), has been considered. The rotor shaft (L = 1.0 m, D_R = 0.05 m) is mounted at the ends in bearings considered as simply supported ends. The aluminium disc (D_D = 0.15 m, t_D = 0.03 m) is put centrally and has an unbalance, U = 10 gm-mm. Following Lesieutre and Mingori (1990) the ATF parameters of aluminium are B = 8000 sec^{-1}, α = 8000 Pa, δ = 4.7766e6 Pa.

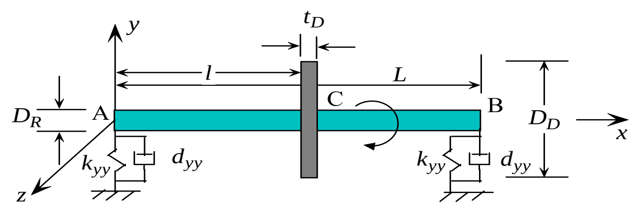

Figure 3 Schematic diagram of the rotor

5.2 Stability limit of spin speed

Stability Limit of the Spin speed (SLS) of the rotor–shaft system (figure 3) has been found out by plotting the maximum real part of all eigen values vs. non-dimensional spin speed (Ω^*) in figure 4(a). SLS corresponds to the non-dimensional spin speed when the maximum value of the real part of all the eigenvalues touches the zero line. Steady state non-dimensional synchronous Unbalance Response Amplitude (UBR) of the disc is plotted in figure 4(b) within the respective stable speed zones of operation (i.e. below the unstable zone marked by UZ) of the rotor-shaft. It may be seen that the value of SLS is very close to 1, which was noted by several researchers [Dimentberg (1961).

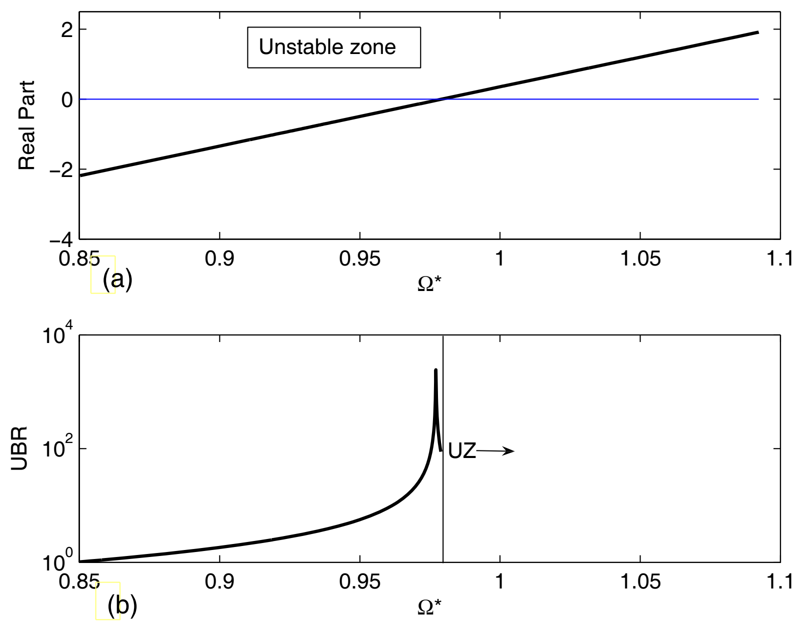

Figure 4 UBR for various spin speeds within stable zone

Figure 5 shows the effect of different positions of the disc along the shaft on the SLS. It is seen that SLS has the lowest value when the disc is placed in the middle of the shaft. This happens as the gyroscopic stiffening due to the disc, increases when the disc is gradually displaced from the mid span to the ends.

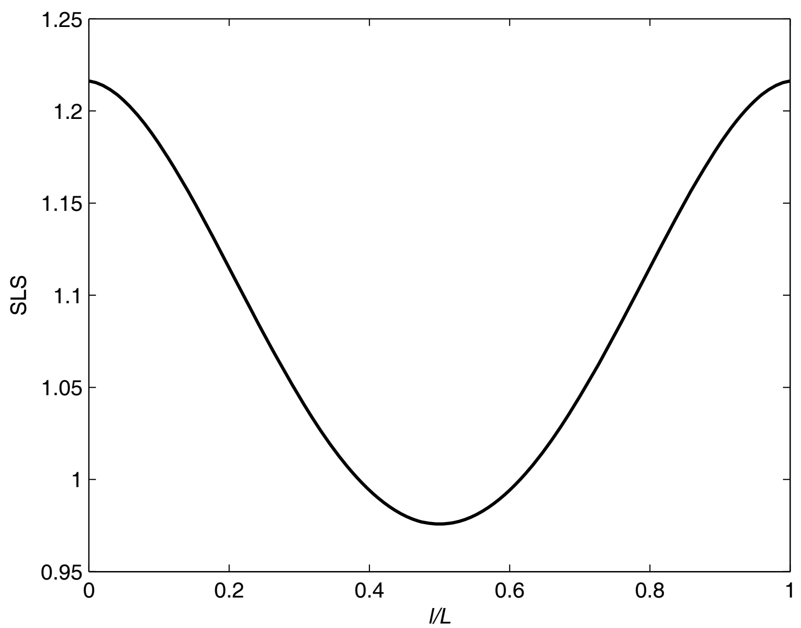

Figure 5 SLS for various disc positions

Transient and steady state time response of the disc due to unbalance have been shown in figure 6 and figure 7 respectively for speeds below and over the SLS, two different spin speed. When the rotor is allowed to rotate with $\Omega^* = 0.9$, figure 6(a) shows the rotor orbit quenchs gradually and in figure 7(a) it is remain constant, because the speed is below the SLS. In both cases the rotor orbit increases in amplitude monotonically for the rotor speed of $\Omega^* = 1.4$, because the speed is over SLS. So beyond this speed the system becomes unstable and does not reach any steady state.

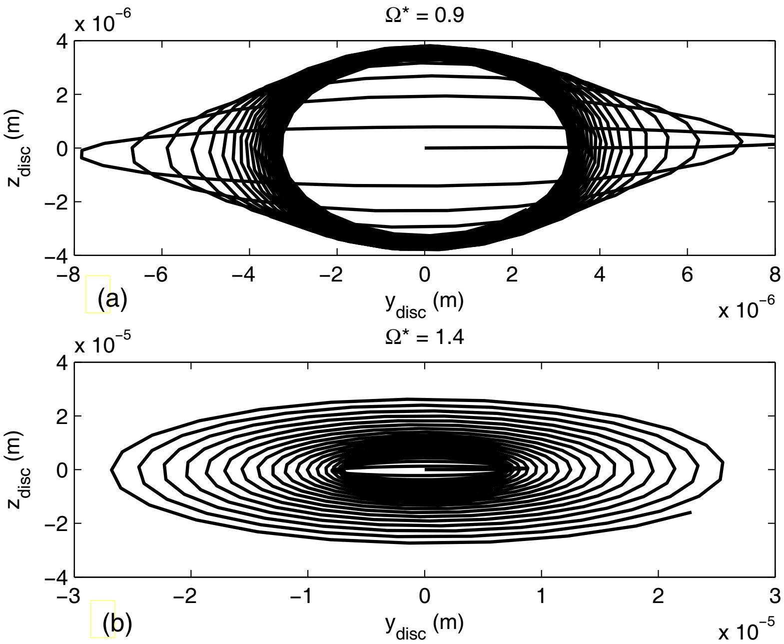

Figure 6 Rotor orbit at the disc due to transient vibration

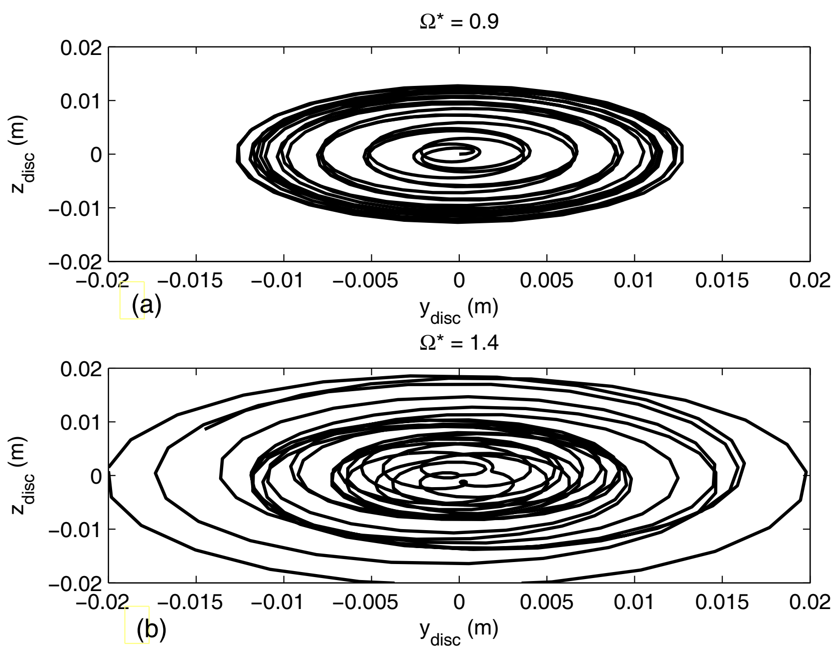

Figure 7 Rotor orbit at the disc due to steady state vibration

5. CONCLUSIONS

This paper has given the equations of motion of a rotor-shaft system having a viscoelastic rotor. The linear viscoelastic rotor-material behaviour is represented in the time domain where the instantaneous stress is obtained by operating the instantaneous strain. The mechanical analogy i.e rheological model is sometime difficult to represent for all viscoelastic materials. The operator may be suitably chosen according to the material model. The formulation has been found very useful to generate equations motion by discretizing the rotor continuum into finite beam elements and study the dynamic behaviour of rotor-shaft systems in terms of stability limit of the spin speed as well as unbalance response of the disc. Temporal variation of disc response has also been plotted as a further verification of stability of the rotor-shaft system. So this work is useful for dynamic analysis of viscoelastic rotors under any type of dynamic forcing function.

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