# Application of Multi-Objective Evolutionary Algorithms in Computational Finance

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***Abstract:***-*Application of Multiobjective evolutionary algorithms (MOEAs) in diversified domains has gained popularity in wide area ranging from engineering and computer science to ecology, sociology* and *medicine field. From these diversified application areas of evolutionary algorithms, computational finance constitutes a very promising field. The use of evolutionary algorithms for solving multi-objective optimization problem emerges as a potential field of research in recent years. This paper deals with application of MOEAs for different problems in computational finance. Different applications are explained briefly and exhaustive simulation study has been carried out for one particular application i.e. investment portfolio optimization. In this paper the Portfolio optimization is solved using two different multi-objective algorithms SPEA2 and NSGA-II. Their performances have been compared in terms of Pareto fronts, the delta, and C and S metrics.*

*Index Terms— Multi-objective optimization problem, Pareto-optimal solutions, Pareto Front, Global optimization, Non-dominated shorting.*

## I. INTRODUCTION

This paper presents a study two different MOEAs and its applications for solving problems in computational finance. One of the main motives of this survey is to attract the attention of EMOO researchers towards this field. Excellent survey has been done by Ma. Guadalupe Castillo Tapia and Carlos A. Coello Coello [2] in this regard. The paper is structured as follows. Section II outlines the basic concept of multi-objective optimization problem. Similarly section III includes many areas of applications of MOEAs in computational finance and their brief description. Section IV outlines two well know multi-objective algorithms i.e. SPEA2 and NSGA II and the investigation of these two MOEAs in Section V. Conclusions and further directions in which research work to be carried out are discussed in the Section VI.

## II. MULTIOBJECTIVE OPTIMIZATION

In a single-objective optimization problem, an optimal solution is the one which optimizes the objective with some associated constraints. It is not possible to find an unique solution for a multiobjective problem and due to the contradictory objectives a number of solutions are obtained. The general multi-objective minimization problem involves minimization of $n$ objective functions:

$$\left\{ f_1\left(\bar{x}\right), f_2\left(\bar{x}\right) \ldots\ldots\ldots, f_n\left(\bar{x}\right) \right\} \text{ Where } n \geq 2 \tag{1}$$

The solution to this problem is a different than the single-objective case and the idea of Pareto-dominance is used to explain it. Let an objective function $\bar{F}\left(\bar{x}\right)$, be represented as a combination of many objective functions.

$$\bar{F}\left(\bar{x}\right) = \left\{ f_1\left(\bar{x}\right), f_2\left(\bar{x}\right) \ldots\ldots\ldots, f_n\left(\bar{x}\right) \right\} \tag{2}$$

A point $\bar{x}_1$ ,with an objective function vector $\bar{F}_1$ , is said to dominate point $\bar{x}_2$ , with an objective function vector $\bar{F}_2$ , if no component of $\bar{F}_1$ is greater than its corresponding component in $\bar{F}_2$ , and at least one component is smaller. Similarly, $\bar{x}_1$ is said to be Pareto-equivalent to $\bar{x}_2$ if some components of $\bar{F}_1$ are greater than $\bar{F}_2$ and some are smaller. Pareto-equivalent points represent a trade-off between the given objective functions and it is difficult to infer that one point is better than another Pareto equivalent point without introducing preferences or relative weighting of the objectives. Hence the solution to a multi-objective optimization problem is a set of vectors which are not dominated by any other vector, and which are Pareto equivalent to each other. By grouping these Pareto optimal set generates a plot, often discontinuous known to be the Pareto front or Pareto border. Its name refers to Vilfredo Pareto [1], who generalized these concepts in 1896.

## III. APPLICATIONS of MOEA

The taxonomy of applications of MOEAs in economics and finance are:

A. Financial time series

B. Forecasting stock prices

C. Stock ranking

D. Risk-Return analysis

E. Economic modeling

F. Model discovery

G. Data mining

A. Investment portfolio optimization

I. Risk management

J. Coevolution

### *A.Finantial Time Series*

In this application, the idea is to find patterns in financial time series, such that predictions can be made regarding the behavior of a certain stock. Different MOEAs have also been reported in this application domain. Ruspini and Zwir [4] used the Niched-Pareto Genetic Algorithm (NPGA) [5] for this purpose. The authors apply their methodology for the identification of significant technical-analysis patterns in financial time series. Two objectives are considered i.e quality of fitness and its extent. Fit measures the extent to which the time-series values correspond to a financial uptrend, downtrend or head-and shoulders interval. Extent measures the length of the interval.

### *B.Forecasting stock prices*

Although long-term forecasting is not possible for the stock market, it is normally possible to perform short-term forecasting with heuristics. The use of genetic programming (GP) in this area has become increasingly popular, since GP can be used for symbolic regression, emulating the tasks traditionally performed by ANNs.

### *C.Stock Ranking*

The aim of this problem is to classify stocks as strong or weak performers based on technical indicators and then use this information to select stocks for investment and for making recommendations to customers. Many MOEAs has been reported in this application area. Mullei and Beling [6] use a GA with a linear combination of weights to select rules for a classifier system adopted to rank stocks based on profitability.

### *D.Risk-Return analysis*

It is slightly different from risk-return trade up which is made in investment portfolio. Credit portfolios handled by banks operate under different rules and therefore they are not modeled using the original Markowitz approach. Schlottmann and Seese [7] use an approach similar to the NSGA-II [8] for solving portfolio selection problems relevant to real-world banking. In the problem studied by the authors, a bank has a fixed supervisory capital budget. There is an upper limit for investments into a portfolio consisting of a subset of assets (e.g., loans to be given to different customers of the bank), each of which is subject to the risk of the default (capital risk). So, in this case, besides having an expected rate of return (as in the original Markowitz problem), each asset also has an expected default probability and a net exposure within a fixed risk horizon. The resulting problem has a discrete constrained search space with many local optima and two conflicting objective functions. Unlike the original NSGA-II, the authors adopt an external archive containing the nondominated solutions found along the search. For validating the approach, the authors adopt data from the Credit- Metrics Technical Document.

### *E.Economic Modelling*

Mardle [9] uses a GA with a weighted goal programming approach to optimize a fishery bioeconomic model. Bioeconomic models have been developed for a number of fisheries as a means of estimating the optimal level of exploitation of the resource and for assessing the effectiveness of the different management plans available.

### *F.Model discovery*

This is an interesting area in econometrics in which non-parametric models are assumed and one tries to use an evolutionary algorithm to derive a model for a certain type of problem (e.g., forecasting nonlinear time series). Normally, artificial neural networks (ANNs) have been used for the model itself, but several researchers have used evolutionary algorithms to find the most appropriate ANN that models the problem of interest.

### *G.Data mining*

The use of data mining techniques for learning complex patterns is a very promising research area in economics and finance. For example, the mining of financial time-series for finding patterns that can provide trading decision models is a very promising topic[10].

### *H.Investment Portfolio Optimization*

One of the most promising fields of application is investment portfolio optimization. It can vary from simple portfolios held by individuals to huge portfolios managed by professional investors. The portfolio contains stocks, bank investments, real estate holdings, bonds, treasury bills etc. The moto of it is to find an optimal set to invest on, as well as the optimal investment for each asset. This optimal selection and weighting is a multi-objective problem where total profit of investment has to be maximized and total risk is to be minimized. There are also different constraints, depending on the type of problem to be solved. For example, the weights normally have lower bounds, upper bounds and many other constraints. This is the so-called optimal investment portfolio that one wishes to obtain by using optimization techniques. This problem is traditionally studied using the Markowitz portfolio selection model [3].

### *I.Risk management*

The study of risk and the reaction of an agent is a very interesting research area. Some researchers have studied, the formation process of risk preferences in financial problems [10].

### *J.Coevolution*

The use of co-evolutionary approaches for certain problems in economics and finance (e.g. for studying artificial foreign exchange markets) is a very interesting topic that certainly deserves attention. Co-evolutionary MOEAs are still not too common, but their potential use in financial areas may boost the interest of researchers in paying more attention to them. Many other possible areas include, the study of consumers patterns, credit scoring, economic growth and auction games.

## IV. MULTIOBJECTIVE EVOLUTIONARY ALGORITHMS

This section deals with the basics of two standard multi-objective algorithms which are used in this paper. The steps involve two different multi-objective algorithms are outlined in sequel.

### *A. The SPEA2 Algorithm*

In SPEA 2 mating selection is used which is based on fitness measure and it uses binary tournament operator [12]. Here the archive update is performed according to the fitness values associated with each of the individual in the archive. All individuals that have fitness less than 1 fill the archive and if the archive size is less than pre-established size, the archive is completed with dominated individuals from current pool. If the archive size exceeds the pre-established size, some individuals are removed from archive using the truncation operator. This operator is based on the distance of an individual to its nearest neighbor.

Input: $N$ (population size), $\bar{N}$ (archive size), $T$ (maximum number of generations)

Output: $A$ (nondominated set)

1. Initialization: Generate an initial population $P_0$ and create the empty archive(external set) $\bar{P}_0 = \phi$ and set $t = 0$.

2. Fitness assignment: Calculate fitness values of individuals in $p_t$ and $\bar{p}_t$ .

3. Environmental selection: Copy all nondominated individuals in $P_t$ and $\bar{p}_t$ to $\bar{P}_{t+1}$ . If size of $\bar{P}_{t+1}$ exceeds $\bar{N}$ then reduce $\bar{P}_{t+1}$ by means of the truncation operator, otherwise if size of $\bar{P}_{t+1}$ is less than $N$ then fill $\bar{P}_{t+1}$ with dominated individuals in $P_t$ and $\bar{p}_t$ .

4. Mating selection: Perform binary tournament selection with replacement on $\bar{P}_{t+1}$ in order to fill the mating pool.

5. Termination: If $t \geq T$ or another stopping criterion is satisfied then set $A$ to the set of decision

vectors represented by the nondominated individuals in $\bar{P}_{t+1}$ .
Stop.

### *B.Nondominated Sorting Genetic Algorithm II*

Dev and Pratab [8] have proposed NSGA II where selection criteria is based on the crowding comparison operator. Here the pool of individuals is splited into different fronts and each front has been assigned a specific rank. All individuals from a front are ordered according to a crowding measure which is equal to the sum of distance to the two closest individuals along each objective. The environmental selection is processed based on these ranks. The archive will be formed by the non-dominated individuals from each front and it begins with the best ranking front.

***NSGA II Algorithm:***

1. Initialize population
2. Generate random parent population $p_0$ of size $N$
3. Evaluate objective Values
4. Assign fitness (or rank) equal to its non dominated level
5. Generate offspring Population $Q_0$ of size $N$ with binary tournament selection, recombination and mutation
6. For $t = 1$ to Number of Generations

6.1 Combine Parent and Offspring Populations

6.2 Assign Rank (level) based on Pareto Dominance.

6.3 Generate sets of non-dominated fronts

6.4 until the parent population is filled do

6.4.1 Determine Crowding distance between points on each front $F_i$

6.4.2 Include the $ith$ non dominated front in the next parent population $(P_{t+1})$

6.4.3 check the next front for inclusion

6.5 Sort the front in descending order using Crowded comparison operator

6.6 Choose the first $N$ - card $(P_{t+1})$ elements from front and include them in the next parent population $(P_{t+1})$

6.7Using binary tournament selection, recombination and mutation create next generation

7. Return to 6

## V. MOEAs APPLICATION TO INVESTMENT PORTFOLIO OPTIMIZATION

Portfolio $p$ consists of $N$ assets. Selection of optimal weighing of assets (with specific volumes for each asset given by weights $w_i$) is to be found. The unconstrained portfolio optimization problem is given as minimizing the variance of the portfolio and maximizing the return of the portfolio as defined in equation 3 and 4 respectively.

$$\rho_p{}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{3}$$

$$\alpha_p = \sum_{i}^{N} w_i \mu_i \tag{4}$$

$$\sum_{i}^{N} w_i = 1 \tag{5}$$

$$0 \le w_i \le 1; \quad \text{and } i = 1,2\ldots,N \tag{6}$$

Where $N$ is the number of assets available, $\mu_i$ the expected return of asset $i$, $\sigma_{ij}$ the covariance between asset $i$ and $j$, and finally $w_i$ are the decision variables gives the composition of the portfolio. $\rho_p$ being the standard deviation of portfolio and $\alpha_p$ being the expected return of portfolio. In this paper we consider a multi-objective portfolio asset selection and optimal weighting of assets where the total profit is maximized while total risk is minimized simultaneously. The present study employs NSGA II and micro-GA for modeling the Pareto front and for optimizing the portfolio performance. The results obtained with these two algorithms are finally compared by performing different numerical experiments.

### *A. Simulation studies*

In this section we present the simulation results obtained while searching the general efficient frontier that resolves the problem formulated in equation 1 and 2. The efficient frontier is computed using SPEA2 and NSGA II. All the computational experiments have been computed with a set of benchmark data available online and obtained from OR-Library being maintained by Prof. Beasley. Five data sets port1 to port5 represent the portfolio problem. Each data set corresponds to a different stock market of the world. The test data comprises of weekly prices from March 1992 to September 1997 from the following indices: Hang Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA and Nikkei in Japan. For each set of test data, the numbers of different assets are 31,85,89,98 and 225. In the paper we have used the first data set which corresponds to Hang Seng stock having 31 assets. The data can be found at http://people.brunel.ac.uk/~mastjjb/jeb/orlib/portinfo.html. The SPEA2 has population size of 100, number of generation 100, crossover rate 0.8 and mutation rate 0.05. The gene length is equal to number of assets. The MOPSO used a population of 100 particles and a repository size of 100 particles. The mutation rate set to 0.5 and 30 divisions for the adaptive grid. The NSGA II has population size of 100, number of generations 100, crossover rate 0.8 and mutation rate 0.05. The number of real-coded variables is equal to number of assets and the selection strategy used is tournament selection.

***B. Performance Measures for Comparison***

***1. S metric and $\Delta$ metric***

The S metric proposed in [11] indicates the extent of objective space dominated by a given nondominated set A. If the S metric of a non dominated front $f_1$ is less than another front $f_2$ then $f_1$ is better than $f_2$. It has been proposed by Zitzler [11]. $\Delta$ is called as spacing metric measures how evenly the points in the approximation set are distributed in the objective space. This formulation introduced by K. Deb [8] is given by

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1} \left| d_i - \bar{d} \right|}{d_f + d_l + (N-1)\bar{d}} \tag{7}$$

Where $d_i$ be the Euclidean distance between consecutive solutions in the obtained nondominated set of solutions. $\bar{d}$ is the average of these distances. $d_f$ and $d_l$ are the Euclidean distance between the extreme solutions and the boundary solutions of the obtained non dominated set and $N$ is the number of solutions from nondominated set. The low value for $\Delta$ indicate a better diversity and hence better is the algorithm.

***2. C metric***

Two sets of non dominated solutions are compared using C metric. The definition of C metric given in [11] for convergence of two sets A and B is given by:

$$C(A,B) = \frac{\left|\{b \in B \mid \exists a \in A : a \succ b\}\right|}{|B|} \tag{8}$$

***C. Simulation results and Pareto fronts of the two MOEA algorithms***

TABLE I

THE RESULTS OBTAINED FOR S AND Δ METRICS

| Algorithm | SPEA 2 | NSGA II |
|---|---|---|
| Metric S | 0.0000067874 | 0.000304616 |
| Metric Δ | 0.8337976192 | 0.5967844252 |

Table I shows the S and $\Delta$ metrics obtained using two algorithms. It may be observed from the Table I that NSGA II performs better as its S and $\Delta$ metric values are less than those obtained by Micro-GA algorithms.

TABLE II

THE RESULTS OBTAINED FOR C METRIC

| | NSGA II | SPEA2 |
|---|---|---|
| NSGA II | — | o.9405 |
| SPEA2 | 0.4543 | — |

Table II demonstrates the results of C metric. A magnitude of 0.9405 on the first line, first column signifies that almost most all of the solutions from final populations obtained by NSGA II dominate the solutions obtained by SPEA2. The standard efficient frontier corresponding to Hang Seng benchmark problem and the unconstraint efficient front (UEF) generated by two algorithms are depicted as:

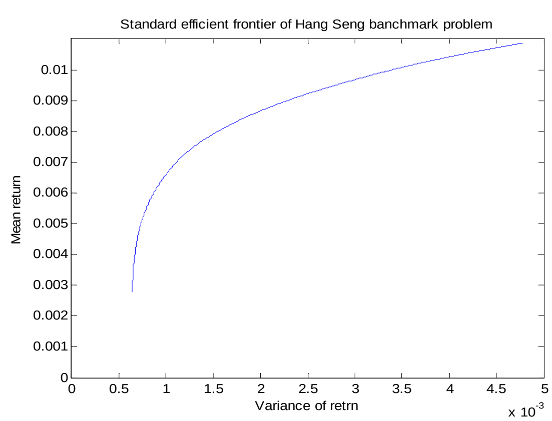

Fig.1.a. Plots of UEF for Hang Sang benchmark problem

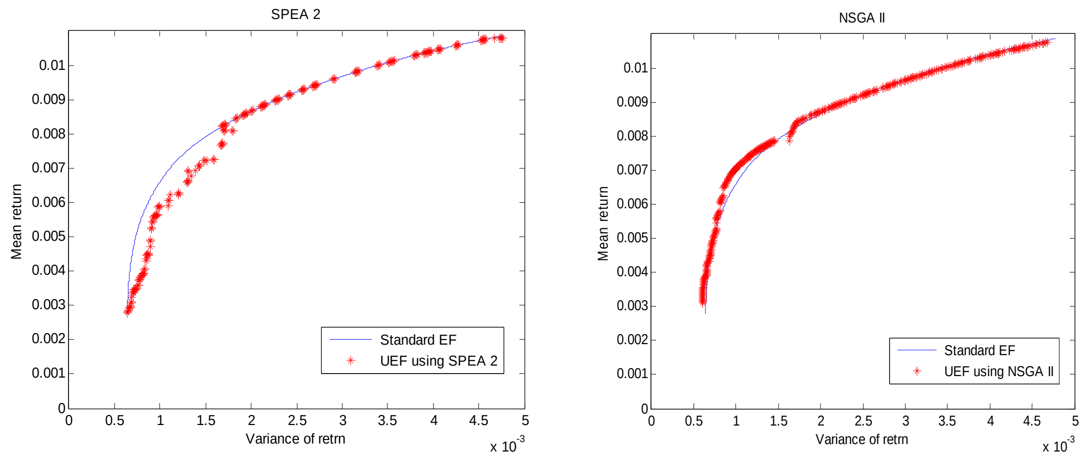


Fig.1.(b) -(c) Unconstraint Efficient Front of two algorithm in Comparison with Pareto efficient front.

## VI.CONCLUSION

This paper has presented a state-of-the-art study on the use of MOEAs for solving problems in economics and finance. We have identified a taxonomy of applications that consists of ten large groups. From these ten groups, the first (investment portfolio optimization) is most popular and here we have shown two multiobjective evolutionary algorithms NSGA II and Micri-GA for solving the bi-objective portfolio optimization problem. The paper makes a comparative study between these two algorithms. Experimental results reveal that the NSGA II algorithm outperforms Micro-GA algorithms in different experiments so conducted. Future work include introduction of different operators for local search in the existing models which allow better exploration and exploitation of the search space when applied to portfolio optimization problem. We expect, this paper can motivate researchers interested in economics and finance to learn more about MOEAs and to apply them in more problems within these areas.

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