Principal Component Analysis in Grey Based Taguchi Method for Optimization of Multiple Surface Quality Characteristics of 6061-T4 Aluminum in CNC End Milling

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Abstract. The present study highlights a multi-objective optimization problem by applying Principal Component Analysis (PCA) coupled with grey based Taguchi method through a case study in CNC end milling of 6061-T4 Aluminum. The study aimed at evaluating the best process environment which could simultaneously satisfy multiple requirements of surface quality. In view of the fact, that traditional Taguchi method cannot solve a multi-objective optimization problem; to overcome this limitation, grey relation theory has been coupled with Taguchi method. Furthermore, to follow the basic assumption of Taguchi method i.e. quality attributes should be uncorrelated or independent; which is not always satisfied in practical situation. To overcome this shortcoming the study applied Principal Component analysis to eliminate response correlation and to evaluate independent or uncorrelated quality indices called Principal Components which were aggregated to compute an overall quality index denoted as overall grey relational grade which was optimized (minimized) finally. The study combined PCA and grey based Taguchi method for predicting optimal setting. Optimal result was verified through confirmatory test.

Keywords: Principal Component Analysis (PCA); grey Taguchi method; CNC end milling
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INTRODUCTION AND PRIOR STATE OF ART

In recent times, computer numerically controlled (CNC) machine tools have been implemented to realize full automation in machining. CNC machine tools provide greater improvements in productivity, and increase the quality of the machined parts and require less operator input. Milling is a common metal removal operation in industry because of its ability to remove material faster with a reasonably good surface quality. It is widely used in a variety of manufacturing industries including aerospace and automotive sectors, where quality is an important factor in the production of slots, pockets, precision moulds and dies.

Literature [Ghani *et al.* (2004), Yang and Chen (2001), Chang and Lu (2007)) depicts that work have been done for optimizing the process parameters and improving the performance measures of CNC end milling process. However, all these studies whether experimental or analytical mostly concentrate on the centre line average roughness R_a value for surface quality. But surface generated by machining is composed of a large number of length scales of superimposed roughness, [Sahoo, (2005)] that are generally characterized by three different types of parameters, viz., amplitude parameters, spacing parameters and hybrid parameters. Thus consideration of centre line average roughness alone is not sufficient to describe surface quality. The other roughness parameters like root mean square roughness (R_q), kurtosis (R_{ku}) and mean line peak spacing (R_{sm}) need to be addressed.

Optimization of various production processes highlighted in literature assumed that individual quality indices are independent to each other i.e. they are not correlated. But in practice the assumption may not be valid always. Therefore, hybrid Taguchi based optimization approaches like grey based Taguchi [Datta el al. (2008)], desirability function based Taguchi [Datta el al. (2006)], utility concept [Walia et al. (2006)] based Taguchi methods those do not account response correlation may lead to erroneous results.

To overcome this limitation the study proposes application of Principal Component Analysis (PCA) to eliminate response correlation and to convert correlated responses into uncorrelated quality indices called principal components. These principal components have been aggregated further to calculate the Multi-response performance index (MPI) called overall grey relational grade [Datta et al. (2010)]. This serves as the single objective function for optimization with the aim to maximize it. Thus, the multi-objective optimization problem has been converted into an equivalent single objective optimization situation which has been solved by Taguchi method by analyzing dispersion statistics of the response data.

EXPERIMENTATION

In the present study; depth of cut, spindle speed and feed rate have been considered as machining parameters while the following four roughness parameters have been selected as the response variables: centre line average roughness (R_a); root mean square roughness (R_q); and mean line peak spacing (R_{sm}). The work piece material was 6061-T4 aluminum. Commercially available CVD coated carbide tools have been used in this investigation. Apart from depth of cut (d, mm), spindle speed (N, rpm) and feed rate (f, mm/min) have been selected as design factors while other parameters have been assumed to be constant over the experimental domain (Table 1). L_{25} Orthogonal Array (OA) has been considered for experimentation. Interaction effect of process parameters has been assumed negligible. The machine used for the milling tests is a 'DYNA V4.5' CNC end milling machine having the control system SINUMERIK 802

D with a vertical milling head. The surface roughness parameters have been measured using the stylus-type profilometer, Talysurf (Taylor Hobson, Surtronic 3+). The measured roughness parameters along with design matrix have been shown in Table 2.

Table 1. Process parameters and domain of experiments

Levels	Aluminum		
	d (mm)	N (rpm)	f (mm/min)
-1	0.10	4500	900
-0.5	0.15	4750	950
0	0.20	5000	1000
+0.5	0.25	5250	1050
+1	0.30	5500	1100

Table 2. Experimental results along with design matrix

Sl. No.	L_{25} Orthogonal Array			Response parameters		
	d	N	f	R_a μm	R_q μm	R_{sm} mm
1	-1.0	-1.0	-1.0	0.611	0.727	0.117
2	-1.0	-0.5	-0.5	0.634	0.769	0.159
3	-1.0	0.0	0.0	0.853	0.987	0.150
4	-1.0	0.5	0.5	0.656	0.807	0.146
5	-1.0	1.0	1.0	0.713	0.847	0.124
6	-0.5	-1.0	-0.5	0.668	0.872	0.111
7	-0.5	-0.5	0.0	0.580	0.700	0.146
8	-0.5	0.0	0.5	0.548	0.673	0.135
9	-0.5	0.5	1.0	0.754	0.910	0.131
10	-0.5	1.0	-1.0	0.514	0.633	0.105
11	0.0	-1.0	0.0	0.678	0.898	0.097
12	0.0	-0.5	0.5	0.678	0.812	0.095
13	0.0	0.0	1.0	0.512	0.641	0.157
14	0.0	0.5	-1.0	0.296	0.379	0.160
15	0.0	1.0	-0.5	0.597	0.745	0.102
16	0.5	-1.0	0.5	0.562	0.719	0.159
17	0.5	-0.5	1.0	0.743	0.888	0.121
18	0.5	0.0	-1.0	0.536	0.664	0.115
19	0.5	0.5	-0.5	0.552	0.676	0.125
20	0.5	1.0	0.0	0.589	0.726	0.127
21	1.0	-1.0	1.0	0.546	0.659	0.091
22	1.0	-0.5	-1.0	0.569	0.708	0.130
23	1.0	0.0	-0.5	0.531	0.639	0.118
24	1.0	0.5	0.0	0.624	0.752	0.126
25	1.0	1.0	0.5	0.669	0.853	0.153

EVALUATION OF OPTIMAL RESULT

Detailed methodology for optimization has been described in the paper by Datta et al. (2010). In this investigation, experimental data have been normalized using (Lower-the-Better) LB criteria. A check has been made to verify whether the responses are correlated or not. It has been observed that, all responses are correlated to each other (Table 3). In order to eliminate response correlations, Principal Component Analysis (PCA) has been applied to derive three independent quality indexes called principal components Z1, Z2 and Z3 (Table 4). The analysis of correlation matrix has been made to calculate the values of three independent principal components in all experimental runs (Table 5). Quality loss of three individual components (compared to the ideal) has been converted into individual grey relational coefficients; which have been aggregated further to compute the overall grey relational grade (Table 6). Thus, the multi-criteria optimization problem has been transformed into a single objective optimization problem using the combination of Taguchi approach and grey relational analyses. Higher is the value of grey relational grade, the corresponding factor combination is said to be close to the optimal. The S/N ratio plot for the overall grey relational grade is represented graphically in Figure 1. The S/N ratio for overall grey relational grade has been calculated using HB (Higher-the-Better) criterion. With the help of the Figure 1, optimal parametric combination has been determined. The optimal factor setting becomes d(0) N(-1) f(-1). [Number indicates level of factors] which been verified by the satisfactory result of confirmatory experiment.

CONCLUSIONS

1. Application of PCA has been recommended to eliminate response correlation by converting correlated responses into uncorrelated quality indices called principal components which have been as treated as independent response variables for optimization.
2. Grey relation theory can combine individual principal components into a single multi-response performance index MPI (overall grey relational grade) to be taken under consideration for optimization. This is really helpful in situations where large number of responses have to be optimized simultaneously.
3. The said approach can be recommended for continuous quality improvement and off-line quality control of a process/product.

Table 3. Check for correlation

Sl. No.	Correlation between	Pearson's correlation coefficient	Comment
1	R_a and R_q	0.996	Both are correlated
3	R_a and R_{sm}	0.114	Both are correlated
5	R_q and R_{sm}	0.112	Both are correlated

Table 4. (Analysis of correlation matrix)
Eigenvalues, eigenvectors, accountability proportion (AP) and cumulative accountability proportion (CAP) computed for the three individual principal components

	Z1	**Z2**	**Z3**
Eigenvalues	2.0209	0.9749	0.0042
Eigenvector	$\begin{vmatrix} -0.669 \\ -0.669 \\ -0.155 \end{vmatrix}$	$\begin{vmatrix} -0.109 \\ -0.110 \\ +0.988 \end{vmatrix}$	$\begin{vmatrix} +0.707 \\ -0.707 \\ -0.001 \end{vmatrix}$
AP	0.674	0.325	0.001
CAP	0.674	0.999	1.000

Table 5. Individual Principal Components

Sl. No.	**Individual Principal Components**		
	Z1	**Z2**	**Z3**
Ideal Situation	**-1.5530**	**0.7690**	**-0.0010**
1	-0.8236	0.6583	-0.0268
2	-0.7595	0.4603	-0.0189
3	-0.6050	0.5194	-0.0268
4	-0.7403	0.5150	-0.0136
5	-0.7167	0.6306	-0.0236
6	-0.7406	0.7139	0.0052
7	-0.8317	0.5006	-0.0226
8	-0.8757	0.5452	-0.0170
9	-0.6732	0.5978	-0.0176
10	-0.9554	0.7277	-0.0170
11	-0.7456	0.8328	0.0094
12	-0.7799	0.8475	-0.0222
13	-0.9072	0.4446	-0.0099
14	-1.4862	0.3430	-0.0006
15	-0.8404	0.7715	-0.0100
16	-0.8253	0.4500	-0.0009
17	-0.6934	0.6527	-0.0208
18	-0.9076	0.6588	-0.0139
19	-0.8796	0.5991	-0.0180
20	-0.8272	0.5957	-0.0145
21	-0.9359	0.8657	-0.0243
22	-0.8463	0.5760	-0.0114
23	-0.9237	0.6359	-0.0260
24	-0.7988	0.6064	-0.0216
25	-0.7121	0.4906	-0.0019

Table 6. Calculation of individual grey relational coefficients

Sl. No.	$\Delta_{0i}(1st\ PC)$	$\Delta_{0i}(2nd\ PC)$	$\Delta_{0i}(3rd\ PC)$	$\Gamma_{0,i}$
1	0.4494	0.6657	0.3359	0.4837
2	0.4267	0.4131	0.4221	0.4206
3	0.3803	0.4658	0.3359	0.3940
4	0.4203	0.4615	0.5098	0.4639
5	0.4127	0.6133	0.3662	0.4641
6	0.4204	0.8038	0.6806	0.6349
7	0.4524	0.4477	0.3768	0.4256
8	0.4697	0.4934	0.4498	0.4710
9	0.3995	0.5609	0.4407	0.4670
10	0.5047	0.8474	0.4498	0.6006
11	0.4220	0.7785	0.5579	0.5862
12	0.4336	0.7393	0.3812	0.5181
13	0.4829	0.4010	0.5963	0.4934
14	1.0000	0.3372	0.9774	0.7716
15	0.4558	1.0000	0.5936	0.6831
16	0.4500	0.4051	1.0000	0.6184
17	0.4055	0.6544	0.3976	0.4858
18	0.4831	0.6668	0.5039	0.5513
19	0.4713	0.5628	0.4348	0.4896
20	0.4507	0.5579	0.4924	0.5003
21	0.4956	0.6958	0.3591	0.5169
22	0.4580	0.5308	0.5579	0.5156

Table 6 (continued)

Sl. No.	$\Delta_{0i}(1st\ PC)$	$\Delta_{0i}(2nd\ PC)$	$\Delta_{0i}(3rd\ PC)$	$\Gamma_{0,i}$
23	0.4902	0.6227	0.3430	0.4853
24	0.4403	0.5737	0.3881	0.4674
25	0.4113	0.4385	0.9420	0.5973

Δ_{0i} = Quality of principal component (w. r. t the ideal) in ith experimental run

$\Gamma_{0,i}$ =Overall grey relational grade in ith experimental run

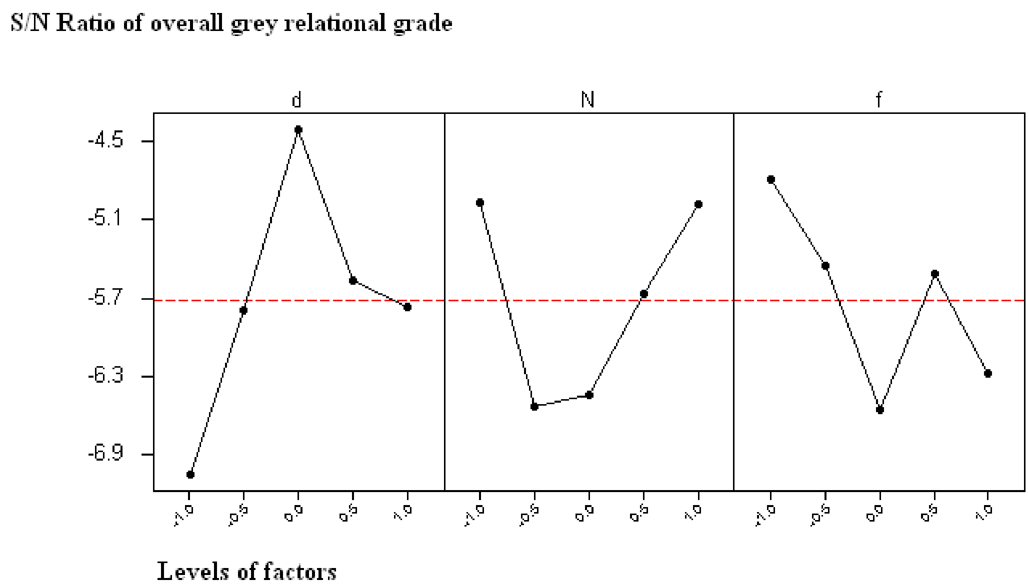

FIGURE 1. Evaluation of optimal setting

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