EXTENSION OF PARIS EQUATION & ITS APPLICATION IN THE STUDY OF FATIGUE PROCESS

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Abstract

In 1964 Prof P. C. Paris developed an equation to estimate the fatigue crack-growth rate. It is generally believed that actual growth rate of fatigue crack deviates from Paris Law when crack propagation becomes very fast during the later part of the fatigue Life. But actually it is due to some geometric factors. Experiments on un-notched cylindrical specimens of mild steel (0.23%C) has shown that if the shift in the neutral axis during fatigue crack growth is taken into account and stress is calculated on the basis of this then the corrected form of Paris Equation can be used till the fracture of the specimen. Actually it has been found the fatigue life predicted by theoretical Calculation based on this equation is very close to the experimental values. In this experiment this has been established for specimens tested at five different stress levels.

Keywords: Paris equation, Geometric factors, Stress intensity factor, Crack growth, Fatigue life

1. INTRODUCTION

If a be the crack length and N be the number of cycles applied, then according to P.C. Paris [1] crack growth rate per cycle (da/dn) = $C (\Delta K)^{m}$
…(1)

Where C & m are two constants and $\Delta K = K_{max} - K_{min}$ where K_{max} & K_{min} are the maximum and minimum values of K, the stress intensity factor. So a plot of log (da/dn) vs. log (Δ K) should result in a straight line. But as shown in Fig. (1), the straight line relationship is believed to be valid only over an intermediate zone of fatigue crack growth and can be considered neither at the initial nor at the final stage of crack growth. The initial part is, however, mainly concerned with fatigue crack initiation rather than its growth.

Actually the number of cycles spent in this region is very small. This part is useful when nucleation of fatigue crack is to be prevented. It is evident from Fig. (1) that apart from giving a measure of the threshold value of Δ K (below which crack growth is not possible), this zone consists only of the portion showing near threshold growth rate which becomes significant only at high temperature and low frequency and that too for a few materials (like Cr-Mo-V steel) [2,3].

However the importance of the final zone of Fig. (1) cannot be neglected as far as fatigue crack growth is concerned. But there is a bit of confusion regarding the appearance of this stage [3,4]. In actual practice it has been found that the value of Δ K at which this deviation appears differs for specimens of same composition but different dimensions. From this it may be assumed that the appearance of the third and final stage of crack growth is due to the presence of some geometric factors and is not related to material property. It then seems reasonable to see whether by correcting some geometric factors the Paris equation can be used upto the fracture. Since the value of stress intensity factor (K) is a function of stress, it is necessary to correct the geometric factors during the calculation of stress.

2. MATHEMATICAL CALCULATIONS

2.1 Correction of Geometric Factors:

Let us consider a flat specimen of width 'b' and thickness 't'. Let 'P' be the load applied at the neutral axis. Let a be the length of crack at any instance. Now generally the stress at this stage is taken to be P / t (b-a). But this is the nominal stress and not the real stress at the crack tip. As shown in Fig. (2) the neutral axis will be shifted by a distance a/2 due to a growth or crack having a length 'a'.

Now this will generate a bending moment (M) = p.a/2. We know that M/I = σ/Y where Y is the distance from neutral axis and I is the moment of inertia.

Now, I = $\frac{t(b-a)^3}{12}$ and y = (b-a) / 2 as obtained from Fig. (2)

$$\therefore \sigma = \frac{\frac{Pa}{2}\,\frac{(b-a)}{2}}{\frac{t(b-a)^3}{12}} = \frac{3Pa}{t(b-a)^2}$$

So the total stress at the crack tip = $\frac{P}{t(b-a)} + \frac{3Pa}{t(b-a)^2}$

$$= \frac{P}{t(b-a)} \left[1 + \frac{3a}{b-a} \right] \qquad \ldots(2)$$

The factor (3a/b-a) can be neglected when the crack length is very small in comparison with the width of the specimen. But during latter stages when the rate of crack growth becomes significant, this factor can not be ignored. In fact the deviation from the straight line relationship (for log da/dn vs. log Δ K plot) takes place for not incorporating this factor in stress calculation.

Now the above calculations are used for axial fatigue. It is to be established whether Paris equation can be used upto fracture in case of cylindrical specimen in reverse bending. As shown in Fig. (3) a shift will occur in the position of the neutral axis due to crack growth in this case also.

Let us consider a strip of length 21 and thickness dx at a distance x from the centre. If 2 α be the angle subtended by the strip at the centre then Cos α = x/R & Sin α = l/R where R is the radius. Now if I be the moment of inertia of the portion of circle remaining after the growth of the crack of length 'a' (about the neutral axis) and the shift in the neutral axis be d, then

$$I = 2\int_{-R}^{R-a} (X+d)^2 \quad l\,dx$$

By actual Calculation it can be shown that

$$d = \frac{R(3Sin\propto_1 - Sin3\propto_1)}{3(Sin2\propto_1 - 2\propto_1 + 2\pi)} \quad \text{where Cos } \propto_1 = \frac{R-a}{R}$$

$$\& \qquad I = \frac{R^4}{16}(Sin4\propto_1 - 4\propto_1 + 4\pi) + \frac{dR^3}{3}(Sin3\alpha_1 - 3Sin\propto_1) + \frac{d^2R^2}{2}(Sin2\propto_1 - 2\propto_1 + 2\pi)$$

the value of y after the growth of crack as obtained from Fig. (3) = d + R – a. Initially when the crack length is negligible

$$I = \pi R^4/4 \text{ and } y = R$$

Once the value of M (bending moment) is known, σ can be obtained at any instance by using the values of I and d.

Since K is a function of σ, the crack growth rate can also be calculated from σ. Now here K is taken to be = $\sigma\sqrt{a}$ (a is the crack length) instead of the most commonly used relationship $K = \sigma\sqrt{\pi a}$. Since π is a constant this factor can be taken into account by adjusting the value of Paris equation constant C.

Now for completely reversed cycle as used in the present case:

$K_{max} = -K_{min} = K$ (say)

$\Delta K = K_{max} - K_{min} = 2K$

So we get $da/dN = C(\Delta K)^m$

$= C(2\sigma\sqrt{a})^m$ …(3)

But here we have used mild steel specimen which has a well defined endurance limit below which fatigue crack does not propagate. So we should incorporate this factor in the Paris equation. Then the corrected form of the equation becomes:

$Da/dn = C\ [2(\sigma - \sigma i) \sqrt{a}]^m$ … (4)

where σi is the endurance limit.

It is obvious that to determine the crack growth rate the values of c, m & σi are to be determined. Actually this was done by computer analysis.

2.2 Theoretical Estimation of Fatigue Life at Different Stress Levels

To calculate the no. of cycles required to cause failure, the stress at any instance is to be calculated from the bending moment and crack length. Then it should be checked whether the stress is greater than the U.T.S. of the material concerned. If the former is less than the latter the value of the crack length is to be compared with the diameter of the specimen. Initial crack size of the specimen was taken to be 1 μm for the un-notched samples used for experiment. This assumption was in accordance with the prediction of Prof M.E. Fine [5]. The value of K is to be calculated provided the diameter is greater than the crack length. The next step is to calculate the crack growth rate (da/dn). The increase in crack length (da) for a particular value of dn (increase in the number of cycles applied) is to be estimated.The value of dn is to be given in accordance with the crack growth rate; initially when crack growth rate is very slow the readings can be taken at long intervals but during the latter stage the value of dn has to be reduced since crack length increases by leaps and bounds even for a small increase in the no. of cycles. The initial value of N must be taken as = O. The value of da is then to be added to the initial value of crack length (a) and the stress is to be recalculated (this time for the new value of 'a'). The process continues till the stress becomes greater than U.T.S. or the crack length exceeds the diameter of the specimen. The results are to be taken when either of these two phenomena takes place.

Now in this process we will get different values of service life for different values of C, m and σi. So first it is necessary to compute the best fit values of these three factors. Another computer programme was developed for this purpose. In that programme the no. of cycles required to cause failure at different stress levels obtained experimentally (the samples were cyclically stressed with the help of Moore's Machine, rotating bending type) should be incorporated and the square of the difference between the experimental and theoretical values of the number of cycles required to cause failure at different stress levels is to be determined. The method of estimation of theoretical values is similar to that described earlier. Provisions are to be made for calculation of the number of cycles to cause failure at different stress levels for different values of C. m and σi. It has been found through actual calculation that for a particular set of values of these three factors, the number of cycles required to cause failure becomes very close to the values obtained experimentally for all the stress levels at which the experiments were performed. So we can accept those values as the best fit values for the material concerned (i.e. mild steel, 0.23% C).

3. EXPERIMENTAL DETAILS

Un-notched Cylindrical specimens of mild steel (the Composition and mechanical properties are shown in Table 1) were used for the experiment. The specimens were annealed, electropolished and then subjected to fatigue loading (in bending) at five different stress levels (70%, 75%, 80%, 85% & 90% of the Yield Strength). The fatigue life determined experimentally was compared with the values obtained by theoretical Calculation.

4. RESULTS, DISCUSSION & CONCLUSION

The best fit values of C, m and σi for mild steel (0.23% C) has been found to be 10^{-29}, 4.25 & 171MPa respectively. The value of the endurance limit (σi) has been found to be 60% of the yield stress.

The number of cycles required to cause failure obtained experimentally for different stress level and those (on the basis of the above mentioned values of C, m and σi) with the help of Paris equation are shown in Table 2.

The results show that the values calculated on the basis of the corrected form of the Paris equation are very close to experimental values for all the applied stress levels. So we can conclude that the Paris equation of crack growth rate can be used till fracture if some geometric factors are corrected. In this regard another information may be given here. With the same set of values of C, m & σi the number of cycles required to cause failure at 90% Y.S. was calculated for axial fatigue of the same material (the sample dimensions were assumed arbitrarily). Obviously the stress was calculated on the basis of eq: (2). The no of cycles to cause failure has been found to be 8.56×10^4 cycles which is very very close to the value obtained for bending fatigue (8.63×10^4). This further establishes the authenticity of the corrected form of the equation.

References

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Table 1 (Material Data)

Composition	Yield Stress (MP_a)	U. T. S. (MP_a)
%C = 0.23		
%Si = 0.05	285	475.5
%Mn = 0.15		

Table 2 (Fatigue Life)

Applied Stress Level (MP_a)	No. of cycles to cause failure Experimental	Theoretical
256.5 (90% Y.S.)	9×10^4	8.63×10^4
242.25 (85% Y.S.)	2×10^5	1.87×10^5
228.0 (80% Y.S.)	4.3×10^5	4.88×10^5
213.75 (75% Y.S.)	2.05×10^6	1.66×10^6
199.5 (70% Y.S.)	8.8×10^6	9.38×10^6