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Application of Neural network for fault diagnosis of cracked cantilever beam

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***Abstract*— This paper discusses neural network technique for fault diagnosis of a cracked cantilever beam. In the neural network system there are six input parameters and two output parameters. The input parameters to the neural network are relative deviation of first three natural frequencies and first three mode shapes. The output parameters of the neural network system are relative crack depth and relative crack location. To calculate the effect of crack depths and crack locations on natural frequencies and mode shapes, theoretical expressions have been developed. Strain energy release rate at the crack section of the beam has been used for calculating the local stiff nesses of the beam. The local stiff nesses are dependent on the crack depth. Different boundary conditions are outlined which take into account the crack location. Several training patterns are derived and the Neural Network has been designed accordingly. Experimental setup has been developed for verifying the robustness of the developed neural network. The developed neural network system can predict the location and depth of the crack in a close proximity to the real results.**

Keywords*— *crack, neural network, beam, vibration, strain energy, stiffness, stress intensity factor, natural frequency, mode shape

I. INTRODUCTION

Dynamics of structures with crack has been studied for last four decades intensively. Natural frequencies and modes shapes undergo variation due to presence of crack. The deviations of natural frequencies and modes shapes mainly dependant on location and intensity of the crack. Scientists are focusing their thoughts to find out the damage location and its intensity. The investigations reported in this regards are jotted below. Measurement of flexural vibrations of a rectangular cross-section cantilever beam having a transverse surface crack extending uniformly along the width of the beam and analytical results are used to relate the measured vibration modes to the crack location and depth [1]. From the measured amplitudes at two points of the structure vibrating at one of its natural modes, the respective vibration frequency and an analytical solution of the dynamic response, the crack location can be found and depth can be estimated with satisfactory accuracy. The method of crack localization and sizing in a beam can be obtained from free and forced response measurements [2]. This method has been illustrated through numerical examples. An iterative algorithm is used to identify the locations and extent of damage in beams using only the changes in their first several natural frequencies [3]. A method using singular value decomposition is developed to handle the ill-conditioned system equations that occur in the experimental investigation by using the measured natural frequencies of the modified structure. An extensive study has been done on diagnosis of fracture damage in structure [4]. The concept of 'fracture hinge' is developed analytically and the same is applied to a cracked section for detecting fracture damage in simple structures. It is experimentally verified that the structural effect of a cracked section can be represented by an equivalent spring loaded hinge.

A nondestructive evaluation procedure is presented for identifying a crack in a structure using modal test data. [5]. The structure is discretized into a set of elements and the crack is assumed to be located within one of the elements. Measured vibration frequencies and mode shapes are used in an identification process to identify the cracked element based on a simple reduced stiffness model. A simple model is proposed that describes the flexural vibration characteristics of a rotating cracked Timoshenko beam [6]. The cracked beam is modeled using two uniform segments connected by a mass less torsional spring at the crack location. The proposed method is verified by finite element analysis. An experimental investigation of the identification of crack location and size has been carried out [7]. By providing the first three natural frequencies through vibration measurements, curves of crack equivalent stiffness versus crack location are plotted, and the intersection of the three curves predicts the crack location and size. Curvature mode shape can be used as a possible candidate for identifying and locating damage in a structure [8]. By using a cantilever and a simply supported analytical beam model, it is shown that absolute changes in the curvature mode shapes are localized in the region of damage and hence can be used to detect damage in a structure.

Artificial neural networks (ANN) can be used as an alternative effective tool for solving the inverse problems because of the pattern-matching capability [9]. The results of ANN are quite encouraging and prove the robustness of the proposed damage assessment algorithm. An artificial neural network-based model has been developed for the fault detection of centrifugal pumping system [10]. The fault detection model is developed by using two different artificial neural network approaches, namely feed forward network with back propagation algorithm and binary adaptive resonance network (ART1). The performance of the developed back propagation and ART1 model is tested for a total of seven categories of faults in the centrifugal pumping system. The modal frequency parameters for the

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flexural vibration of a cantilever beam having a transverse surface crack are analytically computed for various crack locations and depths using a fracture mechanics based crack model [11]. The computed modal frequencies are used to train a neural network to identify both the crack location and depth. This modular neural network architecture can be used as a non-destructive procedure for health monitoring of structures. A damage detection algorithm has been developed using a combination of global (changes in natural frequencies) and local (curvature mode shapes) vibration-based analysis data as input in artificial neural networks (ANNs) for location and severity prediction of damage in beam-like structures [12]. The trained feed-forward back propagation ANNs using the data obtained from the experimental damage case for quantification and localization of the damage is tested.

A novel neural network based-approach is used for detecting structural damage [13]. The proposed approach involves two steps. The first step, system identification, uses Neural System Identification Networks (NSINs) to identify the undamaged and damaged states of a structural system. The second step, structural damage detection, uses the aforementioned trained NSINs to generate free vibration responses with the same initial condition or impulsive force. An experimentally verified crack damage detection algorithm has been proposed using a combination of global (changes in natural frequencies) and local (strain mode shapes) vibration-based analysis data as input in artificial neural networks (ANNs) for location and severity prediction of crack damage in beam-like structures[14]. The structural damage detection has been proposed using frequency response functions (FRFs) as input data to the back-propagation neural network (BPNN) [15]. The analysis results on a cantilevered beam show that, in damage cases the neural network can assess damage conditions with very good accuracy.

In this paper a new methodology has been proposed for prediction of crack intensity and its location using neural network technique. At the beginning theoretical expressions have been developed to find out the effect of crack depth and crack location on natural frequency and mode shapes of the beam structure. Then a set of training patterns are prepared. Those patterns are used to train the neural network for prediction of crack location and crack depth for different natural frequencies and deviation of mode shapes as input. The results are also compared with the experimental results to verify the authenticity of neural technique developed. The comparison shows a very good agreement. This method can be used as an effective tool for prediction of damage location and intensity.

II. THEORETICAL ANALYSIS

The presence of a transverse surface crack of depth 'a_1' and 'a_2' on beam of width 'B' and height 'W' introduces a local flexibility, which can be defined in matrix form, the dimension of which depends on the degrees of freedom. Here a 2x2 matrix is considered. A cantilever beam is subjected to axial force (P_1) and bending moment (P_2), shown in figure 1(a), which gives coupling with the longitudinal and transverse motion. The cross sectional view of the beam is shown in figure 1(b).

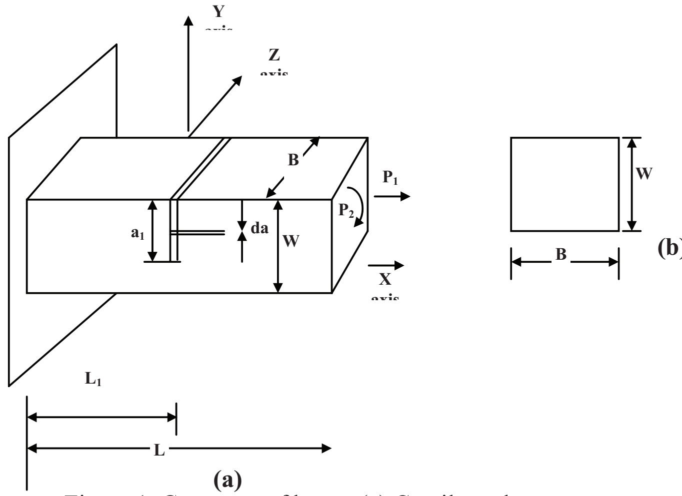

Figure 1. Geometry of beam, (a) Cantilever beam, (b) Cross-sectional view of the beam.

The strain energy release rate at the fractured section can be written as [15];

$$J = \frac{1}{E'}(K_{I1} + K_{I2})^2, \qquad (1)$$

Where $\frac{1}{E'} = \frac{1-v^2}{E'}$ (for plane strain condition);

$\frac{1}{E'} = \frac{1}{E}$ (for plane stress condition)

K_{I1}, K_{I2} are the stress intensity factors of mode I (opening of the crack) for load P_1 and P_2 respectively. The values of stress intensity factors from earlier studies [15] are;

$$K_{I1} = \frac{P_1}{BW}\sqrt{\pi a}\,(F_1(\frac{a}{W})), K_{I2} = \frac{6P_2}{BW^2}\sqrt{\pi a}\,(F_2(\frac{a}{W}))$$

Where expressions for F_1 and F_2 are as follows

$$F_1(\frac{a}{W}), = (\frac{2W}{\pi a}\tan(\frac{\pi a}{2W}))^{0.5} \left\{\frac{0.752 + 2.02(a/W) + 0.37(1-\sin(\pi a/2W))^3}{\cos(\pi a/2W)}\right\} \qquad (2)$$

$$F_2(\frac{a}{W}), = (\frac{2W}{\pi a}\tan(\frac{\pi a}{2W}))^{0.5} \left\{\frac{0.923 + 0.199(1-\sin(\pi a/2W))^4}{\cos(\pi a/2W)}\right\} \qquad (3)$$

Let U_t be the strain energy due to the crack. Then from Castigliano's theorem, the additional displacement along the force P_i is;

$$u_i = \frac{\partial U_t}{\partial P_i} \qquad (4)$$

The strain energy will have the form,

$$U_t = \int_0^{a_1} \frac{\partial U_t}{\partial a} da = \int_0^{a_1} J\, da \qquad (5)$$

Where $J=\frac{\partial U_t}{\partial a}$ the strain energy density function.

From equations (1) and (2), thus we have

$$u_i=\frac{\partial}{\partial P_i}\left[\int_0^{a_1} J(a)\,da\right] \qquad (6)$$

The flexibility influence co-efficient C_{ij} will be, by definition

$$C_{ij}=\frac{\partial u_i}{\partial P_j}=\frac{\partial^2}{\partial P_i \partial P_j}\int_0^{a_1} J(a)\,da \qquad (7)$$

and can be written as

$$C_{ij}=\frac{BW}{E'}\frac{\partial^2}{\partial P_i \partial P_j}\int_0^{\xi_1}(K_{11}+K_{12})^2\,d\xi \quad (8)$$

From the Equation (7), on calculation C_{11}, C_{12} (=C_{21}) and C_{22} we get

$$\overline{C_{11}}=C_{11}\frac{BE'}{2\pi}\;;\qquad \overline{C_{12}}=C_{12}\frac{E'BW}{12\pi}=\overline{C_{21}}\;;$$

$$\overline{C_{22}}=C_{22}\frac{E'BW^2}{72\pi}$$

The local stiffness matrix can be obtained by taking the inversion of compliance matrix. i.e.

$$K=\begin{bmatrix}K_{11} & K_{12}\\ K_{21} & K_{22}\end{bmatrix}=\begin{bmatrix}C_{11} & C_{12}\\ C_{21} & C_{22}\end{bmatrix}^{-1}$$

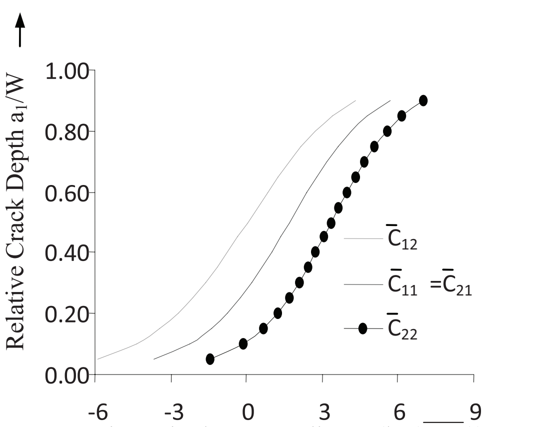

Figure 2: Relative crack depth (a_1/w) vs. dimensionless compliance (ln ($\overline{C_{xy}}$)).

Analysis of Vibration Characteristics of the Cracked Beam

A cantilever beam of length 'L' width 'B' and depth 'W', with a crack of depth 'a_1'and 'a_2' at a distance 'L_1' and 'L_2' respectively from the fixed end is considered (shown in figure 1). Taking $u_1(x, t)$, $u_2(x, t)$, $u_3(x, t)$ as the amplitudes of longitudinal vibration for the sections before, in-between and after the crack and $y_1(x, t)$, $y_2(x, t)$, $y_3(x, t)$ are the amplitudes of bending vibration for the same sections.

The normal function for the system can be defined as

$$\bar{u}_1(\bar{x})=A_1\cos(\bar{K}_u\bar{x})+A_2\sin(\bar{K}_u\bar{x}) \qquad (9)$$

$$\bar{u}_2(\bar{x})=A_3\cos(\bar{K}_u\bar{x})+A_4\sin(\bar{K}_u\bar{x}) \qquad (10)$$

$$\bar{y}_1(\bar{x})=A_5\cosh(\bar{K}_y\bar{x})+A_6\sinh(\bar{K}_y\bar{x})+A_7\cos(\bar{K}_y\bar{x})+A_8\sin(\bar{K}_y\bar{x}) \qquad (11)$$

$$\bar{y}_2(\bar{x})=A_9\cosh(\bar{K}_y\bar{x})+A_{10}\sinh(\bar{K}_y\bar{x})+A_{11}\cos(\bar{K}_y\bar{x})+A_{12}\sin(\bar{K}_y\bar{x}) \qquad (12)$$

Where $\bar{x}=\frac{x}{L}$, $\bar{u}=\frac{u}{L}$, $\bar{y}=\frac{y}{L}$, $\beta=\frac{L_1}{L}$

$$\bar{K}_u=\frac{\omega L}{C_u},\; C_u=\left(\frac{E}{\rho}\right)^{1/2},\; \bar{K}_y=\left(\frac{\omega L^2}{C_y}\right)^{1/2},\; C_y=\left(\frac{EI}{\mu}\right)^{1/2}$$

, $\mu = A\rho$

A_i, (i=1, 12) Constants are to be determined, from boundary conditions. The boundary conditions of the cantilever beam in consideration are;

$$\bar{u}_1(0)=0;\quad \bar{y}_1(0)=0;\quad \bar{y}_1'(0)=0;\quad \bar{u}_2'(1)=0;$$

$$\bar{y}''_2(1)=0;\quad \bar{y}_2'''(1)=0$$

At the cracked section:

$$\bar{u}_1(\beta)=\bar{u}_2(\beta);\; \bar{y}_1(\beta)=\bar{y}_2(\beta)\;;\; \bar{y}_1''(\beta)=\bar{y}_2''(\beta)\;;$$

$$\bar{y}_1'''(\beta)=\bar{y}_2'''(\beta)$$

Also at the cracked section, we have;

$$AE\frac{du_1(L_1)}{dx}=K_{11}(u_2(L_1)-u_1(L_1))+K_{12}\left(\frac{dy_2(L_1)}{dx}-\frac{dy_1(L_1)}{dx}\right) \qquad (13)$$

Multiplying both sides of the above equation by $\frac{AE}{LK_{11}K_{12}}$ we get;

$$M_1M_2\bar{u}'(\beta)=M_2(\bar{u}_2(\beta)-\bar{u}_1(\beta))+M_1(\bar{y}_2'(\beta)-\bar{y}_1'(\beta)) \qquad (14)$$

Similarly,

$$EI\frac{d^2y_1(L_1)}{dx^2}=K_{21}(u_2(L_1)-u_1(L_1))+K_{22}\left(\frac{dy_2(L_1)}{dx}-\frac{dy_1(L_1)}{dx}\right) \qquad (15)$$

Multiplying both sides of the above equation by $\frac{EI}{L^2K_{22}K_{21}}$ we get,

$$M_3M_4\bar{y}_1''(\beta)=M_3(\bar{u}_2(\beta)-\bar{u}_1(\beta))+M_4(\bar{y}_2'(\beta)-\bar{y}_1'(\beta)) \quad (16)$$

Where, $M_1=\dfrac{AE}{LK_{11}}$, $M_2=\dfrac{AE}{K_{12}}$, $M_3=\dfrac{EI}{LK_{22}}$,

$$M_4=\frac{EI}{L^2K_{21}} \quad (17)$$

The normal functions, Equation (9to14) along with the boundary conditions as mentioned above, yield the characteristic equation of the system as;

$$|Q|=0 \quad (18)$$

This determinant is a function of natural circular frequency (ω), the relative location of the crack (β) and the local stiffness matrix (K) which in turn is a function of the relative crack depth (a_1/W).
The results of the theoretical analysis for the first three mode shapes for un-cracked and cracked beam are shown in the figure. 3.

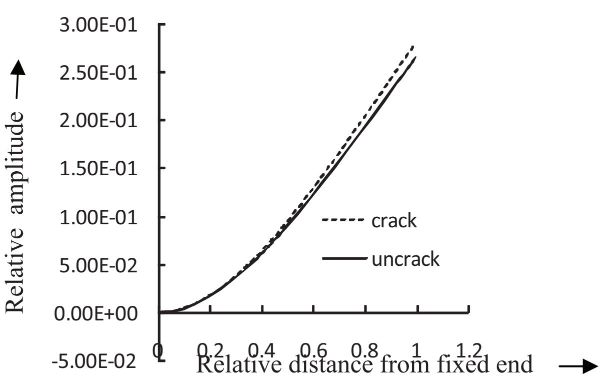

Figure: 3a Relative amplitude vs. relative distance from the fixed end (1st mode of vibration),

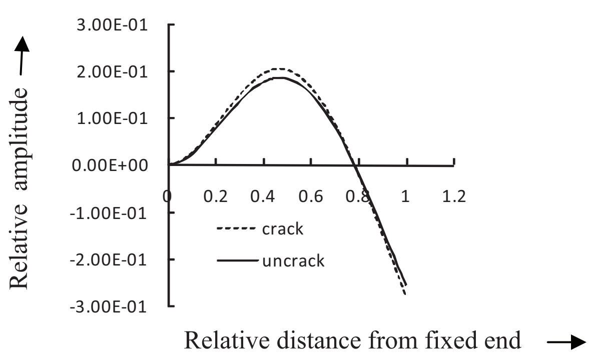

Figure: 3b Relative amplitude vs. relative distance from the fixed end (2nd mode of vibration),

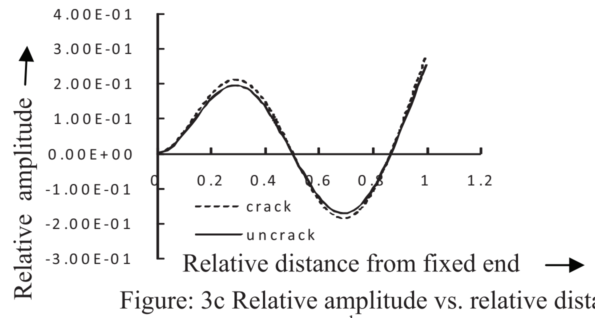

Figure: 3c Relative amplitude vs. relative distance from the fixed end (3rd mode of vibration),

3 ANALYSIS OF NEURAL TECHNIQUE

A back propagation neural network technique has been developed for detection of the relative crack location and relative crack depth (Figure 4). The neural network has got six input parameters and two output parameters. The inputs to the neural network are as follows; Relative first natural frequency = "fnf"; Relative second natural frequency = "snf"; Relative third natural frequency = "tnf"; Relative first mode shape difference = "fmd"; Relative second mode shape difference = "smd" and Relative third mode shape difference = "tmd".The outputs from the neural network are as follows; Relative crack location = "rcl" and Relative crack depth = "rcd"
The back propagation neural network has got ten layers (i.e. input layer, output layer and eight hidden layers). The neurons associated with the input and output layers are six and two respectively. The neurons associated in the eight hidden layers are twelve, thirty-six, fifty, one hundred fifty, three hundred, one hundred fifty, fifty and eight respectively. The input layer neurons represent relative deviation of first three natural frequencies and first three relative mode shape difference. The output layer neurons represent relative crack location and relative crack depth.

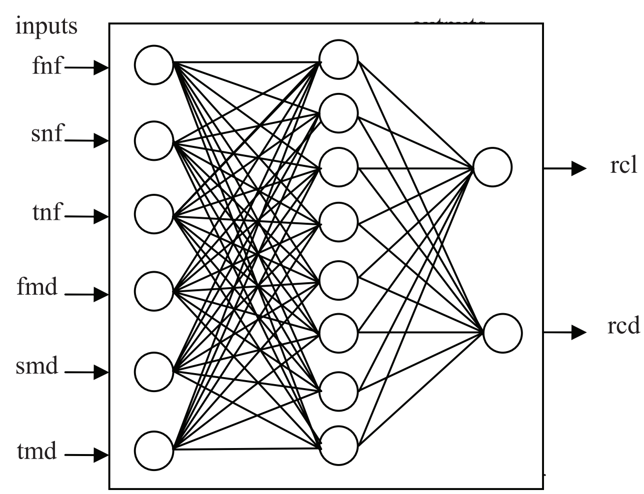

Figure 4. Neural Network Architecture

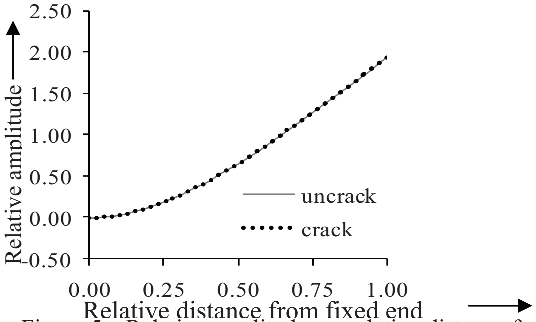

Figure 5a. Relative amplitude vs. relative distance from the fixed end (1st mode of vibration), $a_1/W=0.2$, $L_1/L=0.5128$

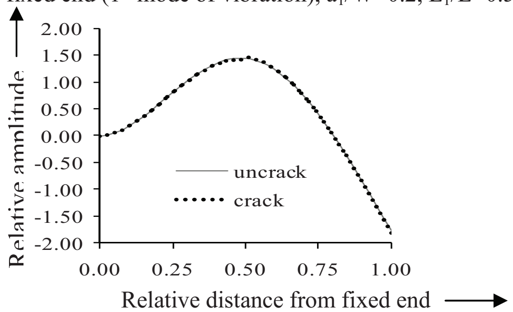

Figure 5b. Relative amplitude vs. relative distance from the fixed end (2nd mode of vibration), $a_1/W=0.2$, $L_1/L=0.5128$

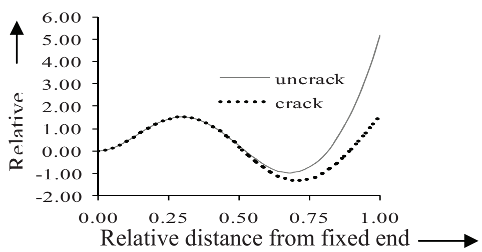

Figure 5c. Relative amplitude vs. relative distance from the fixed end (3rd mode of vibration), $a_1/W=0.2$,

V. EXPERIMENTAL SET UP

Experiments are performed to determine the natural frequencies and mode shapes for different crack depths on Aluminum beam specimen(800x50x6) on the experimental set-up shown in schematic diagram figure 5. The amplitude of transverse vibration at different locations along the length of the Aluminum beam is recorded by positioning the vibration pick-up and tuning the vibration generator at the corresponding resonant frequencies. These results for first three modes are plotted in figure 7. Corresponding numerical results and results from fuzzy method for the cracked and un-cracked beam are also presented in the same graph for comparison. A clear cut deviation is observed between the corresponding natural frequencies and mode shapes of the cracked and un-cracked Aluminum beam specimen.

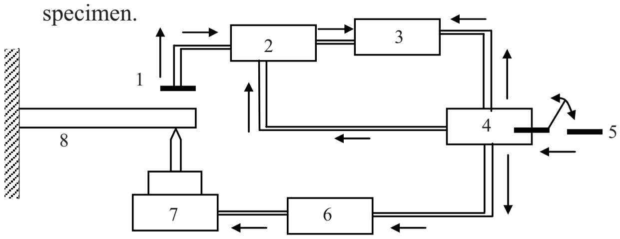

Figure 6. Schematic diagram of Experimental set-up

1. Vibration Pick-up. (Accelerometer)	4. Distribution box	7. Vibration amplifier
2. Vibration analyzer	5. Power supply	8. Vibration Exciter
3. Vibration indicator with software	6. Function Generator	9. Cantilever beam (Specimen)

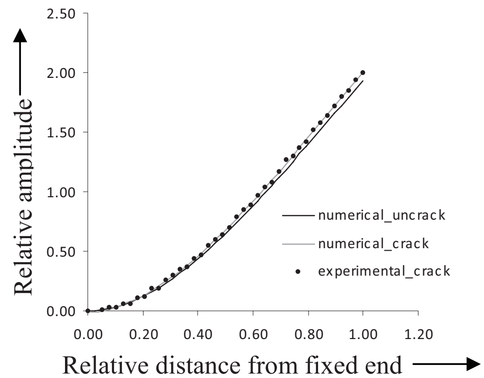

Figure 7a. Relative amplitude vs. relative distance from the fixed end (1st mode of vibration), $a_1/W=0.4$, $L_1/L=0.026$

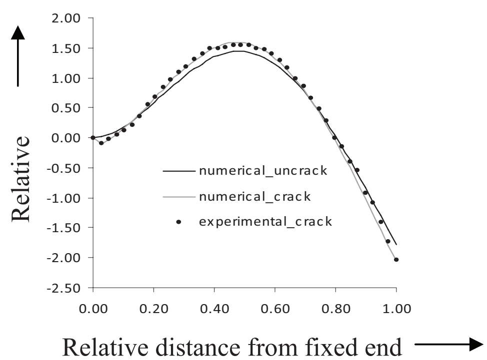

Figure 7b. Relative amplitude vs. relative distance from the fixed end (2nd mode of vibration), $a_1/W=0.4$, $L_1/L=0.026$

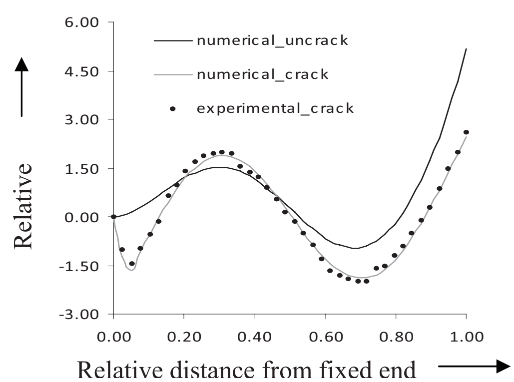

Figure 7c. Relative amplitude vs. relative distance from the fixed end (3rd mode of vibration), $a_1/W=0.4$, $L_1/L=0.026$

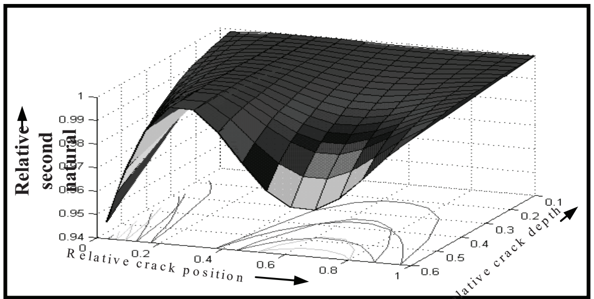

Figure 8a. Three dimensional cum contour plot for relative natural frequency

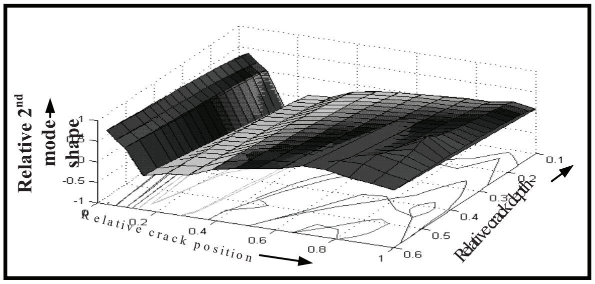

Figure 8b. Three dimensional cum contour plot for relative mode shape difference

VI. DISCUSSION

From the predicted results of developed neural back-propagation technique, numerical and experimental analyses, the following points are drawn.

It is evident from the Figure 2 that as the relative crack depth increases the compliances (C_{11}, $C_{12=}$, C_{21}, C_{22}) increase. For different relative crack locations (0.5128, 0.2564) and relative crack depths (0.2, 0.3), first three mode shapes are drawn pictorially in Figure 3 and Figure 5. It is observed from Figure 3 and Figure 5 that there are reasonable changes in mode shapes when compared between the cracked and uncracked beam. The experimental and numerical results and their comparisons between the cracked and uncracked beam are represented graphically in Figure 7. Figure 8 represent the variation of relative natural frequencies and relative mode shapes in three dimensional forms along with the contour plot with respect to relative crack location and relative crack depth respectively. Ten Examples of the training patterns out of the 800 training patterns for training of the Neural Network. Some of the predicted results by the developed Neural Network Controller and their comparison with corresponding numerical and experimental results which show very good agreement.

VII. CONCLUSION

Following conclusions can be drawn on the basis of the results and discussions made from the present investigations. Due to the presence of crack, the beam structure undergoes remarkable changes in natural frequencies and mode shapes. Again these changes depend upon the crack location and its intensity. The neural network technique considered here is used to predict the crack location and its intensity by using relative deviation of first three natural frequencies and first three mode shapes as inputs. The neural network predicted results are reasonably acceptable and in agreement with the experimental data. The successful detection of crack and its intensity in cantilever beam demonstrates that the new technique developed in the present study can be used efficiently and effectively in crack identification in different beam-type structures. This developed neural network technique can be used as a smart fault detecting tool for different types of vibrating structures. Further research can be made to generate hybrid neural network technique for more efficient fault identification.

VIII. REFERENCES

[1] P.F. Rizos, N. Aspragathos and A.D.Dimarogonas, **"**Identification of crack location and magnitude in a cantilever beam from the vibration modes", *Journal of Sound and Vibration*, ,Vol. 138(3), 381-388. 1990.

[2] M.A. Karthikeyan,R.A. Tiwari and S.B.Talukdar, "Crack localization and sizing in a beam based on the free and forced response measurement", Mechanical Systems and Signal processing, Vol. 21(3), 1362-1385, 2007.

[3] G.Y. Xu, W.D. Zhu and B.H. Emory, **"**Experimental and Numerical Investigation of Structural Damage Detection Using Changes in Natural Frequencies" *Journal of Vibration and Acoustics*, Vol. 129(6), 686-700, 2007.

[4]M. Akgun, F.D. Ju. and T.L. Paez, "Fracture Diagnosis in Beam Frame Structures using Circuit Analogy" *Recent Advances in Engineering Mechanics and Their Impact on Civil Engineering* Practice, , Vol. 2, 767-769, 1983.

[5] T.Y. Kam .and T.Y. Lee, "Detection of cracks in structures using modal test data", *Engineering Fracture Mechanics*, ,Vol. 42(2), 381-387, 1992

[6] S.M. Al -Said, M. Naji and A.A. Al-Shukry, "Flexural vibration of rotating cracked Timoshenko beam", *Journal of Vibration and Control*, Vol. 12 (11), 1271-1287, 2006.

[7]X.F. Chen, Z.J. H and J.W. Xiang, " Experiments on crack identification in cantilever beams", *Experimental Mechanics*, vol. 45 (3), 295-300, 2005.

[8]. A.K.Pandey, M. Biswas and M.M.Samman, "Damage detection from changes in curvature mode shapes", *Journal of Sound and Vibration*, Vol. 145(2), 321-332, 1991.

[9] B. Sahoo, and D. Maity, "Damage assessment of structures using hybrid neuro-genetic algorithm", *Applied Soft Computing*, Vol. 7(1), 89-104, 2007.

[10] S. Rajakarunakaran, P. Venkumar, D. Devaraj and K. Surya Prakasa Rao, **"**Artificial neural network approach for fault detection in rotary system", *Applied Soft Computing*, , Vol. 8 (1), 740–748, 2008.

[11] S. Suresh, S.N. Omkar, R. Ganguli, and V.Mani, **"**Identification of crack location and depth in a cantilever beam using a modular neural network approach", *Smart Material & Structure*, Vol. 13, 907-915, 2004.

[12] M. Sahin, and R.A. Shenoi, "Quantification and localisation of damage in beam-like structures by using artificial neural networks with experimental validation", *Engineering Structures*, Vol. 25(14), 1785-1802, 2003.

[13] C.Y. Kao. and S.L. Hung, "A Neural Network-based Approach for Detection of Structural Damage", *Proceeding, Modelling and Simulation*, Vol. 23.459-468, 2005.

[14] L. Huijian, H. Changjun, J. Jialin, H.Wang and H. Caizhe, "Crack damage detection in beam-like structures using RBF neural networks with experimental validation", *International Journal of Innovative Computing, Information and Control*, Vol. 1(4),625-634, 2005.

[15] H. Tada, PC Paris, Irwin GR, The stress analysis of cracks hand book. Del Research Corp. Hellertown, pensylvania,1973.