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A New Algorithm for Motion Control of Acyclic Minimally Persistent Formation of Mobile Autonomous Agents

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Abstract— **This paper presents a motion control algorithm for a multiple mobile autonomous agents using the concept of a rigid and persistent formation with a Leader-Follower structure. Initially, a triangular formation was started for three agents and subsequently Henneberg Sequence [4] is applied to extend for multi-agent system. A decentralized control for each agent considering the shortest path between its initial & final position during the motion of formation is developed using SQP-a nonlinear optimization technique.**

Keywords— **Formation Control, Autonomous Agents, Leader-Follower, Directed Graph**

I. INTRODUCTION

In formation control of autonomous agents such as robots, the motion control strategies may be either a centralized or decentralized. In centralized mode of control [1], the command for all agents of the group are assigned by the central command control board or a designated group leader for monitoring and control of all agents to guide them be placed at desired position. The centralized formation control could be a good scheme for a small group of robots, when it is implemented with a single computer and a single sensor to monitor and control the entire group. For controlling large number of robots decentralized control scheme is used instead to reduce greater computational complexity and large amount of communication. In the decentralized mode of control [3,5], one agent of the group can be a leader and others are followers (or each agent of the group can be a leader and follower except a designated group leader and the two outmost agents) and as a follower each agent generates its own commands autonomously (i.e. control law for each agent is provided in distributed way such that each agent works autonomously) based on the relative measurement only from its neighbours without need of an external supervisor and whole purpose of the formation motion is achieved.

Motion control scheme for formation may be modified depending upon some factors like agent dynamics, inter-agent information exchange structure, control goals in different applications, etc.

Formations are modelled using *formation graph.* The pattern of information exchange between agents leads to two different types of graphs namely directed & dundirected [2].

In this paper development of control strategy uses directed graph based formation. Digraph is called acyclic when no cycle is present in its sensing pattern. In these papers [6, 7] it is defined that minimally persistent formation of autonomous agents may be formed in two ways. First one is *leader-follower* graph architecture constructed from an initial leader-follower seed by Henneberg Sequence with standard vertex additions or edge splitting [4]. Leader-follower type minimally persistent graph is always acyclic. Another type of construction by sequence of specific operation [8] such that every intermediate construction is also persistent.In this paper control strategies for only leader-follower type formation constructed from sequence of vertex addition is described.

In [5], formation control strategy of leader-follower and three co-leader structures is set up based on discrete-time motion equations considering decentralized approach. However a leader-follower structure type persistent formation control using optimization of some distances (in continuous domain) has been proposed in this paper.

It may be noted that for each agent both the objective function and the constraints are different. The control scheme proposed considers a decentralized approach. With advent of high speed computational platforms the solution associated with optimization procedure in the control generation is possible.

II. PROBLEM FORMULATION

In this work a triangular formation of leader-follower structure with three mobile autonomous point agents, in plane is considered. In Fig.1, R-1, R-2, R-3 denote leader, first follower & the ordinary follower respectively. It may be noted that R-1 has no outgoing edge i.e. free to move along a specified trajectory. R-2 needs to maintain one distance constraint d_1 i.e. to R-1 and R-3 needs to maintain two distance constraints d_2 & d_3 w.r.t. R-2 & R-1 respectively.

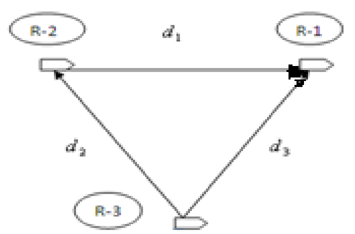

Fig.1 Triangular formation of leader-follower structure.

We assume the desired distances among the agents satisfy triangle inequalities given by

$$d_1 < d_2 + d_3 \; ; \; d_2 < d_1 + d_3 \; ; \; d_3 < d_1 + d_2 \qquad (1)$$

Assumption 1:

(i) For each agent 'i', the kinematic model [5] of unicycle non-holonomic point agent is considered as

$$(\dot{x}_i, \dot{y}_i) = (v_i \cos\theta_i, v_i \sin\theta_i) \;\&\; \dot{\theta}_i = \omega_i \qquad (2)$$

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where $p_i(t) = (x_i(t), y_i(t))$ with $i = 1,2,3$ denotes, the position of i^{th} agent and θ_i (t) & $v_i(t)$ denote the orientation and translational velocity of the i^{th} agent at each instant of time (t), respectively. $(x_i(t), y_i(t))$ is with respect to an earth-fixed coordinate coordinate system.

(ii) each agent can measure its position with respect to the earth-fixed coordinate system by proper arrangement of a navigation sensor in each agent

(iii) first follower has relative distance and bearing information of leader only and ordinary follower has relative distance and bearing information of leader & first follower both using an active sensor (e.g. sonar) for each agent(first follower and ordinary follower) with respect to their respective cartesian coordinate system fixed on the respective point body.

(iv) using positions for first follower and ordinary follower with respect to the earth-fixed coordinate system and relative distance and bearing information in (ii) with respect to respective cartesian coordinate system fixed on their (first follower and ordinary follower) point body, can be converted with respect to the earth-fixed coordinate system.

It is intended to provide a control scheme for this triangular formation (starting from a *non-collinear* arrangement) such that during the motion of three robots for any mission, desired inter-agent distances are preserved.

III. DEVELOPMENT OF CONTROL LAW

The objective of the controller design is to develop set of decentralized control laws for overall formation. Hence, for each agent, a separate control law for continuous movement in autonomous manner based on local knowledge only of direction of its neighbours has been considered. The control law for each agent is derived by optimizing the corresponding objective function with given constraints including the desired distance constraints of formation. The unknown variables to be solved, are *x* and *y* coordinates (which are continuous function of time) of position (with respect to the global coordinate system) of corresponding agent for which objective function is derived.

***Assumption 2*:**

(i) Inter-agent distances are sufficiently large so that initial collision among robots can be avoided for almost continuous motion of robots.

(ii) Initially positions of robots are not collinear i.e. condition (1) is satisfied.

(iii) During motion of robots no communication failure occurs among the robots

(iv)There is no time delay and uncertainty in sensing the information

(v)Control input in the form of translational velocity and angular velocity (discussed later in this section) calculated from final and initial position of any agent must be generated by controller of each agent. Now at the outset, we proceed for development of control law.

When all the agents of given formation completes movement to a new set of position coordinates from an old set of position coordinates during certain period of time such that desired distances among agents are preserved for both set of positions and not any other distance preserving position set is available in between these two position sets during motion, then the movement of formation from the old set to new set of positions is called one complete displacement of formation.

Before proceeding to develop the control laws for all agents, it is assumed that at any time t each agent is maintaining its own distance constraints. Then how these agents move to their new positions for a complete displacement is discussed below.

Control Law for Leader: Leader doesn't need to maintain any distance constraint from any other agent in formation. A specified control action is provided for its dynamics such that it moves along a specified path (trajectory) i.e. each position (at each instant of time t) coordinate is known (preprogrammed) to computational system of the leader. Suppose, at time t initial position coordinate for leader is assumed as ($(x_{1In}(t), y_{1In}(t))$. Let the leader moves to a new position $(x_{1f}(t), y_{1f}(t))$ i.e. the final position (rest point), in very small period of time dt *such that continuity preserves between* $(x_{1In}(t), y_{1In}(t))$ & $(x_{1f}(t), y_{1f}(t))$ *i.e. the distance between these two positions is sufficiently small.* This motion of the leader and corresponding movement of the first follower is shown in Figure.2.According to Fig.2 $d\vec{s}_1 = dx_1\hat{i} + dy_1\hat{j}$,where $\vec{s}_1$ is a function of t and $\hat{i}$ and $\hat{j}$ are unit vectors along x and y direction of earth-fixed coordinate system. $dx_1 = x_{1f} - x_{1In}$ and $dy_1 = y_{1f} - y_{1In}$ .Therefore, control input to reach its final position is

$$\vec{v}_1(t) = \frac{d\vec{s}_1(t)}{dt} = \frac{dx_1}{dt}\hat{i} + \frac{dy_1}{dt}\hat{j} = v_{1x}(t)\hat{i} + v_{1y}(t)\hat{j} \quad (3)$$

where, $\frac{dx_1}{dt} = v_{1x}(t)$ and $\frac{dy_1}{dt} = v_{1y}(t)$

The translational velocity control input during $dt = \|\vec{v}_1\| = \sqrt{(v_{1x}(t))^2 + (v_{1y}(t))^2}$ meter/sec.

The angular velocity control input (rad. /sec) during same period of time is

$$= \omega_1(t) = \tan^{-1}\left|\frac{dy_1}{dx_1}\right| \text{ when } dx_1 \text{ is +ve, } dy_1 \text{ is +ve}$$

$$= \left(\pi - \tan^{-1}\left|\frac{dy_1}{dx_1}\right|\right) \text{ when } dx_1 \text{ is –ve, } dy_1 \text{ is +ve}$$

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$= -\left(\pi - \tan^{-1}\left|\frac{dy_1}{dx_1}\right|\right)$ when dx_1 is –ve, dy_1 is –ve

$= -\tan^{-1}\left|\frac{dy_1}{dx_1}\right|$ when dx_1 is +ve, dy_1 is –ve

$= -\pi/2$ or $\pi/2$ when $dx_1 = 0$ and dy_1 is -ve or +ve

Control Law for First Follower: Initial and final position coordinates for the leader are $(x_{1In}(t), y_{1In}(t))$, $(x_{1f}(t), y_{1f}(t))$ respectively. Then first follower senses the disturbance in position of leader, i.e. it senses error in desired distance constraint (d_1) by sensing the final position of the leader staying at its initial position $(x_{2In}(t), y_{2In}(t))$ .It tries to satisfy this distance constraint to leader. Therefore, suppose, it moves to a rest point (final position) $(x_{2f}(t), y_{2f}(t))$ at the next instant of time dt after the instant during which the leader moves to its final position. During this movement of first follower, leader is assumed to be stationary at the position $(x_{1f}(t), y_{1f}(t))$. A condition is given to first follower such that only due to disturbance in position of leader; first follower changes its position .To maintain the cohesive motion with the leader distance of the first follower from the leader must be d_1 at final position of both the agents. It can be formulated as follows:

$$(x_{1f}(t) - x_{2f}(t))^2 + (y_{1f}(t) - y_{2f}(t))^2 - d_1^2 = 0 \qquad (4)$$

where the values of x_{1f}, y_{1f}, d_1 are known to computational system of first follower. But it is clear from eq. (4) that locus of the position of first follower is a circle. Hence, its rest position may be at anywhere on this circle. First follower may take its rest position for which it crosses over the leader and may collide with leader. Undesirable consequence of this phenomenon is ordinary follower may collide with leader and first follower both for maintaining the distant constraints from both of them. Hence to provide a control avoiding this unsafe situation, a restriction to the motion of first follower must be imposed, such that it reaches to a rest point on the circle so that distance between the new and initial position of it is minimum.

In Fig.2, $\|d\vec{s}_2\|$ is defined as the distance between the final and initial position of first follower. Here $\vec{s}_2$ is function of t .Then, we have,

$$\|d\vec{s}_2\|^2 = (x_{2f}(t) - x_{2In}(t))^2 + (y_{2f}(t) - y_{2In}(t))^2 \qquad (5)$$

Therefore, $\|d\vec{s}_2\|$ must be minimum such that first follower moves along the shortest path to its final position. Hence, the first follower follows the leader maintaining safe motion. Now we intend to propose a control law for motion of first

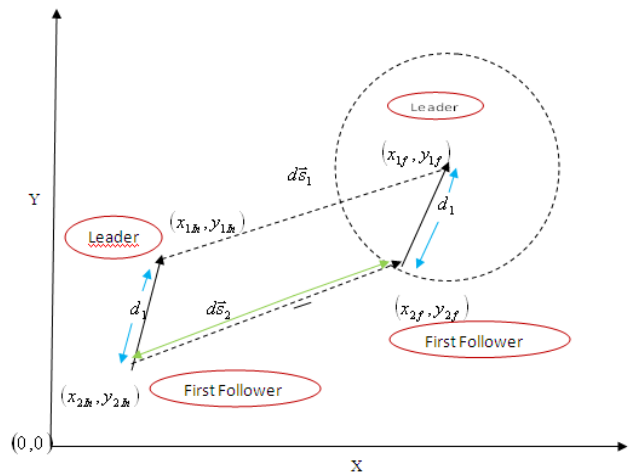

Figure 2. Motion of Leader and First Follower for a very small duration of time (dt)

follower satisfying aforesaid conditions. Actually the whole problem may be treated as an optimization problem where minimization of objective function (5) under equality constraint (4) should be performed. And a control law based on this optimization is presented as

$$\vec{v}_2(t) = \frac{d\vec{s}_2}{dt} = \frac{dx_2}{dt}\hat{i} + \frac{dy_2}{dt}\hat{j} = v_{2x}(t)\hat{i} + v_{2y}(t)\hat{j} \qquad (6)$$

where, $\frac{dx_2}{dt} = v_{2x}(t)$ and $\frac{dy_2}{dt} = v_{2y}(t)$ and $\hat{i}$ and $\hat{j}$ are unit vectors along x and y direction of the earth-fixed coordinate system.

The translational velocity control input is

$= \|\vec{v}_2\| = \sqrt{(v_{2x}(t))^2 + (v_{2y}(t))^2}$ m/s.

The angular velocity (rad. /s) control input is

$= \omega_2(t) = \tan^{-1}\left|\frac{dy_2}{dx_2}\right|$ when dx_2 is +ve, dy_2 is +ve

$= \left(\pi - \tan^{-1}\left|\frac{dy_2}{dx_2}\right|\right)$ when dx_2 is –ve, dy_2 is +ve

$= -\left(\pi - \tan^{-1}\left|\frac{dy_2}{dx_2}\right|\right)$ when dx_2 is –ve, dy_2 is –ve

$= -\tan^{-1}\left|\frac{dy_2}{dx_2}\right|$ when dx_2 is +ve, dy_2 is –ve

$= -\pi/2$ or $\pi/2$ when $dx_2 = 0$ and dy_2 is -ve or +ve

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Control Law for Ordinary Follower: Let the leader and first follower have been placed at their corresponding final positions. The ordinary follower senses the disturbances in position of leader and first follower i.e. it senses error in desired distance constraints d_2 and d_3 by sensing the final position of the first follower and leader respectively. It tries to satisfy these distance constraints to first follower and leader. Therefore, suppose, the ordinary follower moves to its final position (rest point) at the next time instant of the time (dt) after the instant during which first follower moves to its final position. During this movement of ordinary follower, leader and first follower are assumed to be stationary at the position $(x_{1f}(t), y_{1f}(t))$ and $(x_{2f}(t), y_{2f}(t))$ respectively. At the final position ordinary follower satisfies its distance constraints .Here a condition is given to the ordinary follower such that only when disturbances in positions of both leader as well as first follower (not merely leader) occur; ordinary follower changes its position to final one. This final position is assumed $\left(x_{3f}(t), y_{3f}(t)\right)$ .We assume the initial position of the ordinary follower is $\left(x_{3In}(t), y_{3In}(t)\right)$ Now according to the desire distance constraints for it two conditions are to be satisfied, as given below:

$$(x_{1f}(t)-x_{3f}(t))^2+(y_{1f}(t)-y_{3f}(t))^2-d_3^2=0 \qquad (7)$$

$$\left(x_{2f}(t)-x_{3f}(t)\right)^2+\left(y_{2f}(t)-y_{3f}(t)\right)^2-d_2^2=0 \qquad (8)$$

where, $x_{1f}, y_{1f}, x_{2f}, y_{2f}, d_2, d_3$ are known to the computational system of ordinary follower. Hence, $\|d\vec{s}_3\|$ is defined as distance between the final and initial position of ordinary follower. Where $\vec{s}_3$ is function of time. Then we define,

$$\|d\vec{s}_3\| = \left(x_{3f}(t)-x_{3In}(t)\right)^2+\left(y_{3f}(t)-y_{3In}(t)\right)^2 \qquad (9)$$

Actually equations (7) and (8) are equations of two circles. They meet at two different points.

The ordinary follower will follow the leader and first follower maintaining safe motion and moves to any one meeting point such that $\|d\vec{s}_3\|$ is minimum. By maintaining $\|d\vec{s}_3\|$ minimum, ordinary follower moves along shortest path to its final position. Now it is the need to propose a control law for motion of first follower satisfying aforesaid conditions. Actually the whole problem may be treated as an optimization problem where minimization of objective function (9) under equality constraint (7) & (8) should be performed. And a control law based on this optimization is presented as:

$$\vec{v}_3(t)=\frac{d\vec{s}_3}{dt}=\frac{dx_3}{dt}\hat{i}+\frac{dy_3}{dt}\hat{j}=v_{3x}(t)\hat{i}+v_{3y}(t)\hat{j} \qquad (10)$$

where $\frac{dx_3}{dt}=v_{3x}(t)$ and $\frac{dy_3}{dt}=v_{3y}(t)$ and $\hat{i}$ and $\hat{j}$ are unit vectors along x and y direction of the earth-fixed coordinate system.

The translational velocity control input is

$$=\|\vec{v}_3\|=\sqrt{(v_{3x}(t))^2+(v_{3y}(t))^2} \text{ m/s}$$

The angular velocity (rad. /s) control input is

$$=\omega_3(t)=\tan^{-1}\left|\frac{dy_3}{dx_3}\right| \text{ when } dx_3 \text{ is +ve}, dy_3 \text{ is +ve}$$

$$=\left(\pi-\tan^{-1}\left|\frac{dy_3}{dx_3}\right|\right) \text{ when } dx_3 \text{ is –ve}, dy_3 \text{ is +ve}$$

$$=-\left(\pi-\tan^{-1}\left|\frac{dy_3}{dx_3}\right|\right) \text{ when } dx_3 \text{ is –ve}, dy_3 \text{ is –ve}$$

$$=-\tan^{-1}\left|\frac{dy_3}{dx_3}\right| \text{ when } dx_3 \text{ is +ve}, dy_3 \text{ is –ve}$$

$$=-\pi/2 \text{ or } \pi/2 \text{ when } dx_3=0 \text{ and } dy_3 \text{ is -ve or +ve}$$

Remarks: Hence, from the previous discussion it is concluded that for *each complete displacement* of considered triangular formation, at the end of first instant of time dt the leader moves to its desired final position. Then at the end of next instant dt (which is the second one) the first follower moves to its final desired position to maintain distance constraints to the leader, during which leader is kept stationary. At end of another instant dt (which is third one) the ordinary follower reaches to its final position to maintain distance constraints to both leader and first follower. Therefore the agents are not reaching their corresponding final position exactly at the same time .Consequently during the period from "after the starting of first instant" and "before the end of third instant" desired distances are not preserved among the agents, rather the first follower and ordinary follower try to form up.So it may be concluded that after every 3* dt time, the desired formation is obtained. Therefore, each agent start to move to its new position (to be in a new position set) after every 2* dt time. That is there is a discontinuous motion occurs for every agent. Therefore, for formation of n number of agents, after every $n * dt$ time the desired formation is maintained. Each agent start to move to its new position (to be in a new position set) after every $(n-1)$ * dt time. However, *if the dt is chosen as very small we may assume that all the agents reach their corresponding final (new) positions during first instant of time dt (almost same time taken by leader to reach its desired final position) during each complete displacement of formation and continuous motion of formation is maintained .Consequently we may also assume all the agents moves with continuous motion. Simulation results in the next section are also done based on this assumption.*

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IV. SIMULATION AND RESULTS

The control laws (3), (6), (10) for different agents have been tested successfully via two cases of simulations for specified formation with consideration of $d_1 = d_2 = d_3 = 2$ meters.

These control laws require the final position of the corresponding agent at each instant of time during their motion. For the leader, the final position at each instant of time is available as the path is specified for it, but for other agents, these positions must be calculated. To find out the final position at each instant of time t, the controller in each case requires optimization(for our algorithm) of a quadratic objective function under one or two quadratic equality constraints as described in section III. Several optimization methods are available for this purpose. Our choice here is to exploit Sequential Quadratic Programming (SQP) as it is one of the most popular and robust algorithms for nonlinear continuous optimization. The method is based on solving a series of subproblems designed to minimize a quadratic model of the objective subject to a linearization of the constraints. In the proposed controller, the objective functions are chosen as quadratic whilst the constraints are taken as non-linear quadratic which can be linearized during course of optimization procedure. At the beginning of each instant of time t (i.e.at the beginning of a complete displacement of the whole formation), the position of each agent is used as the initial position in control law of that particular agent. This position coordinate is also assumed as starting point of that agent's complete iterative procedure (in optimization process using SQP) for finding out its final position.

Specific assumptions in different cases:

$i)$ Length of each time instant is considered as 0.001 second for simulation during optimization

$ii)$ Translational velocity input to the leader is 1 m/s,

$iii)$ In first case initial position coordinates are assumed as (1.5, 1.732), (0.5, 0), (2.5, 0) respectively for leader, first follower and ordinary follower.

$iv)$ In the second case initial position coordinates are assumed as respectively (0, 2), (0, 0), (1.732, 1) for leader, first follower and ordinary follower.

$v)$ Distance travelled by leader is assumed to be 1.5 m in each case

Description of results in different cases:

Case A**:** A linear trajectory motion with constant velocity of 1m/s is provided to the leader. The paths followed by all other agents along with leader during motion of formation are shown in Fig.3. Distances are observed maintained at specified values after every small instant of time (0.001 sec) during motion. Plots of d_1 vs. t, d_2 vs. t and d_3 vs. t are shown in Fig.4, Fig.5, Fig.6 respectively for motion of formation during first 1.6 second travelling of formation.

Case B**:** A circular trajectory motion is provided to the leader. For this purpose leader is provided with 0.00157 rad/s angular velocity input and constant translational velocity input at 1 m/s.Paths of all agents during motion are shown in Fig.7.From the simulation studies of case B, it is found that the distances among agents are maintained at the desired values after every small instant of time (0.001 sec) during the motion of the formation as in case of case-A. Due to space limitation plots of d_1 vs. t, d_2 vs. t and d_3 vs. t are not shown for this case.

V. CONCLUSION

A new algorithm using a set of decentralized control laws based on optimization (using Sequential Quadratic

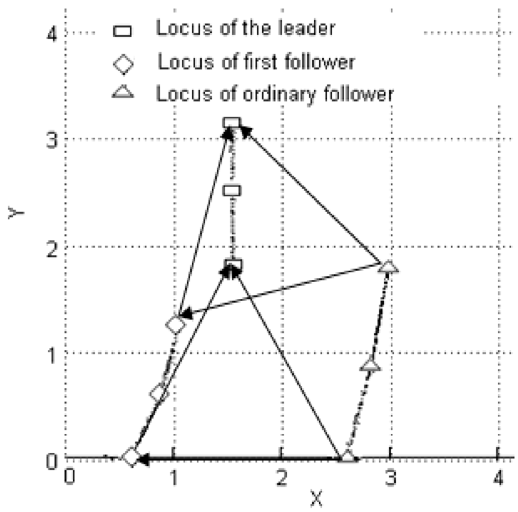

Fig.3. Motion of triangular formation with a linear trajectory motion and constant velocity provided to the leader

Programming) of distance constraints has been proposed for the motion control of a ***leader-follower*** structure type formation of multiple mobile autonomous agents. The effectiveness of the proposed control schemes have been demonstrated through a set of simulation results. During the motion of formation, the inter-agent distances are maintained at desired values after every specified duration of time and inter-agent collisions are absolutely overcome.

The above described control design for formation of three autonomous agents may be extended to a large number of agents in the form of leader-follower structure in which each agent observes the distances of only two or fewer number of neighboring agents to which it needs to maintain distance constraints. This control strategy is almost suitable for time-critical mission where all vehicles must arrive their final positions exactly at the same time. The immediate task

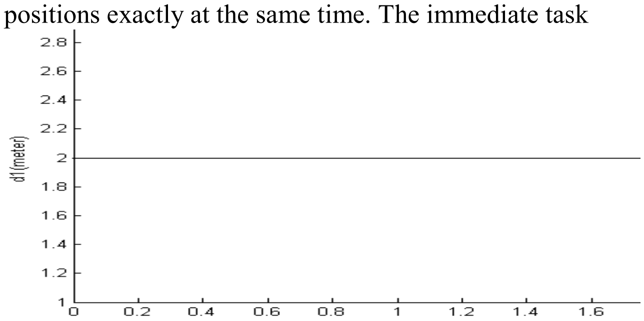

Fig.4.Distance between leader & 1st follower vs. time

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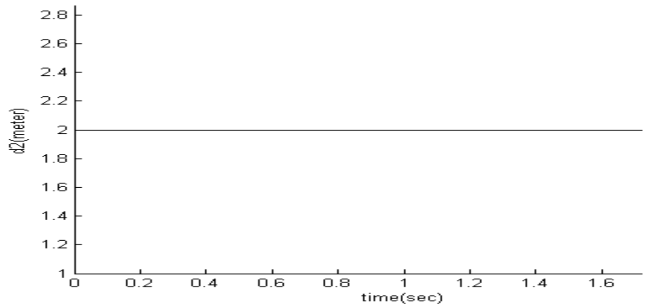

Fig.5.Distance between first follower & ordinary follower vs. time

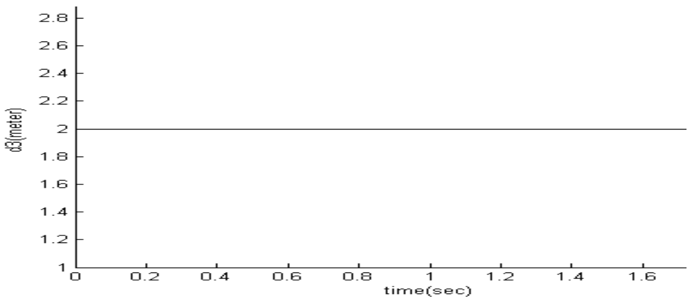

Fig.6. Distance between leader and ordinary follower vs. time

is to develop a control scheme for purely time-critical mission of this type, using our algorithm with some improvement.

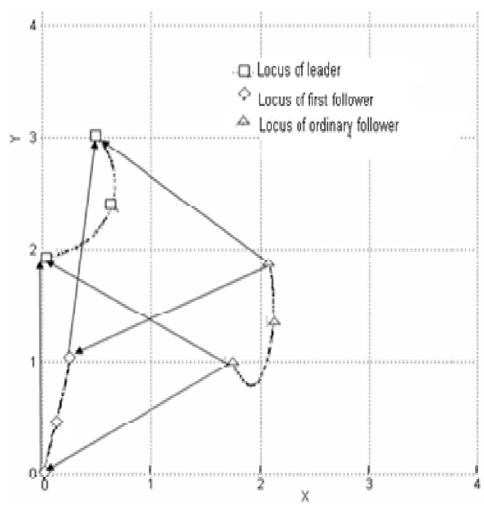

Fig. 7. Motion of triangular formation with circular trajectory motion provided to the leader

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