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Determination of fatigue crack growth rate from experimental data: A new approach

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Abstract

The determination of crack growth rate from laboratory observations of crack length and number of cycles is certainly a tedious job in order to considerably reduce the scatter in the test results. There are several curve-fitting methods currently in use including the standard ASTM methods. In this work, an alternative technique has been presented which has been found to be efficient in determining the crack growth rate in 2024 T3 and 7020 T7 aluminum alloy specimens.

*Key-words***:** Fatigue crack growth rate; Specific growth rate; Incremental polynomial; Exponential equation.

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Nomenclature

a	crack length measured from edge of the specimen (mm)
a_i	crack length (initial) corresponding to the 'i^{th}' step (mm)
a_j	crack length corresponding to the 'j^{th}' step (mm)
B	plate thickness (mm)
COD	crack opening displacement
da/dN	crack growth rate (mm/cycle)
F	remotely applied load (kN)
$f(g)$	geometrical factor
K	stress intensity factor ($\mathrm{MPa}\sqrt{\mathrm{m}}$)
ΔK	stress intensity factor range ($\mathrm{MPa}\sqrt{\mathrm{m}}$)
m_{ij}	specific growth rate corresponding to the the interval i-j
N	number of cycles or fatigue life
N_i	number of cycles (initial) corresponding to the 'i^{th}' step
N_j	number of cycles corresponding to the 'j^{th}' step
w	plate width (mm)
ϖ	exponent in the Frost and Dugdale law

1. Introduction

Fatigue crack propagation, a continuous physical process of material damage, is characterized by the analysis of the rate of change of crack length (a) with change in number of cycles (N). It requires a discrete set of crack length vs. number of cycles data from measurement on laboratory specimens. Unlike monotonic test, fatigue test data contain a large amount of experimental scatter. The crack growth rate (da/dN) obtained from raw laboratory data contain a still larger amount of scatter. Hence, it is necessary to have some means of data smoothening. In recent years, many crack growth models have been proposed to predict fatigue life under various loading conditions which primarily deal with the relationships between fatigue crack growth rate and different crack driving forces (as well as other parameters like material properties). However, in majority of instances, the method of determination of crack growth rates from *a–N* data is not being explicitly mentioned. The most widely used techniques for crack growth rate determination are:

a) calculating finite differences between successive data points and making a linear interpolation to estimate the gradient at the mid-point (Mukherjee, 1972);
b) fitting best smooth curve through *a–N* data and taking gradients of the slope (Smith, 1973);
c) fitting an analytical curve (e.g. polynomial) through all or a part of the data (Davies and Feddersen, 1973);
d) using orthogonal polynomial method for fitting cubic expressions to equidistantly spaced crack length measurements (Munro, 1973);
e) by spline technique both for interpolation and data smoothening (Polak and Knesl, 1975) etc.
f) by incremental polynomial method fitting a second-order polynomial (parabola) to sets of (2n+1) successive data points, where n is usually 1, 2, 3, or 4. (ASTM E 647-08)

The test results of constant amplitude fatigue crack growth reveal that there is an increase in crack length with number of loading cycles. This increase in crack length is exponential in nature and can be expressed by simple log-linear relationship (Eq. 1) as per the observation of Frost and Dugdale (Frost and Dugdale, 1958).

$$Ln(a) = \varpi N + Ln(a_i) \qquad \text{or,} \quad a = a_i e^{\varpi N} \tag{1}$$

where, N is the fatigue life, ϖ is a parameter that depends on the geometry, material and load scenario, a is the crack length and a_i is the initial flaw size. Other researchers have also observed the apparent exponential rate of crack growth for both micro- and macro-cracks (Wang, 1982; Zhang, 2000; Mohanty, Verma, and Ray, 2008, 2009a, 2009b). Further, it is known that various types of non-linear functions such as logarithmic, exponential or some special types of functions can be fitted to the scattered experimental data and then the least squares method can be easily applied to get the smooth curve. All these concepts encouraged the authors to work out and present a new approach to determine the crack growth rate (d*a*/d*N*) from raw laboratory data.

2. Experimental procedure

The study was conducted on two Aluminum alloys, namely 7020 T7 and 2024 T3. The 7020 Al-alloy, procured in the as-fabricated condition, was subjected to T7 heat-treatment while 2024 Al-alloy was procured in T3 heat-treated condition. The chemical compositions and the mechanical properties of the two alloys are summarized in Tables 1 and 2 respectively. Single-edge notched, SEN specimens having a thickness of 6.48 mm were used for conducting the fatigue test. The specimens were made in the LT plane, with the loading aligned in the longitudinal direction. The detail geometry of the specimens is given in Fig 1.

The experiments were performed in *Instron*-8502 machine with 250 kN load cell capacity. All tests were conducted in air and at room temperature. Both the surfaces of the test specimen were mirror polished and marked at every one mm interval in order to measure crack length by visual method. It was then fatigue pre-cracked under mode-I loading to an a/w ratio of ~0.30 and were subjected to constant load test maintaining a load ratio of 0.1. The sinusoidal loads were applied at a frequency of 6 Hz. The load scenarios are presented in Table 3. The crack growth was monitored by using an optical method with a 20× magnification for 2024 Al-alloy specimens while for 7020 T7 specimens it was monitored by using a COD gauge mounted on the face of the machined notch. The following equation was used to determine stress intensity factor '*K*' (Brown and Srawley, 1966)

$$K = \mathrm{f}(g).\frac{F\sqrt{\pi a}}{wB} \qquad (2)$$

where, $\mathrm{f}(g) = 1.12 - 0.231(a/w) + 10.55(a/w)^2 - 21.72(a/w)^3 + 30.39(a/w)^4$

3. Procedure for the determination of crack growth rate

3.1 By using proposed exponential equation

Earlier the authors had used Exponential model to predict retardation parameters (Mohanty, Verma, and Ray, 2008a) and fatigue life (Mohanty, Verma, and Ray, 2008b; 2008c) using 7020 T7 and 2024 T3 Al-alloys. However, the method of obtaining crack growth rate was not explicitly mentioned. In the present case, the procedure for calculation of da/dN is enumerated with data on 7020 T7 Al. alloy.

Based on the concept of exponential nature of crack growth, the crack length vs. number of cycles data have been fitted by an exponential equation of the form:

$$a_j = a_i e^{m_{ij}(N_j - N_i)} \tag{3}$$

where, a_i and a_j = crack length in i^{th} step and j^{th} step in 'mm' respectively,

N_i and N_j = No. of cycles in i^{th} step and j^{th} step respectively,

m_{ij}= specific growth rate in the interval i-j,

i = No. of experimental steps,

and $j = i+1$

The procedures of the method are outlined below with the help of Table 4 (since fatigue test data are very large in number, only a small part of data is presented in Table 4 for the purpose of explaining the procedure of smoothening the *a–N* curve):

1. The exponent 'm_{ij}'(i.e. specific growth rate) is the important controlling parameter in the proposed exponential equation. The specific growth rate m is not a constant quantity. It depends on a number of factors. Its significance and dependence on various crack driving parameters are given elsewhere (Mohanty, Verma, and Ray, 2008b). The specific growth rate 'm_{ij}' is derived by taking logarithm of equation (3) as follows:

$$m_{ij} = \frac{\ln\left(\frac{a_j}{a_i}\right)}{\left(N_j - N_i\right)} \tag{4}$$

2. The raw values of specific growth rate (m) from experimental *a–N* data (columns A and B, Table 4) are calculated using equation (4) and are given in column C of Table 4. These are fitted with corresponding crack lengths by a polynomial curve-fit.
3. To get a better result, crack lengths at small increments (0.005 mm in the present case) are tabulated in column D and the corresponding values of m are obtained using polynomial equation (column E).
4. The above values of specific crack growth rates are used to get the smoothened values of the number of cycles (column F, Table 4) as per the following equation:

$$N_j = \frac{\ln\left(\frac{a_j}{a_i}\right)}{m_{ij}} + N_i \tag{5}$$

5. The crack growth rates (da/dN) are calculated directly from the above calculated values of 'N' as follows:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{\left(a_j - a_i\right)}{\left(N_j - N_i\right)} \tag{6}$$

The scatter of specific growth rate calculated piecewise and that obtained after data smoothening are shown in Fig. 2.

3.2 By incremental polynomial method

This method is based on nine point incremental polynomial as per ASTM standard [ASTM E647-08]. It involves fitting a second-order polynomial (parabola) to sets of nine successive data points so as to minimize the square of the deviations between observed and fitted values of crack sizes (least squares method). The crack growth rates are obtained from the first derivative of the fitted equation. The calculated crack growth rates for the present case are presented in Fig. 3 along with the results of proposed exponential equation method for comparison.

4. Discussion and Conclusion

Several methods are in use to determine crack growth rate from raw experimental *a–N* data as already highlighted. Every method has its own merits and demerits. However, the most attractive technique is to fit a polynomial through the experimental data and differentiating it to obtain crack growth rates. Its applicability is questioned by Davis and Feddersen (Davies and Feddersen, 1973) because of the requirement of higher order polynomial for entire data range which suffers from several inflexions leading to large deviations from the expected monotonic behavior of the growth rates. Although a higher order polynomial gives a better value of regression coefficient (R^2 value), the value of the standard error of estimate increases due to the presence of the inflection points. Therefore, Davis and Feddersen asked for a different class of functions suitable for smooth curve fitting. The piece-wise curve-fit by five or seven point incremental polynomial method suggested in ASTM standard may partially overcome these difficulties. However, it is time consuming and also requires much effort to get better smoothness in crack growth rate curve.

The proposed exponential equation is proved useful for calculating crack growth rate. The advantages of this method are:

1. An exponential equation fits well with most crack growth studies and covers both slow crack growth region as well as accelerated crack growth region. A

single middle order polynomial equation is sufficient to fit the entire data range from slow growth region to accelerated growth region. Hence, instead of fitting a polynomial equation to the basic *a–N* curve, it is more appropriate to use exponential equation.

2. Using an exponential equation has some physical basis. The specific crack growth rate (*m*) has physical significance and has been correlated with several crack driving parameters in authors' earlier work (Mohanty, Verma, and Ray, 2008b).
3. The procedure of obtaining crack growth rates (d*a*/d*N*) can be easily performed with the help of a digital computer.
4. It requires less computational time and effort than standard ASTM method.
5. The crack growth rate (d*a*/d*N*) values obtained by this method give much smoother curve (Fig. 3).

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Table 1 – Chemical Composition (wt.%) of 7020 T7 and 2024 T3 Al alloys

Matls.	Al	Cu	Mg	Mn	Fe	Si	Zn	Cr	Others
7020-T7 Al. alloy	Main constituent	0.05	1.2	0.43	0.37	0.22	4.6	-	-
2024-T3 Al. alloy	90.7– 94.7	3.8 – 4.9	1.2–1.8	0.3 – 0.9	0.5	0.5	0.25	0.1	0.15

Table 2 – Mechanical Properties of 7020 T7 and 2024 T3 Al alloys

Material	Tensile strength (σ_{ut}) MPa	Yield strength (σ_{ys}) MPa	Young's modulus (E) MPa	Poisson's ratio (v)	Plane Strain Fracture toughness (K_{IC}) MPa$\sqrt{m}$	Plane Stress Fracture toughness (K_C) MPa$\sqrt{m}$	Elongation
7020-T7 Al. alloy	352.14	314.7	70,000	0.33	50.12	236.8	21.54 % in 40 mm
2024-T3 Al. alloy	469	324	73,100	0.33	37.0	95.31	19 % in 12.7 mm

Table 3 – Load scenarios of the tested specimens

Alloy	F_{max} KN	F_{min} KN	a_i mm	a_f mm	W mm	B mm	σ_{max} MPa	σ_{min} MPa
7020 T7	8.89	0.89	16.1	30.29	51.88	6.19	27.68	2.768
2024 T3	7.20	0.72	15.4	37.40	51.90	6.47	21.44	2.144

Table 4 – Comparison of experimental and smoothed sets of fatigue growth data

A	B	C	D	E	F
No. of cycles (expt)	Crack length (expt) (mm)	Sp. Growth rate (calculated piecewise)	Crack length incremented by 0.055 mm	Sp. Growth rate (calculated from polynomial equation)	No. of cycles (calculated from equation)
66950	20.01	7.7E-06	20.005	8.45E-06	67590
67970	20.2	5.72E-06	20.225	8.6E-06	68871
68980	20.41	1.37E-05	20.445	8.74E-06	70116
70000	20.58	9.73E-06	20.665	8.88E-06	71328
71020	20.77	2.12E-05	20.885	9.03E-06	72508
72040	20.96	9.19E-06	21.105	9.19E-06	73656
73060	21.18	7.27E-06	21.325	9.35E-06	74773
74080	21.4	9E-06	21.545	9.52E-06	75859
75100	21.63	1.07E-05	21.765	9.7E-06	76914
76120	21.79	7.07E-06	21.985	9.89E-06	77939
77140	21.97	7.01E-06	22.205	1.01E-05	78932
78160	22.12	3.48E-06	22.425	1.03E-05	79895
78920	22.27	5.39E-06	22.59	1.05E-05	80596
79940	22.53	1.54E-05	22.81	1.08E-05	81503
80960	22.7	7.05E-06	23.03	1.11E-05	82379
81980	22.95	6.71E-06	23.25	1.14E-05	83223
83000	23.23	9.95E-06	23.47	1.17E-05	84036
84010	23.5	1.36E-05	23.69	1.21E-05	84816
85030	23.79	1.35E-05	23.91	1.25E-05	85565
86050	24.09	9.98E-06	24.13	1.29E-05	86283
87070	24.43	1.31E-05	24.35	1.34E-05	86969
88090	24.77	1.62E-05	24.57	1.39E-05	87624
88600	24.94	1.29E-05	24.68	1.42E-05	87940

Fig. 1 – Single Edge Notch (SEN) specimen geometry

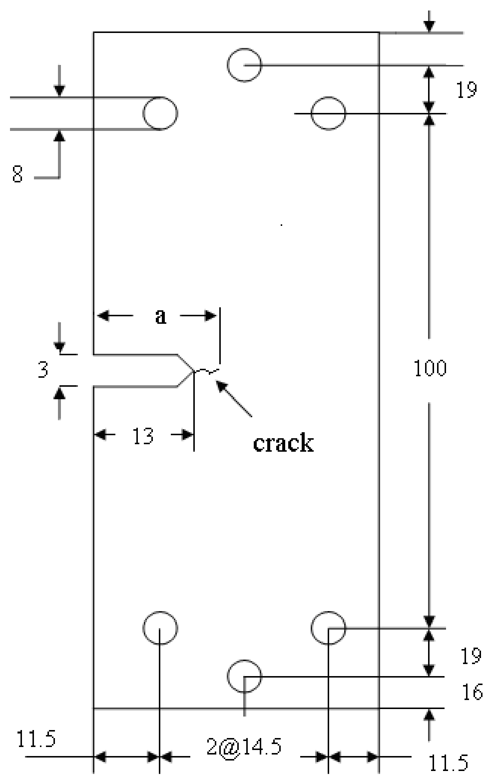

Fig. 2 – Specific crack growth rate vs. crack length

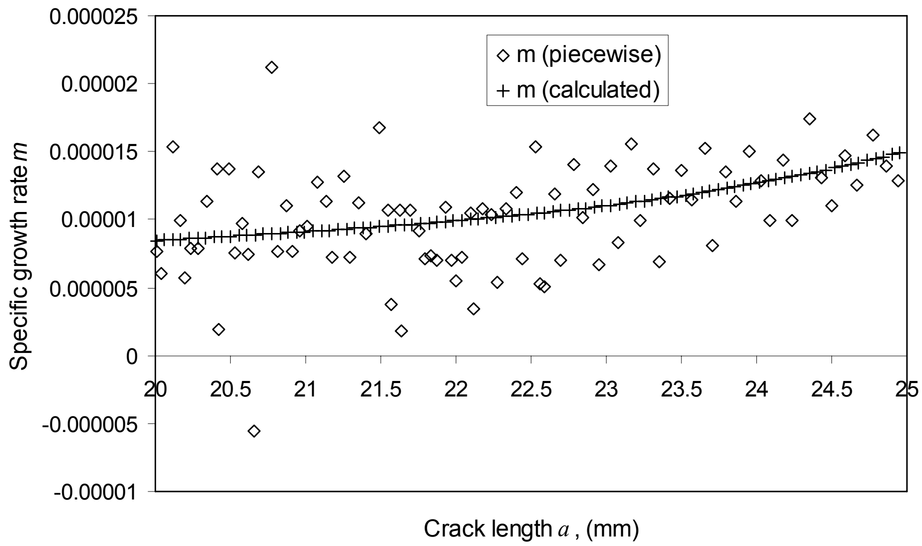

Fig. 3 – Comparison of exponential and polynomial crack growth rate (da/dN) with stress intensity factor range ΔK