Elimination of multi-response correlation while applying Taguchi philosophy in optimization of submerged arc weld

[1] **Dr. Saurav Datta** [2] **Prof. Siba Sankar Mahapatra** [3]**Prof. Asish Bandyopadhyay**

[1]Assistant Professor, Department of Mechanical Engineering,
National Institute of Technology, Rourkela, Orissa-769008, India
Email: sdattaju@gmail.com

[2] Professor, Department of Mechanical Engineering,
National Institute of Technology, Rourkela, Orissa-769008, India
Email: mahapatrass2003@yahoo.com

[3] Professor, Department of Mechanical Engineering,
Jadavpur University, Kolkata-700075, India
Email: asishbanerjee@yahoo.com

Abstract

There are several bead geometry parameters which indicate quality of submerged arc weldment. These includes, bead height, penetration depth, bead width, percentage dilution etc. Achieving an optimal weld, with desired quality features, is really a challenging job. Because, these quality features are highly correlated and are expected to be influenced directly or indirectly by the direct effect of process parameters or their interactive effects (i.e. on process environment). However, the extents of significant influence of the process parameters are different for different responses. Therefore, SAW is a multi-factor, multi-objective metal fabrication process. It is desired that, an optimal weld should confirm lesser bead height and width, to reduce excess weld metal consumption; deeper penetration and higher parentage of dilution, in order to increase joint strength. Therefore, to solve such a multi-objective optimization problem, it is felt necessary to identify the optimal parametric combination, following which all objectives could be optimized simultaneously. In this context, it is essential to convert all the objective functions into an equivalent single objective function or overall representative function to meet desired multi-quality features of the weldment. The required multi-quality features may or may not be conflicting in nature. The representative single objective function, thus calculated, would be optimized finally. In the present work, Design of Experiment (DOE) with Taguchi L_{16} Orthogonal Array (OA) has been explored to produce 16 weld specimens on mild steel plates by SAW. Collected data related to weld bead geometry have been utilized for optimization. Principal Component Analysis (PCA) has been applied to eliminate correlation among the responses and to evaluate independent or uncorrelated quality indices called principal components. Based on quality loss of individual principal components with respect to the ideal condition, an overall grey relational grade of the weldment has been calculated to serve as the single objective function for optimization. Finally, Taguchi method has been adopted for searching optimal process condition to yield desired quality of weld bead geometry. Result of the aforesaid optimization procedure has been verified through confirmatory test. The study illustrates the detailed methodology of PCA based grey-Taguchi method and its effectiveness for multi-response optimization in SAW.

Key words: SAW, PCA, Taguchi method, overall grey relational grade

1. Introduction

Submerged arc welding (SAW) is a useful metal joining process in fabrication industry. Several process parameters influence directly or indirectly on various aspects of submerged arc weldment. Work, to a far extent, has already been done to study the effects of the parameters like voltage, current, electrode stick-out, wire feed rate and traverse speed on geometry and quality of the weld bead produced by submerged arc welding on mild steel. But the search is still being continued. Control of the above parameters, in a more precise manner, can essentially improve the quality of the weldment, enhance the possibility of increased deposition rate and economize the process of submerged arc welding.

In many of the cases, quality of the weld is left dependent on operators past experience and working skill. But, with the advent of automation, it is now possible to design a machine capable of selecting optimal process parameters to provide desired quality weld. However, this requires reliable data of knowledge.

Literature depicts that work has been explored on various aspects of modeling, simulation and process optimization in submerged arc welding [1, 2, 3, 4, 5] on mild steel and many other materials. The common approaches to tackle optimization problem in welding include multiple regression analysis, Response Surface Methodology (RSM), Artificial Neural Network (ANN) modeling and Taguchi method, [6, 7, 8, 9]. In most of the cases the optimization was carried out using single objective function. For a multi-response process, while applying the optimal setting of control factors, it can be observed that, an increase and (or) improvement of one response may cause change in another response value, beyond the acceptable tolerance limit. Thus for solving a multi-criteria optimization problem, it is convenient to convert all the objectives into an equivalent single objective function. This equivalent objective function, which is the representative of all the quality characteristics of the product, is to be optimized finally.

Taguchi's philosophy, developed by Dr. Genichi Taguchi, a Japanese quality management consultant, is an efficient tool available for the design of high quality manufacturing system. However, traditional Taguchi method cannot solve a multi-objective optimization problem [10]. Therefore, Taguchi method coupled with grey relational analysis has been firmly recommended in literature [11]. In this method, a multiple response process optimization problem is converted to a single response optimization problem where overall grey relational grade serves as the single objective function or response function to be optimized (maximized). Tarng, Y. S. *et al.* [12] applied grey-based Taguchi methods for optimization of Submerged Arc Welding process parameters in hardfacing. They considered multiple weld qualities and determined optimal process parameters based on grey relational grade from grey relational analysis proposed by Taguchi method.

The disadvantage of grey based Taguchi approach is the unrealistic assumption of non-existence of correlation among the responses and they are treated as uncorrelated or independent. To overcome these shortcomings, the present study explores the use of Principal Component Analysis (PCA) to convert correlated responses into uncorrelated quality indices called principal components. PCA is an efficient statistical technique while studying multi-quality characteristics; those are highly correlated [13, 14]. The PCA allows data which contain information of multi-quality characteristics to be converted into several independent quality indicators. Part of these indicators is then selected to construct a composite quality indicator, which is the representative of multi-quality features of the process output [13].

In the present work, PCA has been used to eliminate correlation among the responses and to evaluate independent quality indicators called principal components. These have been accumulated to calculate the overall grey relational grade which is for replacement of four correlated responses viz. bead height, depth of penetration, bead width and percentage dilution of submerged arc weld. Finally, Taguchi method has been applied to search an optimal process condition by optimizing the overall grey relational grade.

2. Principal Component Analysis (PCA)

Principal Component Analysis (PCA) is a way of identifying patterns in data, and expressing the data in such a way so as to highlight their similarities and differences. The main advantage of PCA is that once the patterns in data have been identified, the data can be compressed, i.e. by reducing the number of dimensions, without much loss of information. The methods involved in PCA are discussed below, [15, 16]:

1. Getting some data
2. Normalization of data
3. Calculation of covariance matrix.
4. Interpretation of covariance matrix.

The normalized data have then been utilized to construct a variance-covariance matrix M , which is illustrated as below:

$$M = \begin{bmatrix} N_{1,1} & N_{1,2} & . & . & . & N_{1,u} \\ N_{2,1} & N_{2,2} & . & . & . & N_{2,p} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ N_{q,1} & N_{q,2} & . & . & . & N_{q,p} \end{bmatrix} \tag{1}$$

Here $N_{k,l} = \dfrac{Cov(Y^*_{i,k}, Y^*_{i,l})}{\sqrt{Var(Y^*_{i,k})Var(Y^*_{i,l})}}$ (2)

In which u stands for the number of quality characteristics and p stands for the number of experimental runs. Then, eigenvectors and Eigenvalues of matrix M can be computed, which are denoted by $\overline{V}_j$ and λ_j respectively.

In PCA the eigenvector $\overline{V}_j$ represents the weighting factor of j number of quality characteristics of the jth principal component. For example, if Q_j represents the jth quality characteristic, the jth principal component ψ_j can be treated as a quality indicator with the required quality characteristic.

$$\psi_j = V_{1j}Q_1 + V_{2j}Q_2 + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + V_{jj}Q_j = \overline{V}_j^{'}\overline{Q} \qquad (3)$$

It is to be noted that every principal component ψ_j represents a certain degree of explanation of the variation of quality characteristics, namely the accountability proportion (AP). When several principal components are accumulated, it increases the accountability proportion of quality characteristics. This is denoted as cumulative accountability proportion (CAP). In the present work, the composite principal component ψ has been defined as the sum/ linear combination of principal components with their individual Eigenvalues. Thus, the composite principal component represents the overall quality indicator as shown below:

$$\psi = \sum_{j=1}^{k} \psi_j \qquad (4)$$

If a quality characteristic Q_j strongly dominates in the jth principal component, this principal component becomes the major indicator of such a quality characteristic. It should be noted that one quality indicator may often represent all the multi-quality characteristics. Selection of individual principal components (ψ_j), those to be included in the composite quality indicator ψ, depends on their individual accountability proportion.

3. Experimentation and data collection

Bead-on-plate SAW welding on mild steel plates (thickness 10 mm) has been carried out as per Taguchi's L_{16} Orthogonal Array design with 16 combinations of voltage (OCV), wire feed rate, traverse speed and stick-out to be varied in four discrete levels within the scope for factorial adjustments in the setup used. Table 1 represents domain of experiments. Design of experiment is furnished in Table 2. Domain of experiment has been selected from the knowledge of the work carried out by Gunaraj and Murugan [2, 3]. Copper coated electrode wire of diameter 3.16 mm (AWS A/S 5.17:EH14) has been used during the experiments. Welding has been performed with flux (AWS A5.17/SFA 5.17) with grain size 0.2 to 1.6 mm with basicity index 1.6 (Al_2O_3+MnO_2 35%, CaO+MgO 25% and SiO_2+TiO_2 20% and CaF_2 15%). The experiments have been performed on Submerged Arc Welding Machine- INDARC AUTOWELD MAJOR Maker: IOL Ltd., India. Bead geometry (macrostructure) has been observed in Optical Trinocular Metallurgical Microscope (Make: Leica, GERMANY, Model No. DMLM, S6D & DFC320 and Q win Software). It consists of an image analyzer. The macrograph is obtained on the computer interface which has the provision of selecting two points or some region in order to calculate relative distance between said points as well as area of the region under consideration respectively. Like this way bead height, penetration depth and bead width have been calculated. %dilution is the ratio of area of penetration to the area of reinforcement measured on a bead cross section. These areas have been obtained from the said microscope and their ratio has been computed accordingly. The experimental data of different quality indicators relating bead geometry have been listed in Table 2. Aforesaid data bank has been utilized in PCA based hybrid Taguchi method for identification of optimal factors setting.

Table 1: Domain of experiment (limits of factors)

Parameters	Unit	Notation	Level 1	Level 2	Level 3	Level 4
OCV	Volt	V	32	34	36	38
Wire feed rate	m/min	Wf	0.93	1.16	1.39	1.62
Traverse speed	m/min	Tr	0.51	0.59	0.67	0.75
Stick-out	mm	N	30	32	34	36

Table 2: Design of experiment and collected response data

	L_{16} OA				Response values related to bead geometry			
Sl. No.	V	Wf	Tr	N	Bead height (mm)	Penetration depth (mm)	Bead width (mm)	Dilution (%)
1	1	1	1	1	1.1730	2.1990	10.6520	46.0090
2	1	2	2	2	1.9370	2.4970	9.4280	46.8320
3	1	3	3	3	2.7510	2.6570	9.3560	45.6350
4	1	4	4	4	3.6150	2.6790	10.4360	42.4180
5	2	1	2	3	1.6220	2.3110	10.2090	43.5030
6	2	2	1	4	2.5120	1.9800	11.4400	45.1840
7	2	3	4	1	1.2080	2.5830	11.2030	49.5450
8	2	4	3	2	2.0980	3.4800	10.1300	47.1860
9	3	1	3	4	1.8170	1.4490	10.0140	44.7810
10	3	2	4	3	1.9420	2.5560	8.9400	48.8120
11	3	3	1	2	1.8930	3.6330	12.7580	52.5390
12	3	4	2	1	1.3420	3.9880	12.2360	52.5300
13	4	1	4	2	2.3210	1.9190	7.9230	53.0510
14	4	2	3	1	1.6580	2.7350	10.4280	57.9400
15	4	3	2	4	1.8850	1.9090	11.2330	54.3930
16	4	4	1	3	1.7980	3.4770	13.1860	55.2420

2. Methodology for optimization

Assuming, the number of experimental runs in Taguchi's OA design is m, and the number of quality characteristics is n. The experimental results can be expressed by the following series: $X_1, X_2, X_3, ..., X_i, ..., X_m$

Here,

$$X_1 = \{X_1(1), X_1(2).........X_1(k).....X_1(n)\}$$
.
.
.
.
$$X_i = \{X_i(1), X_i(2).........X_i(k).....X_i(n)\}$$
.
.
.
.
$$X_m = \{X_m(1), X_m(2).........X_m(k).....X_m(n)\}$$

Here, X_i represents the $i\,th$ experimental results and is called the comparative sequence in grey relational analysis.

Let, X_0 be the reference sequence:

Let, $X_0 = \{X_0(1), X_0(2).........X_0(k).....X_0(n)\}$

The value of the elements in the reference sequence means the optimal value of the corresponding quality characteristic. X_0 and X_i both includes n elements, and $X_0(k)$ and $X_i(k)$ represent the numeric value of $k\,th$ element in the reference sequence and the comparative sequence, respectively, $k = 1, 2, ..., n$. The following illustrates the proposed parameter optimization procedures in detail [18].

Step 1: Normalization of the responses (quality characteristics)

When the range of the series is too large or the optimal value of a quality characteristic is too enormous, it will cause the influence of some factors to be ignored. The original experimental data must be normalized to eliminate such effect. There are three different types of data normalization according to whether we require the LB (lower-the-better), the HB (higher-the-better) and NB (nominal-the-best). The normalization is taken by the following equations.

(a) LB (lower-the-better)

$$X_i^*(k) = \frac{\min X_i(k)}{X_i(k)} \tag{5}$$

(b) HB (higher-the-better)

$$X_i^*(k) = \frac{X_i(k)}{\max X_i(k)} \tag{6}$$

(c) NB (nominal-the-best)

$$X_i^*(k) = \frac{\min\{X_i(k), X_{0b}(k)\}}{\max\{X_i(k), X_{0b}(k)\}} \tag{7}$$

Here, $\begin{array}{l} i = 1, 2, ..., m; \\ k = 1, 2, ..., n \end{array}$

$X_i^*(k)$ is the normalized data of the $k\,th$ element in the $i\,th$ sequence.

$X_{0b}(k)$ is the desired value of the $k\,th$ quality characteristic. After data normalization, the value of $X_i^*(k)$ will be between 0 and 1. The series $X_i^*, i = 1, 2, 3, ..., m.$ can be viewed as the comparative sequence used in the grey relational analysis.

Step 2: Checking for correlation between two quality characteristics

Let,
$$Q_i = \{X_0^*(i), X_1^*(i), X_2^*(i), \ldots\ldots\ldots\ldots, X_m^*(i)\} \quad where,\ i = 1, 2, \ldots\ldots., n. \tag{8}$$

It is the normalized series of the $i\,th$ quality characteristic. The correlation coefficient between two quality characteristics is calculated by the following equation:

$$\rho_{jk} = \frac{Cov(Q_j, Q_k)}{\sigma_{Q_j} \times \sigma_{Q_k}}, \tag{9}$$

here, $j = 1, 2, 3\ldots\ldots, n.$ $k = 1, 2, 3, \ldots\ldots.., n.,$ $j \neq k$

Here, ρ_{jk} is the correlation coefficient between quality characteristic j and quality characteristic k; $Cov(Q_j, Q_k)$ is the covariance of quality characteristic j and quality characteristic k; σ_{Q_j} and σ_{Q_k} are the standard deviation of quality characteristic j and quality characteristic k, respectively.

The correlation is checked by testing the following hypothesis:

$$\begin{cases} H_0 : \rho_{jk} = 0 & (There\ is\ no\ correlation) \\ H_1 : \rho_{jk} \neq 0 & (There\ is\ correlation) \end{cases} \tag{10}$$

Step 3: Calculation of the principal component score

(a) Calculate the Eigenvalue λ_k and the corresponding eigenvector $\beta_k\ (k = 1, 2, \ldots\ldots, n)$ from the correlation matrix formed by all quality characteristics.

(b) Calculate the principal component scores of the normalized reference sequence and comparative sequences using the equation shown below:

$$Y_i(k) = \sum_{j=1}^{n} X_i^*(j)\beta_{kj}, \quad i = 0,1,2,.......,m; k = 1,2,........,n. \tag{11}$$

Here, $Y_i(k)$ is the principal component score of the $k\,th$ element in the $i\,th$ series. $X_i^*(j)$ is the normalized value of the $j\,th$ element in the $i\,th$ sequence, and β_{kj} is the $j\,th$ element of eigenvector β_k.

Step 4: Calculation of the individual grey relational grades

Calculation of the individual grey relational coefficients

Use the following equation to calculate the grey relational coefficient between $X_0(k)$ and $X_i(k)$.

$$r_{0,i}(k) = \frac{\Delta_{\min} + \xi\Delta_{\max}}{\Delta_{0,i}(k) + \xi\Delta_{\max}}, \quad i = 1,\ 2,..........,m; \quad k = 1,\ 2,.....,n. \tag{12}$$

Here, $r_{0,i}(k)$ is the relative difference of $k\,th$ element between sequence X_i and the comparative sequence X_0 (also called grey relational grade), and $\Delta_{0,i}(k)$ is the absolute value of difference between $X_0(k)$ and $X_i(k)$.

$$\Delta_{0,i}(k) = \begin{cases} \left|X_0^*(k) - X_i^*(k)\right|, & \textit{no significant correlation between quality characteristics} \\ \left|Y_0(k) - Y_i(k)\right|, & \textit{Significant correlation between quality characteristics} \end{cases} \tag{13}$$

$$\Delta_{\max} = \begin{cases} \max\limits_{i} \max\limits_{k} \left|X_0^*(k) - X_i^*(k)\right|, & \textit{no significant correlation between quality characteristics} \\ \max\limits_{i} \max\limits_{k} \left|Y_0(k) - Y_i(k)\right|, & \textit{Significant correlation between quality characteristics} \end{cases} \tag{14}$$

$$\Delta_{\min} = \begin{cases} \min\limits_{i} \min\limits_{k} \left| X_0^*(k) - X_i^*(k) \right|, \textit{no significant correlation between quality characteristics} \\ \min\limits_{i} \min\limits_{k} \left| Y_0(k) - Y_i(k) \right|, \quad \textit{Significant correlation between quality characteristics} \end{cases}$$

(15)

Note that ξ is called the distinguishing coefficient, and its value is in between 0 to 1. In general it is set to 0.5, [17].

(5) Calculation of the overall grey relational grade

After the calculation of the grey relational coefficient and the weight of each quality characteristic, the grey relational grade is determined by:

$$\Gamma_{0,i} = \sum_{k=1}^{n} w_k r_{0,i}(k), \qquad i = 1, 2, ..., m. \tag{16}$$

In this paper, the multiple quality characteristics are combined to one grey relational grade, thus the traditional Taguchi method can be used to evaluate the optimal parameter combination. Finally the anticipated optimal process parameters are verified by carrying out the confirmatory experiments.

(6) Optimization using Taguchi method

The overall grey relational grade is then optimized (maximized) using Taguchi method [11]. Taguchi's HB (Higher-the-Better) criterion has been explored to maximize the overall grey relational grade (Equation 17).

$$SN\,(Higher - the - better) = -10\log\left[\frac{1}{t}\sum\nolimits_{i=1}^{t}\frac{1}{y_i^2}\right] \tag{17}$$

Here t is the number of measurements, and y_i the measured $i\,th$ characteristic value i.e. $i\,th$ quality indicator.

4. Evaluation of optimal setting

Experimental data (Table 2) have been normalized first. Normalized data have been furnished in Table 3. For reinforcement, bead width Lower-the-Better (LB) and for penetration, percentage dilution Higher-the-Better (HB) criterion has been selected (Equation 5 and 6 respectively). After normalization the data have been checked for correlation. Table 4 shows existence of correlation among the responses (coefficient of correlation evaluated using Equation (8) and (9) became non-zero value). Principal component analysis has been applied to eliminate correlation among the responses. Table 5 represents results of analysis of correlation matrix. Correlated responses have been converted to four independent quality indices denoted as principal components: ψ_1, ψ_2, ψ_3 and ψ_4 respectively by using equation 11. Principal components in all L_{16} OA experimental observations have been shown in Table 6. Quality loss estimates $\Delta_{0i}(k)$ for all principal components have been computed using equations 13, 14 and 15. The values have been furnished in Table 7. Individual grey relational coefficients have been calculated using equation 12.

Table 3: Normalized data

	Normalized Response data related to bead geometry			
Sl. No.	Bead height	Penetration depth	Bead width	Dilution
Ideal condition	1.0000	1.0000	1.0000	1.0000
1	1.0000	0.5514	0.7438	0.7941
2	0.6056	0.6261	0.8404	0.8083
3	0.4264	0.6662	0.8468	0.7876
4	0.3245	0.6718	0.7592	0.7321
5	0.7232	0.5795	0.7761	0.7508
6	0.4670	0.4965	0.6926	0.7798
7	0.9710	0.6477	0.7072	0.8551
8	0.5591	0.8726	0.7821	0.8144
9	0.6456	0.3633	0.7912	0.7729
10	0.6040	0.6409	0.8862	0.8425

11	0.6197	0.9110	0.6210	0.9068
12	0.8741	1.0000	0.6475	0.9066
13	0.5054	0.4812	1.0000	0.9156
14	0.7075	0.6858	0.7598	1.0000
15	0.6223	0.4787	0.7053	0.9388
16	0.6524	0.8719	0.6009	0.9534

Table 4: Test for correlation among the responses

Sl. No.	Correlation between responses	Coefficient of correlation	Comment
1	Penetration and reinforcement	+0.293	Both are correlated
2	Penetration and bead width	-0.171	Both are correlated
3	Penetration and dilution	+0.462	Both are correlated
4	Reinforcement and bead width	-0.038	Both are correlated
5	Reinforcement and dilution	+0.384	Both are correlated
6	Bead width and dilution	+0.035	Both are correlated

Table 5: (Analysis of correlation matrix)
Eigenvalues, eigenvectors, accountability proportion (AP) and cumulative accountability proportion (CAP) computed for the four major quality indicators

	ψ_1	ψ_2	ψ_3	ψ_4
Eigenvalue	0.051316	0.026943	0.012819	0.004896
Eigenvector	$\begin{vmatrix} -0.720 \\ -0.653 \\ 0.079 \\ -0.219 \end{vmatrix}$	$\begin{vmatrix} 0.680 \\ -0.716 \\ 0.153 \\ -0.047 \end{vmatrix}$	$\begin{vmatrix} -0.065 \\ 0.132 \\ 0.975 \\ 0.169 \end{vmatrix}$	$\begin{vmatrix} 0.120 \\ 0.207 \\ 0.146 \\ -0.960 \end{vmatrix}$
AP	0.536	0.281	0.134	0.051
CAP	0.536	0.815	0.949	1.000

Table 6: Principal components in all L_{16} OA experimental observations

Sl. No.	Principal components ψ_1 (1st PC) to ψ_4 (4th PC)			
	(1st PC) ψ_1	(2nd PC) ψ_2	(3rd PC) ψ_3	(4th PC) ψ_4
Ideal condition	-1.5130	0.0680	1.2110	-0.4870
1	-1.1952	0.3602	0.8672	-0.4196
2	-0.9555	0.0524	0.9993	-0.4510
3	-0.8476	-0.0962	1.0190	-0.4434
4	-0.7727	-0.1801	0.9315	-0.4140
5	-1.0022	0.1588	0.9131	-0.4007

6	-0.7765	0.0300	0.8423	-0.4887
7	-1.2535	0.2631	0.8564	-0.4671
8	-1.0889	-0.1648	0.9790	-0.4199
9	-0.8088	0.2620	0.9080	-0.4738
10	-0.9679	0.0461	1.0518	-0.4743
11	-1.1906	-0.1797	0.8387	-0.5163
12	-1.4297	-0.0664	0.8597	-0.4639
13	-0.7996	0.1071	1.1604	-0.5727
14	-1.1162	0.0578	0.9543	-0.6222
15	-0.9105	0.1428	0.8691	-0.6245
16	-1.2004	-0.1347	0.8197	-0.5688

Overall grey relational grade has been calculated using equation 16. While calculating overall grey relational grade it has been assumed that all responses are equally important. Therefore 25% weightage has been assigned to prioritize four responses. Table 9 shows overall grey relational grade for all experimental runs and corresponding S/N ratio calculated using Taguchi's Higher-the-Better (HB) criterion (Equation 17). Optimal parameter setting has been determined from Figure 1. The predicted optimal setting becomes **V3 Wf2 Tr4 N2**.

Table 10 represents mean values of overall grey relational grade. It indicates the order of factors (ranking) representing the extent of significance on the overall grey relational grade. After evaluating the optimal parameter settings, the next step is to predict and verify the enhancement of quality characteristics using the optimal parametric combination. Table 11 reflects the satisfactory result of confirmatory experiment.

Table 7: Calculation of $\Delta_{0i}(k)$ for all principal components

Sl. No.	$\Delta_{0i}(1st\ PC)$	$\Delta_{0i}(2nd\ PC)$	$\Delta_{0i}(3rd\ PC)$	$\Delta_{0i}(4th\ PC)$
1	0.3178	0.2922	0.3438	0.0674
2	0.5575	0.0156	0.2117	0.0360
3	0.6654	0.1642	0.1920	0.0436
4	0.7403	0.2481	0.2795	0.0730
5	0.5108	0.0908	0.2979	0.0863

6	0.7365	0.0380	0.3687	0.0017
7	0.2595	0.1951	0.3546	0.0199
8	0.4241	0.2328	0.2320	0.0671
9	0.7042	0.1940	0.3030	0.0132
10	0.5451	0.0219	0.1592	0.0127
11	0.3224	0.2477	0.3723	0.0299
12	0.0833	0.1344	0.3513	0.0231
13	0.7134	0.0391	0.0506	0.0857
14	0.3968	0.0102	0.2567	0.1352
15	0.6025	0.0748	0.3419	0.1375
16	0.3126	0.2027	0.3913	0.0818

5. Conclusion

Grey based Taguchi method is generally adopted for solving multi-attribute decision making problems (multi-response optimization). The method is based on the assumption that all response features must be uncorrelated or independent. However, this assumption may not be valid in practical situation. For example, in the present case, it has been observed that quality features thus selected viz. bead height, penetration depth and bead width are highly correlated with %dilution. So, traditional grey-Taguchi technique fails to overcome this problem. In order to solve this shortcoming the present work highlights application of Principal Component Analysis (PCA) in combination with Taguchi technique to solve this correlated multi-response optimization problem. The correlated quality indices have been transformed into uncorrelated independent quality indices called principal components. These independent multi-indices have been accumulated to calculate the composite principal component which has been optimized finally by Taguchi technique. From the foregoing study the following conclusions can also be made.

1. PCA can provide a representative quality indicator which can replace correlated quality characteristics of the process output.

2. Grey based Taguchi method coupled has been found appropriate to tackle a multi-objective optimization problem with PCA to eliminate response correlation and to convert a multi-objective optimization problem to a single response optimization by accumulating the principal components into the overall grey relational grade.

3. It can be recommended that the PCA based hybrid Taguchi method is good, for example, in case of processes (chemical and pharmaceutical) industries when there are hundreds of response variables.

Table 8: Calculation of individual grey relational coefficients

Sl. No.	Individual grey relational coefficients			
	(1*st PC*)	(2*nd PC*)	(3*rd PC*)	(4*th PC*)
1	0.6591	0.3566	0.4565	0.5174
2	0.4888	0.9668	0.6045	0.6725
3	0.4379	0.5037	0.6352	0.6270
4	0.4083	0.3965	0.5183	0.4969
5	0.5147	0.6599	0.4989	0.4544
6	0.4098	0.8490	0.4363	1.0000
7	0.7201	0.4581	0.4475	0.7943
8	0.5709	0.4125	0.5759	0.5187
9	0.4221	0.4595	0.4939	0.8596
10	0.4954	0.9301	0.6939	0.8646
11	0.6548	0.3969	0.4336	0.7140
12	1.0000	0.5571	0.4502	0.7671
13	0.4185	0.8440	1.0000	0.4561
14	0.5912	1.0000	0.5444	0.3454
15	0.4662	0.7076	0.4581	0.3416
16	0.6642	0.4481	0.4195	0.4681

Table 9: Calculation of overall grey relational grade and corresponding S/N ratio

Sl. No.	$\Gamma_{0,i}$	S/N Ratio
1	0.4974	-6.06588
2	0.6832	-3.30904
3	0.5509	-5.17854
4	0.4550	-6.83977
5	0.5320	-5.48177
6	0.6739	-3.42809
7	0.6050	-4.36489

8	0.5195	-5.68829
9	0.5588	-5.05487
10	0.7460	-2.54522
11	0.5498	-5.19591
12	0.6936	-3.17782
13	0.6796	-3.35493
14	0.6203	-4.14796
15	0.4934	-6.13602
16	0.5000	-6.02060

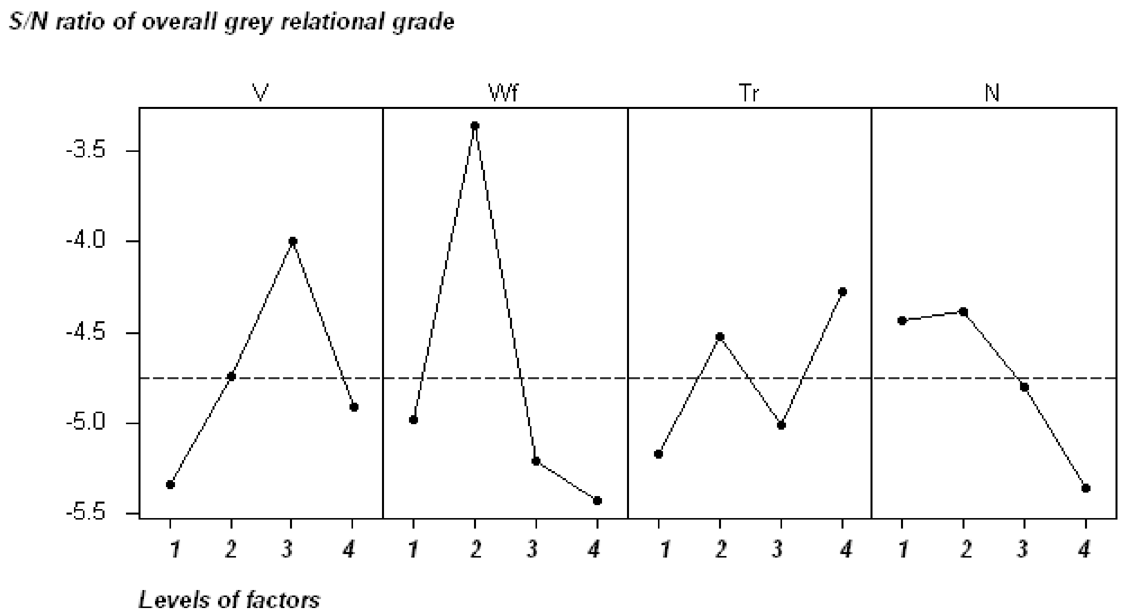

Figure 1: S/N ratio plot for overall grey relational grade

Table 10: Mean value table of overall grey relational grade

Level	V	Wf	Tr	N
1	0.546625	0.566950	0.555275	0.604075
2	0.582600	0.680850	0.600550	0.608025
3	0.637050	0.549775	0.562375	0.582225
4	0.573325	0.542025	0.621400	0.545275
Delta	0.090425	0.138825	0.066125	0.062750
Rank	2	1	3	4

Table 11: Results of confirmatory experiment

	Optimal setting	
	Prediction	Experiment
Level of factors	**V3 Wf2 Tr4 N2**	**V3 Wf2 Tr4 N2**
S/N ratio of Overall grey relational grade	-1.76623	-1.75600
Overall grey relational grade	0.91495	0.91594

5. References

1. Gupta, S. R. and Arora, N., 1993, "Influence of Flux Basicity on Weld Bead Geometry and HAZ in Submerged Arc Welding", Journal of Material Processing Technology, Volume 39, Issues 1-2, pp. 33-42.
2. Gunaraj, V. and Murugan, N., 1999, "Application of Response Surface Methodology for Predicting Weld Bead Quality in Submerged Arc Welding of Pipes"; Journal of Materials Processing Technology, Volume 88, pp.266-275.
3. Gunaraj, V. and Murugan, N., 1999, "Prediction and Comparison of the Area of the Heat Affected Zone for the Bead-no-Plate and Bead-on-Joint in Submerged Arc Welding of Pipes", Journal of Materials Processing Technology, Volume 95, Issues 1-3, pp. 246-261.
4. Gunaraj, V. and Murugan, N., October 2000, "Prediction and Optimization of Weld Bead Volume for the Submerged Arc Process- Part-1", Welding Research Supplement, pp. 286s-294s.
5. Gunaraj, V. and Murugan, N., November 2000, "Prediction and Optimization of Weld Bead Volume for the Submerged Arc Process- Part-2", Welding Research Supplement, pp. 331s-338s.

6. Unal, R., and Dean, Edwin B., 1991, "Taguchi Approach to Design Optimization for Quality and Cost: An Overview", Presented at the Annual Conference of the International Society of Parametric Analysis".
7. Rowlands, H., Antony, J. and Knowles, G., 2000, "An Application of Experimental Design For Process Optimization", The TQM Magazine, Volume 12, Number 2, pp. 78-83.
8. Antony, J. and Antony, F., 2001, "Teaching the Taguchi Method to Industrial Engineers", Work Study, Volume 50, Number 4, pp. 141-149.
9. Maghsoodloo, S., Ozdemir, G., Jordan, V. and Huang, C-H., 2004, "Strengths and Limitations of Taguchi's Contributions to Quality, Manufacturing, and Process Engineering", Journal of Manufacturing Systems, Volume 23, Number 2, pp. 73-126.
10. Jeyapaul, R., Shahabudeen, P. and Krishnaiah, K., 2005, "Quality Management Research by Considering Multi-Response Problems in the Taguchi Method-A Review", International Journal of Advanced Manufacturing Technology, Volume 26, pp. 1331-1337.
11. Datta S, Bandyopadhyay A, Pal PK (2008) Grey based Taguchi method for optimization of bead geometry in submerged arc bead-on-plate welding. International Journal of Advanced Manufacturing Technology 39:1136-1143.
12. Tarng, Y. S., Juang, S. C., Chang, C. H., 2002, "The Use of Grey-Based Taguchi Methods to Determine Submerged Arc Welding Process Parameters in Hardfacing", Journal of Materials Processing Technology, Volume 128, pp. 1-6.

13. Antony J (2000) Multi-response optimization in industrial experiments using Taguchi's quality loss function and Principal Component Analysis. Quality and Reliability Engineering International 16:3-8.
14. Su CT, Tong LI (1997) Multi-response robust design by principal component analysis. Total Quality Management 8(6):409-416.
15. Johnson, Richard, A. and Wichern, Dean, W., "Applied Multivariate Statistical Analysis", Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632.
16. MINITAB (Release 13.1) State College, PA, USA. User manual.
17. Deng J (1989) Introduction to grey system. Journal of grey system 1(1):1-24.
18. Yu-Min Chiang and Hsin-Hsien Hsieh, (Article in press), The use of the Taguchi method with grey relational analysis to optimize the thin-film sputtering process with multiple quality characteristic in color filter manufacturing, Computers & Industrial Engineering, DOI:10.1016/j.cie.2007.12.020.