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SOME APPLICATIONS OF FUZZY LOGIC IN TRANSPORTATION ENGINEERING

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ABSTRACT

Fuzzy logic is able to express any natural phenomenon especially related to human decision making efficiently using linguistic variables, which is not promising using any mathematical expression. In this paper details of fuzzy sets and fuzzy inference are presented. Fuzzy logic proved its wide applications in many areas. A review of important applications of fuzzy logic in transportation engineering (involving traffic flow modeling, transportation planning, and traffic control mechanisms and so on) has been presented in this paper, from which it is observed that fuzzy logic is an important tool for understanding and solving various problems related to transportation engineering.

INTRODUCTION

"*The power (in a set–theoretic sense) of our thinking and feeling is much higher than the power of living language. If in turn we compare the power of a living language with the logical language, then we will find that logic is even poorer*." — Zimmermann [1].

From the above quotation it can be said that expressing any natural phenomenon by mathematical expressions does not always guarantee exact capturing of the phenomenon itself. So, if any phenomenon, especially related to human decision making can be expressed linguistically chance of proper capturing of the phenomenon increases. Fuzzy logic is such a logic, which is based on approximate reasoning and can be expressed linguistically to capture the inherent vagueness of human mind. So, it can be applied to the areas which involve human decision making like supervision, monitoring, planning, scheduling etc. In this Paper basic properties of fuzzy logic have been described in the beginning and then its applications to transportation engineering have been presented. Transportation engineering involves human decision making in terms of route choice, choice of speed etc. and some control mechanisms (as for example, signalized intersection) where prevailing situations are imprecise in nature and the outcome of any adopted control strategy is unknown. So, there are plenty of scopes of application of fuzzy logic in transportation engineering.

This paper is broadly divided into five sections, of which this is the first to introduce the matter. The next section discusses in detail about fuzzy sets; while, the third section is on fuzzy inference. The fourth section concentrates on the review of the case studies of applications of fuzzy logic in transportation engineering. Finally, in the fifth section conclusions on the theory of fuzzy logic and its applications to Transportation engineering problems are presented.

FUZZY SET

A set is a collection of similar elements having a common group property. When the belonging to the group is complete without any doubt, the set is called a classical or crisp set. A crisp set A can be defined like this:

$$A = \{x \mid x \in X\} \tag{1}$$

where, x is an element of the set and X is the common property of the set.

The concept of fuzzy set was introduced by Zadeh [2]. He defined fuzzy set as a "class of objects with a continuum of grades of membership." A fuzzy set is such kind of a set, where belonging to that group may not be complete. In a fuzzy set an element can belong to any group either completely or partially and can also belong to any other group partially. The difference between a crisp set and a fuzzy set lies in the nature of their boundary. In a crisp set, the boundary is crisp, i.e., well defined. Whereas, in a fuzzy set, the boundary is a vague region. The degree of belonging to a set is defined by a membership value, which is obtained using some membership function. For a crisp set, if an element belongs to it, the membership value is 1 and if does not it is 0. For a fuzzy set, it is any value between 0 to 1.

So, a fuzzy set $\widetilde{A}$ can be defined like this:

$$\widetilde{A} = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in X\} \tag{2}$$

where, $\mu_{\tilde{A}}$ is called the membership function or grade of membership or degree of compatibility or degree of truth of x in $\tilde{A}$ that maps X to the membership space $M(0 \le M \le 1)$ [1, 3].

Some Definitions

Support: If $\tilde{A}$ is a fuzzy set, then its support $S(\tilde{A})$ is a crisp set of all $x \in X$ having non–zero membership function values in $\tilde{A}$, i.e.,

$$S(\widetilde{A}) = \{(x, \mu_{\widetilde{A}}(x)) \mid x \in X; \mu_{\widetilde{A}}(x) > 0\} \tag{3}$$

α–level set: The crisp set which contains all the elements having membership function values $\ge \alpha$ of fuzzy set $\tilde{A}$, is α–level set or α–cut, i.e.,

$$A_{\alpha} = \{x \in X \mid \mu_{\widetilde{A}}(x) \ge \alpha\} \tag{4}$$

Height: Height of a fuzzy set $\tilde{A}$ is the highest membership function value among all of its elements, i.e.,

$$Hgt(\widetilde{A}) = \vee_{\forall X}[\mu_{\widetilde{A}}(x)] \tag{5}$$

Normality: For a fuzzy set $\tilde{A}$, if and only if there exists an element having complete belonging to the set, i.e., membership function value as 1, the fuzzy set is called normal.
Modality: For a fuzzy set $\tilde{A}$, the number of elements having membership function value as height of $\tilde{A}$, i.e., $Hgt(\tilde{A})$ is called its modality.
Cardinality: For a fuzzy set cardinality is the summation of belonging of all its elements, i.e., summation of membership function values of all its elements, i.e.,

$$|\widetilde{A}| = \sum_{x \in X} \mu_{\widetilde{A}}(x) \tag{6}$$

Convexity: A fuzzy set is said to be convex if and only if

$$\mu_{\widetilde{A}}.(\lambda.x_1 + (1.0 - \lambda).x_2 \ge Min\{\mu_{\widetilde{A}}(x_1), \mu_{\widetilde{A}}(x_2)\} \tag{7}$$

where, $x_1, x_2 \in X$ and $\lambda \in [0, 1]$.

Fuzzy number: A fuzzy number $\widetilde{M}$ is a convex, normal and uni–modal fuzzy set of the real line $\mathbf{R}$, having its membership function $\mu_{\tilde{A}}(x)$ piecewise continuous. If the constraint of uni–

modality is relaxed, i.e., the complete belonging is spread over an interval instead of a point it is called *fuzzy interval*.

Fuzzy Set Operations

The three basic set operations namely union, intersection and complement for fuzzy set are as follows:

Union: If there are two fuzzy sets $\widetilde{A}$ and $\widetilde{B}$, then their union $\widetilde{C}$, is such that the membership function of $\widetilde{C}$ is maximum between the membership functions of $\widetilde{A}$ and $\widetilde{B}$, i.e.,

$$\mu_{\widetilde{C}}(x) = Max\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)\} \tag{8}$$

Intersection: In the same line as union, membership function of the intersection of $\widetilde{A}$ and $\widetilde{B}$, say $\widetilde{D}$ is such that the membership function of $\widetilde{D}$ is the minimum of the membership functions of $\widetilde{A}$ and $\widetilde{B}$, i.e.,

$$\mu_{\widetilde{D}}(x) = Min\{\mu_{\widetilde{A}}(x), \mu_{\widetilde{B}}(x)\} \tag{9}$$

Complement: For a fuzzy set $\widetilde{A}$, its complement $\overline{\widetilde{A}}$ is such that,

$$\mu_{\overline{\widetilde{A}}}(x) = 1 - \mu_{\widetilde{A}}(x) \tag{10}$$

As stated before boundary of $\widetilde{A}$ is not crisp, so boundary of $\overline{\widetilde{A}}$ is also not crisp. Thus, $\widetilde{A}$ and $\overline{\widetilde{A}}$ are not mutually exclusive to each other. So,

$$\widetilde{A} \cap \overline{\widetilde{A}} = \phi \tag{11}$$

where, ϕ is the Null set.

Fuzzy Arithmetic

The arithmetic applicable to fuzzy numbers is referred to as fuzzy arithmetic. Arithmetics namely addition, subtraction, multiplication and division are explained in [4]. Here, one arithmetic namely weighted average is described. If there are N number of fuzzy numbers, $\widetilde{M}_i$ (i = 1 to N), in $\boldsymbol{R}$, having weight w_i (say); then the weighted average is

$$\overline{M} = \frac{\sum_{i=1}^{N} w_i.\widetilde{M}_i}{\sum_{i=1}^{N} w_i} \tag{12}$$

FUZZY INFERENCE

Fuzzy logic deals with linguistic variables through approximate reasoning. The rule structure is based on implications like "if A is the scenario then B is the action". Now, for the decision maker if the scenario is A, he will take the action B. If the prevailing scenario is in between any two defined in the rule base, the action will be in between the two corresponding actions [1, 5]. Fuzzy inference is based on approximate reasoning. According to Zadeh, fuzzy inference is "the process or processes by which a possibly imprecise conclusion is deduced from a collection of imprecise premises" [6]. Before going to describe about approximate reasoning it is necessary to describe the constituents of the fuzzy rule base.

Proposition and Truth Value

According to classical logic proposition is a statement which is either true or false. Fuzzy logic is the generalized version of classical logic. Fuzzy logic uses infinite logic variables including true and false. The rest lie in between, i.e., having some degree of truth or truth value. This truth value lies in [0, 1]. If the proposition is true, the value is 1 and if it is false the value is 0. Any value in between indicates its degree of truth.

Logical Connectives

Multiple propositions are connected by 'logical connectives'. The term AND and OR are two logical connectives. These connectives in fuzzy logic can be defined like this:

$$|\, x_1 \text{ is } A_1 \text{ AND } x_2 \text{ is } A_2 \,| = |\, x_1 \text{ is } A_1 \,| \wedge |\, x_2 \text{ is } A_2 \,| \text{ and}$$
$$|\, x_1 \text{ is } A_1 \text{ OR } x_2 \text{ is } A_2 \,| = |\, x_1 \text{ is } A_1 \,| \vee |\, x_2 \text{ is } A_2 \,| \qquad (13)$$

where, "x_i is A_i" is a proposition, and $\wedge$ and $\vee$ denote the MIN and MAX operators, respectively.

Premise Variable

In fuzzy logic, the proposition/s representing the prevailing condition/s is represented as a linguistic variable, named as premise variable. It certainly can carry a value with it. But, that value does not always guarantee its exact grouping.

Consequence Variable

This represents the course of action corresponding to a particular combination of premise variables. It is a fuzzy number representing the approximate value of the course of action. This fuzzy number is approximately equal to a crisp value.

Implication and Reasoning

In the fuzzy rule based inference system, more than one propositions (which are captured by the premise variables) are connected by logical connectives to define the course of action (which is captured by the consequence variable). For a particular set of premise variables, there is a unique value of consequence variable connected by a fuzzy rule denoting a particular corresponding action. This action is the conclusion of the fuzzy logic. If the *prevailing conditions* (*Input*) are such that, satisfy the compatibility with the *premise variables* of any particular rule; the inferred *course of action* (*Output*) will be the same as that defined by the *consequence variable* of that particular rule (see Figure 1). But, if the prevailing conditions are such that, satisfy the compatibility with the premise variables of more than one rule, the course of action is determined by the weighted average of the consequence variables of all those rules.

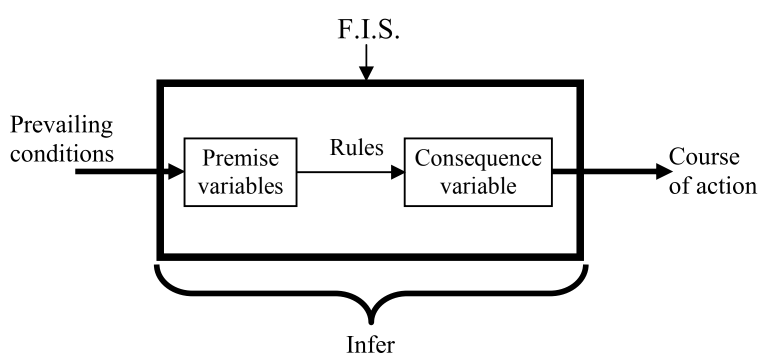

Figure 1: Fuzzy inference system

Details of fuzzy logic can be obtained from [1, 4, 5].

APPLICATIONS OF FUZZY LOGIC IN TRANSPORTATION ENGINEERING

Transportation engineering, when concentrated on road transportation is concerned about construction and maintenance of roads and movement of vehicles therein. The second part, i.e., movement of vehicles requires scheduling and routing of vehicles and a lot of control mechanisms. These all depend on availability of resources (like, number of vehicles available, whether proper electronic devices are available for signal controlling at intersection or not etc.) and the demand of people for transportation. For a particular resource availability condition, the task is to effort for catering the demand efficiently. For this it is of utmost necessity to have proper control mechanism and scheduling and routing strategy. Most importantly, it is necessary to understand the motion of vehicles on roads, i.e., the traffic flow theory. Various models have been proposed to understand traffic flow using various methods till now. On the operation side, lots of problems are encountered everyday, namely, delay, traffic jam, accidents and so on. Studies have been conducted on analysis of delay and queue on roads and also on reasons and mitigations of accidents.

There are some areas in transportation engineering where imprecise perception involves. One of them is modeling the behavior of drivers (which is a human thought process). Modeling driver behavior involves to predict how a driver of a particular vehicle responds (in terms acceleration) according to the behavior of other drivers (in terms acceleration) and the environment. This environment may be static (roadway features) as well as dynamic (other vehicles). So, fuzzy logic has been applied into modeling driver behavior, which is imprecise in nature. Fuzzy logic has also been applied into transportation planning for deciding mode choice and route choice. Control devices, like signal control at signalized intersection (where the perception of the condition is not clear and the consequence of decision is not known) are suitable places for applicability of fuzzy logic.
Fuzzy logic has been applied into traffic modeling, transportation planning, traffic control and some more areas in transportation engineering. In the modeling and planning side fuzzy inference system has been developed to capture the imprecise decision making of drivers; whereas, in the control and application side, the concept of fuzzy logic has been implemented into machines.

Application of Fuzzy Logic in Traffic Flow Modeling: Car Following Behavior

Modeling traffic flow involves studying the behavior of vehicles in response to the behavior of other vehicles. Generally, a vehicle considers the vehicle just ahead and behaves accordingly. This behavior is called car following behavior, which is observed during congested traffic flow; because at free flow condition a driver need not to bother about the vehicles far ahead. Study of this behavior is necessary for adaptive cruise control for automated highway system (AHS). Some models have been proposed by various researchers to capture the car following behavior; among them one worth mentioning model is by General Motors (GM) [7]. This model uses the following equation to represent the behavior of the following vehicle (n+1$^{\text{th}}$ vehicle) in terms of acceleration (where, n^{th} vehicle is the leading vehicle)

$$\ddot{x}_{n+1}(t+\Delta t) = \{\frac{\alpha(l,m)(\dot{x}_{n+1}(t+\Delta t))^m}{(x_n(t)-x_{n+1}(t))^l}\}[\dot{x}_n(t)-\dot{x}_{n+1}(t)] \tag{14}$$

where, $x_i(t)$, $\dot{x}_i(t)$ and $\ddot{x}_i(t)$ are the distance (from some arbitrary upstream point), speed and acceleration/deceleration of the i^{th} vehicle at time t, respectively; Δt is the perception reaction time; l and m are some constants; and $\alpha(l,m)$ is another constant dependent on l and m.
This model because of its deterministic nature can not explain some of the properties of imprecise decision making process of car following behavior. Chakroborty and Kikuchi [8] proposed a car following model based on fuzzy inference system. In an abstract way the model can be represented like this:

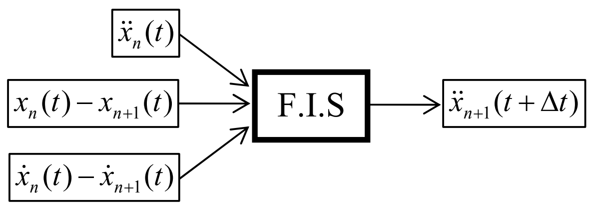

Figure 2: Schematic of the fuzzy inference based car following model

This model because of being based on imprecise decision making process of drivers can explain all the properties of car following behavior, i.e., can overcome the lacunae of the GM model. Later, they also calibrated the fuzzy membership function using artificial neural network (ANN) to strengthen the fuzzy inference based car following model [9].

Application of Fuzzy Logic in Transportation Planning

Transportation planning consists four phases. In the first the demand for transportation is studied, i.e., how many people want to go out of their homes and what are their purposes. This phase is named as *trip generation*. In the second phase (named as *trip distribution*) the places where these demands are satisfied are studied and accordingly from every location how many people move to other locations are determined. The third phase (named as *modal split*) is intended to know which modes of transportation (bus, train etc.) are used by how much fraction of people. Finally, the last is concentrated on which routes are followed by how many vehicles. This phase is named as *route choice*. These are dependent on human decisions, like whether to go for a particular purpose or not, in which place to go for that purpose and by which mode to go and which route to choose. The outcome of these is vehicles moving on roads. This is also variable in time. Also, depending upon the situation of the roads some vehicles may shift to some alternate routes, which is not known beforehand. So, this involves a lot of uncertainties, thus, both the inputs and outputs are imprecise in nature. So, to study the dynamic route choice phase, fuzzy logic is an appropriate tool and has been applied.

Vincent Henn [10] proposed a fuzzy rule based route choice model considering the imprecisions and the uncertainties lying in the dynamic route choice. This model is able to make more accurate description of the route choice process than the existing deterministic and stochastic models. The input to the fuzzy model is existing condition of each route and the outcome is the attractiveness of that route. Finally, a driver chooses a route having highest attractiveness. Hoogendoorn et al. [11] proposed a multi–modal travel choice model based on fuzzy logic.

Application of Fuzzy Logic in Traffic Control at Signalized Intersection

When two roads criss–cross, there is an intersection. In an intersection cross directional traffic obstruct each other. So, some strategies are followed to make time sharing between cross directional traffic, i.e., say, when vehicles in North–South road are in motion, vehicles in

East–West road are at a halt and vice–versa. An intersection can be unsignalized (drivers knowing the rules about their time sharing) or signalized (time sharing is controlled by signals). This apportionment of time for cross directional traffic is made in such a way to achieve the optimal utilization of the intersection. Various approaches have been adopted to apportion the signal timing. Among them fuzzy logic proved one of the efficient tool.

Signal control is done based on some information like the demand of users, their urgency and so on, which are imprecise in nature. In addition, the outcome, i.e., whether the delay to vehicles and queue length at the intersection scenario will improve by using a particular signal controller is not known beforehand. Also, the idea about which vehicle is to be given priority at a particular time is not clear. Thus, there remains a lot of uncertainty in signal control at intersection. Since, fuzzy logic is capable of handling uncertainties at imprecise decision making scenarios; it is suitable for application in signal controller at roadway intersection.

Wei et al. [12] have used fuzzy logic for signal timing at four–approach intersection to ascertain whether the existing signal phasing is okay or to change it. Their model is able to adjust signal timing in response to observed changes in demand. They used a multi–objective genetic algorithm to find a set of optimal parameters for the fuzzy inference based signal controller. Niittymaeki and Maeenpaeae [13] proposed a structure of fuzzy inference based signal control system and tested its rule bases.

Application of Fuzzy Logic in Ramp–metering

In the intersection two roads criss–cross (as mentioned in the previous subsection); whereas, ramp is the merging of a minor road into a major road. Here, also some control mechanism is required to ascertain when the vehicles from the minor road should be allowed to mingle into the major road, so that there is no disturbance or a little disturbance to the vehicles of the major road. Here also some signal control device is used for regulation of the entry of vehicles from minor road to the major road, which is called ramp–metering. Fuzzy logic has also been applied in ramp–metering.

Middelham developed a fuzzy ramp–metering. Based on the results of this study, the Dutch Ministry of Transport carried out a pilot study on ramp–metering. It was found that the fuzzy controller performed better than the existing controllers [11]. Chen et al. [15] developed a fuzzy ramp–metering control algorithm using procedural knowledge of traffic operators.

Study on Parking Garage

In the parking space there are some fixed numbers of positions for parking of vehicles, beyond which vehicles can not be placed. So, there should have some idea about occupancy status of parking space; which can only be done by parking demand information; which is again an uncertain matter. So, the task of prediction of available space in a parking garage can be tackled using fuzzy logic.
Hellendoorn and Baudrexl [16] developed a fuzzy–inference based forecasting system to predict the available space in a parking garage.

Traffic Monitoring and State Estimation

Kirschfink *et al*. [17] developed a traffic network data analysis system using fuzzy logic. This system helped to improve communication. Busch *et al*. [18] used fuzzy logic to detect an incident related to traffic flow, which is efficient in the sense that by their method, there is a three minute decrease in the time needed to detect an incident in comparison to the existing system.

CONCLUSIONS

Fuzzy logic uses linguistic variables to draw imprecise conclusions from imprecise input conditions. It is suitable to capture the inherent vagueness of human mind and determine the course of action, when the prevailing conditions are not clear and the consequence of the course of action is not known. Fuzzy logic has been applied into various areas including transportation engineering. In the area of transportation engineering it has been applied to model traffic flow, transportation planning, and traffic control and so on. In all of these applications fuzzy logic has proved to be successful in overcoming the drawbacks of other methods in the respective areas.

REFERENCES

1. Zimmermann, H.J. (2001), “Fuzzy set theory and its applications”, Fourth edition, Kluwer Academic Publishers, Boston/Dordrecht/London.
2. Zadeh, L.A. (1965), “Fuzzy sets”, *Information and Control* , vol. 8, pp 338–353.
3. Bellman, R.F. and Zadeh, L.A. (1970), “Decision making in a fuzzy environment” *Management Science*, vol. 17, pp 14–164.
4. Kaufmann, A. and Gupta, M.M. (1985), “Introduction to fuzzy arithmetic theory and its applications”, Van Nostrand Reinhold Company, New York.
5. Klir, G.J. and Folger, T.A. (1988), “Fuzzy sets, uncertainty, and information”, Prentice Hall, Englewood Cliffs, New Jersey.
6. Zadeh, L.A. (1979), “A theory of approximate reasoning” *Machine Intelligence*, vol. 9, pp 149–194.
7. Gazis, D.C., Herman, R. and Rothery, R.W. (1961), “Nonlinear follow–the–leader models of traffic flow” *Operations Research*, vol. 9, pp 545–567.
8. Chakroborty, P. and Kikuchi, S. (1999). “Evaluation of the General Motors based car-following models and a proposed fuzzy inference model”, *Transportation Research Part C*, vol. 7, pp. 209–235.
9. Chakroborty, P. and Kikuchi, S. (2003), “Calibrating the membership functions of fuzzy inference system: Instantiated by Car–following Data” *Transportation Research Part C*, vol. 11, pp. 91–119.
10. Henn, V. (2000), “Fuzzy route choice model for traffic assignment”, *Fuzzy Sets and Systems*, vol. 116, pp 77–101.
11. Hoogendoorn–Lanser, S. (1998), “Structured modelling of multi – modal urban travel choice behaviour”, 4th TRAIL PhD Congress.
12. Wei, W., Zhang, Y., Mbede, J.B., Zhang, Z. and Song, J. (2001), “Traffic signal control using fuzzy logic and moga”, *IEEE*, vol. 2, pp 1335–1340.
13. Niittymaeki, J. and Maeenpaeae, M. (2001), “The role of fuzzy logic public transport priority in traffic signal control” *Traffic engineering & control*, vol. 42, pp 22–26.
14. Hoogendoorn, S., Hoogendoorn–Lanser, S. and Schuurman, H. (1998), “Fuzzy perspectives in traffic engineering”, Research Report, TRAIL Research School, Delft, report on behalf of Dutch Ministry of Transport.
15. Chen, L.C., May, A.D. and Auslander, D.M. (1990), “Freeway ramp control using fuzzy set theory for inexact reasoning”, *Transportation Research*, vol. 24A, pp 15–25.
16. Hellendoorn, H., and Baudrexl, R. (1995), “Fuzzy neural traffic control and forecasting”. *International Joint Conference Of The 4th IEEE International Conference On Fuzzy Systems and The 2nd International Fuzzy Engineering Symposium IEEE International Conference On Fuzzy Systems*, vol. 4, IEEE, Piscataway, pp 2187–2194.
17. Kirschfink, H., Lange, R. and Jansen, B. (1997), “Monitoring, control and management on the motorway network in Hessen using intelligent traffic modeling”, *Proceedings of the ’97 ITS Word Conference*, Berlin.

18. Busch, F., Cremer, M., Ghio, A. and Henninger, T. (1994), “A multi-model approach for traffic state estimation and incident detection on motorways”, *Proceedings Of The First World Congress On Applications Of Transport Telematics And Intelligent Vehicle-Highway Systems*, Paris, vol. 3, pp 1245–1252.