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A Robust H_∞ Load-Frequency Controller Design Using LMIs

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Abstract— **A robust H_∞ controller design in an LMI framework has been carried out for a load-frequency control problem of a single-area uncertain power system model. For carrying out this design, the uncertainties have been restructured by considering norm-bounded uncertainty instead of rank-1 uncertainty structure of [16]. The proposed design is simple and can be easily solved to obtain optimal values of the controller gain. It is concluded by simulation that the proposed approach may improve dynamic performance and disturbance rejection property over the existing controllers.**

Key words: *LMI (Linear matrix Inequality), Load-Frequency control (LFC), Single-area power system,* H_∞ *Controller Norm bounded uncertainty, robust controller.*

I. Introduction

Load frequency control (LFC) is used to maintain electric supply frequency around a desired value with respect to load variation in electric power systems. Several control strategy such as PI control, optimal control and variable structure control have been used to control the frequency of the output electric power. One major advantage of PI controller is that it reduces the steady state error to zero. Since these conventional controllers are generally designed at some nominal operating point of the actual uncertain system, these do not perform well under varying operating conditions and exhibits poor dynamic performance. To improve the performances in presence of the load disturbances we require robust controllers. Several types of controller with ability to handle uncertainties and (or) disturbances have been developed in literature, e.g., H_∞ control, adaptive control, Neuro-control schemes [1],[3],[5],[12],[13]. In this paper, we consider the H_∞ controller with the objective to reject load disturbances.

In [6],[8],[9], and [15] the issues of load-frequency control with time-delays have been discussed in the framework of LMI and H_∞ controller. Traditionally, the area control error (ACE) signal is used as the input for the load frequency controller (LFC) whose objective is to control the frequency deviation. In [16] the robust LFC design has been carried out for an uncertain power system model with rank-1 uncertainty in the system matrix and simply bounded uncertainties in the input and disturbance input matrices. The uncertainty is parametric one i.e, due to varying operating conditions. The load disturbance rejection part was not considered in the controller design, also the simulation of the considered system was carried out for constant disturbance input only. The controller design was based on Riccati equation approach. In [17] robust LFC design was based on μ synthesis technique and in [4] an LMI based decentralized H_∞ controller was designed for multi area power system.

In this paper, we have reformulated the uncertain power system model in [16] by restructuring the uncertainties in norm bounded fashion instead of rank-1 type. The effect of this consideration may be seen as the ability to improve performance of the designed robust H_∞ load-frequency controller in LMI framework. This improvement is validated by closed loop simulation results.

II. LFC Model

We consider a of linearized single-area power system model with time-varying norm bounded uncertainties in the system and input matrices. The state space representation of it is ([16])

$$\dot{x}(t) = A(t)\,x(t) + B\,u(t) + L\,w(t) \qquad (1)$$

where the state vector is $x(t) = [\Delta f(t) \quad \Delta P_g(t) \quad \Delta X_g(t) \quad \Delta E(t)]^T$, $\Delta f(t)$ is the incremental frequency deviation (in Hz), $\Delta P_g(t)$ is the incremental change in generator output (in p.u. MW), $\Delta X_g(t)$ is the incremental change in governor valve position (in p.u. MW), $\Delta E(t)$ is the incremental change in integral control; $u(t) \in \Re$ is the input vector; the disturbance scalar $w(t) = \Delta P_d(t)$, $\Delta P_d(t)$ is the load disturbance (in p.u. MW). $A(t), B(t)$ and $L(t)$ are uncertain time-varying matrices with appropriate dimensions. The matrices $A(t)$ and $B(t)$ are having nominal and uncertain components as:

$$A(t) = A + \Delta A(t) \text{ and } B(t) = B + \Delta B(t) \qquad (3)$$

The time-varying uncertain matrices $\Delta A(t)$ and $\Delta B(t)$ are norm bounded and may be decomposed as

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$$\Delta A(t) = D_a F_a(t) E_a \text{ and } \Delta B(t) = D_b F_b(t) E_b \quad (4)$$

where $F_a(t)$ and $F_b(t)$ satisfy $F_a^T(t) F_a(t) \le I$, $F_b^T(t) F_b(t) \le I$. The decomposition of the uncertain matrices as in (4) may be used to exploit the uncertainty structure and to normalize the time-varying components $F_a(t)$, $F_b(t)$ to have effective result.

III. CONTROLLER DESIGN

Consider the system (1) with feedback control law $u(t) = K x(t)$. Then one can write (1) as

$$\dot{x}(t) = (A(t) + B(t) K) x(t) + L w(t) \quad (5)$$

Our objective is to design the control gain K. To have this, we now present the following Lemmas which will be used to design K.

Lemma 1 [13]: Given matrices $Q = Q^T, H, E$ and $R = R^T > 0$ with appropriate dimensions, and $Q + HFE + E^T F^T H^T < 0$, for all F satisfying $F^T F \le R$, if and only if there exists some $\lambda > 0$, such that $Q + \lambda H H^T + \lambda^{-1} E^T R E < 0$.

Lemma 2 [7]: For any $z, y \in \Re^n$ and for nay symmetric positive definite matrix $X \in \Re^{n\times n}$, following inequality is true

$$-2 z^T y \le z^T X^{-1} z + y^T X y$$

Lemma 3 (Schur Complement) [2]: For any matrices Q, R and S

$$\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} < 0$$

is equivalent to

$$R < 0 \text{ and } Q - S R^{-1} S^T < 0.$$

Lemma 4: If there exists a matrix $Y = Y^T > 0$ and positive scalar ε_1, ε_2 and γ such that

$$\Lambda = \begin{bmatrix} \Lambda & Y^T E_a^T & X^T E_b^T & Y C^T & L Y \\ * & -\varepsilon_1 & 0 & 0 & 0 \\ * & 0 & -\varepsilon_2 & 0 & 0 \\ * & 0 & 0 & -I & 0 \\ * & 0 & 0 & 0 & -\gamma^2 \end{bmatrix} < 0 \quad (6)$$

where

$$\Lambda = Y A^T + A^T Y + X^T B + B X + \varepsilon_1 D_a D_a^T + \varepsilon_2 D_b D_b^T$$

then the system states of (5) subject to the assumptions in section II satisfies a H_∞ norm-bound of γ with respect to the disturbance input.

Proof: The proof is based on Lyapunov's direct method. We consider a Lyapunov function as

$$V(t) = x^T(t) \, P \, x(t) \quad (7)$$

Finding the time-derivative of (7) one obtains

$$\dot{V}(t) = \dot{x}^T(t) P x(t) + x^T(t) P \dot{x}(t) \quad (8)$$

Along the trajectory of (5) one may write (8) as

$$\begin{aligned} \dot{V}(t) &= [(A(t) + B(t) K) x(t) + L w(t)]^T P x(t) \\ &\quad + x^T(t) P [(A(t) + B(t) K) x(t) + L w(t)] \\ &= x^T(t) A^T(t) P x(t) + x^T(t) K^T(t) B^T(t) P x(t) \\ &\quad + w^T(t) L^T P x(t) + x^T(t) P A(t) x(t) \\ &\quad + x^T(t) P B(t) K x(t) + x^T(t) P L w(t) \end{aligned} \quad (9)$$

Now,

$$\begin{aligned} \dot{V}(t) &= x^T(t) A^T(t) P x(t) + x^T(t) K^T(t) B^T(t) P x(t) \\ &\quad + x^T(t) P A(t) x(t) + x^T(t) P B(t) K x(t) \\ &\quad + 2 x^T(t) P L w(t) \end{aligned} \quad (10)$$

Substituting the values of $A(t)$ and $B(t)$ as defined above and using Lemma 2, (10) may be written as

$$\begin{aligned} \dot{V}(t) &\le x^T(t)[A^T P + P A + K^T B P + P B K + E_a^T F_a^T D_a^T P \\ &\quad + P D_a F_a E_a + P D_b F_b E_b K + K^T E_b^T F_b^T D_b^T P \\ &\quad + \gamma^{-2} P L L^T P] x(t) + \gamma^2 w^T(t) w(t) \end{aligned} \quad (11)$$

Using Lemma 1 in (11) one can obtain

$$\dot{V}(t) \le x^T(t) \Omega x(t) + \gamma^2 w^T(t) w(t) \quad (12)$$

where

$$\begin{aligned} \Omega &= A^T P + P A + K^T B P + P B K + \varepsilon_1 P D_a (P D_a^T) \\ &\quad + \varepsilon_1^{-1} E_a{}^T E_a + \varepsilon_2 P D_b (P D_b)^T + \varepsilon_1^{-1} K^T E_b^T E_b K + \gamma^{-2} P L L^T P \end{aligned}$$

Now defining a cost function

$$J = \int_0^\infty [x^T(t) x(t) - \gamma^2 w^T(t) w(t)] dt \quad (13)$$

Note that, if $J_{yw} < 0$ then the system states achieve a H_∞ norm of γ with respect to the disturbance input $w(t)$ since $\|x(t)\|_2 \le \gamma \|w(t)\|_2$.

Substituting $\gamma^2 w^T(t) w(t)$ from (12) into (13) one may write

$$J \le \int_0^\infty [x^T(t)\Sigma x(t) - \dot{V}(t)]\,dt \qquad (14)$$

where $\Sigma = I + \Omega$.

Clearly if $\Sigma < 0$ then $J < 0$ is satisfied assuming $V(\infty) - V(0) > 0$ which is obvious in case of $V(0) = 0$. Therefore, $\Sigma < 0$ implies the system states are bounded with H_∞ norm of γ . Finally, pre- and post-multiplying Σ by $Y = P^{-1}$, letting $KY = X$ and using Lemma 3 one obtains the LMI criterion (6). ■

IV. NUMERICAL RESULTS

We consider a power system model as given in (1) with the following nominal parameters [16]

$$A = \begin{bmatrix} -0.0665 & 8 & 0 & 0 \\ 0 & -3.663 & 3.663 & 0 \\ -6.86 & 0 & -13.736 & -13.736 \\ 0.6 & 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 13.736 \\ 0 \end{bmatrix}$$

and $L = \begin{bmatrix} -8 & 0 & 0 & 0 \end{bmatrix}^T$.

The structural decomposition matrices of the uncertain parameters are

$$D_a = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, D_b = I, E_b = \begin{bmatrix} 0 \\ 0 \\ 13.736 \\ 0 \end{bmatrix},$$

$$E_a = \begin{bmatrix} -0.01675 & 2 & 0 & 0 \\ 0 & -0.5495 & 0.5495 & 0 \\ -0.3779 & 0 & -0.4121 & -0.4121 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

A. Open-loop simulation:

The open-loop simulation for the proposed uncertain system is carried out for the system model (1) with $u(t) = 0$ and the proposed uncertainty structure in (3) and (4) with $F_a(t)$ and $F_b(t)$ matrices are taken to be identity matrices, whereas the uncertain system of [16] has been simulated by considering the values of nominal and uncertain matrices given in section 4, equation (14) of [16] with $u(t) = 0$. The simulation results presented in Fig. 1 and 2 shows that, both the uncertainty structure effect the system dynamics in a similar fashion.

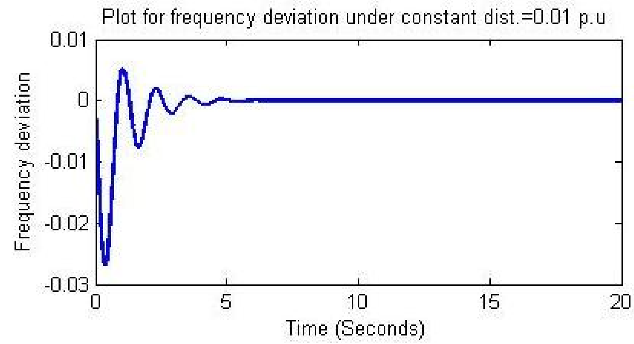

Fig.1: Plot for frequency deviation under open-loop simulation for the model in [16]

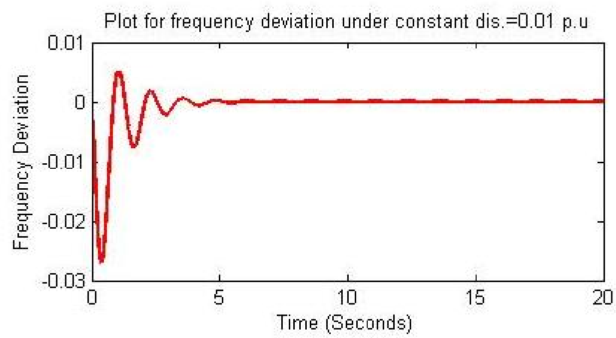

Fig. 2: Plot for frequency deviation under open-loop simulation for the proposed model.

B. Closed-loop simulation:

The closed-loop simulation is carried out for the proposed uncertain system model in (5) with the uncertainty structure in (3) and (4) with $F_a(t)$ and $F_b(t)$ matrices taken to be identity matrices. The gain matrix $(K = XY^{-1})$ has been obtained by solving the LMI (6) in Lemma 4 with the objective to minimize γ using standard algorithm available with LMI toolbox of MATLAB [11]. The closed loop simulation for both the uncertain power system model has been carried for two different types of load disturbance (i) for a constant load disturbance of 0.01 p.u and (ii) for a sinusoidal time-varying load disturbance ($0.1\sin 2\pi f t$) with $f = 0.1\text{Hz}$. Closed loop simulations with the present controller and with that obtained in [16] is shown in Fig. 3 and 4. A comparison of these shows the proposed controller provides better attenuating performance for the simulated cases.

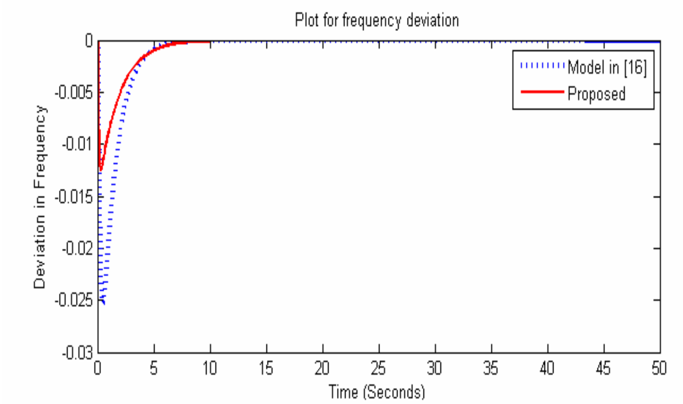

Fig.3: Plot for frequency deviation under constant load disturbance input of 0.01 p.u

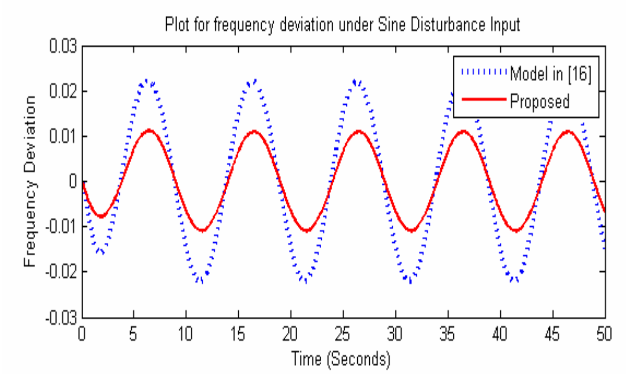

Fig.4: Plot for frequency deviation under Sinusoidal load disturbance input with f=0.1 Hz.

V. CONCLUSION

In this paper a H_∞ controller has been designed for an uncertain single area LFC power system in LMI framework. The uncertain system matrices are considered to be norm-bounded rather than the rank-1 type as considered in [16]. This reformulation helps in improving performance of the designed controller. Moreover, such an advantage may be utilized to further designing the controller by considering more practicality in the system model, e.g., actuator saturation.

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