Archives of Control Sciences
Volume 19(LV), 2009
No. 3*, pages 245–259*

Design of a Path Following Controller for an Underactuated AUV

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Abstract: This paper describes a tracking control strategy for an underactuated autonomous underwater vehicle (AUV) on a two dimensional plane ($\Re^2$).Based on a smooth, inertial, 2D reference trajectory curve, the proposed algorithm uses vehicle dynamics to generate the reference orientation and body-fixed velocities. Following these, required error dynamics are developed. Error dynamics are then stabilized using inverse dynamics control strategy, forcing the tracking error to an arbitrarily small neighborhood of zero. Circular path, as a constant velocity reference trajectory, has been chosen for simulation studies. Simulation results are included to demonstrate the tracking performance of the controller.

Key Words: Kinematics, Reference Path, Tracking control, Inverse Dynamics Control

1. Introduction

For the last twenty years or more, a great amount of research has been directed on the operation of AUVs. AUVs are extremely useful in exploration of living and nonliving resources in the oceans. Some of the important applications of AUVs include oceanographic observations, bathymetric surveys, ocean floor analysis, military applications, recovery of lost man-made objects, under water inspections, under water constructions etc. [1, 2]. AUVs pose many challenging control problems as they are underactuated, i.e., number of available actuated inputs is less than number of controllable degrees of freedom (DOF), imposing nonintegrable acceleration constraints. It is noted that AUVs' kinematic and dynamic models are highly nonlinear and coupled thus AUV control design is complicated [1]. Full state feedback linearization control design is not possible by the conventional full-state linearization control. It is to be noted that that when moving on a horizontal plane, AUVs present similar dynamic behavior to underactuated surface vessels [1, 4, 5].

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Received 08.09.2009.

Stabilization problem for AUVs has been addressed by various researchers[4,6,7,8,9,10].Standard linear (or nonlinear) methods are used for development of control schemes for AUVs [1]. Kaminer et al. [11] proposed a control algorithm where the linearization of the vehicle's error dynamics is done by trimming the trajectories i.e. racking with constant required velocities. Stability issues of path tracking controller has been studied in [12]. In [5], the formation control of a group of underactuated surface vessels has been considered. Although, inverse dynamics based control design is a very popular approach to robotics control, but it has not been applied to AUVs. Thus, the authors feel to attempt the control of AUVs employing inverse dynamic control strategy which is described in this work considering a simple AUV model.

This paper describes tracking control for an under-actuated AUV moving on the horizontal plane. The trajectories are generated as per the required reference inputs to the motion control system. This ensures that the vehicle executes the planned path. We have studied the inverse dynamics based path control problem of AUV by considering a circular reference path.

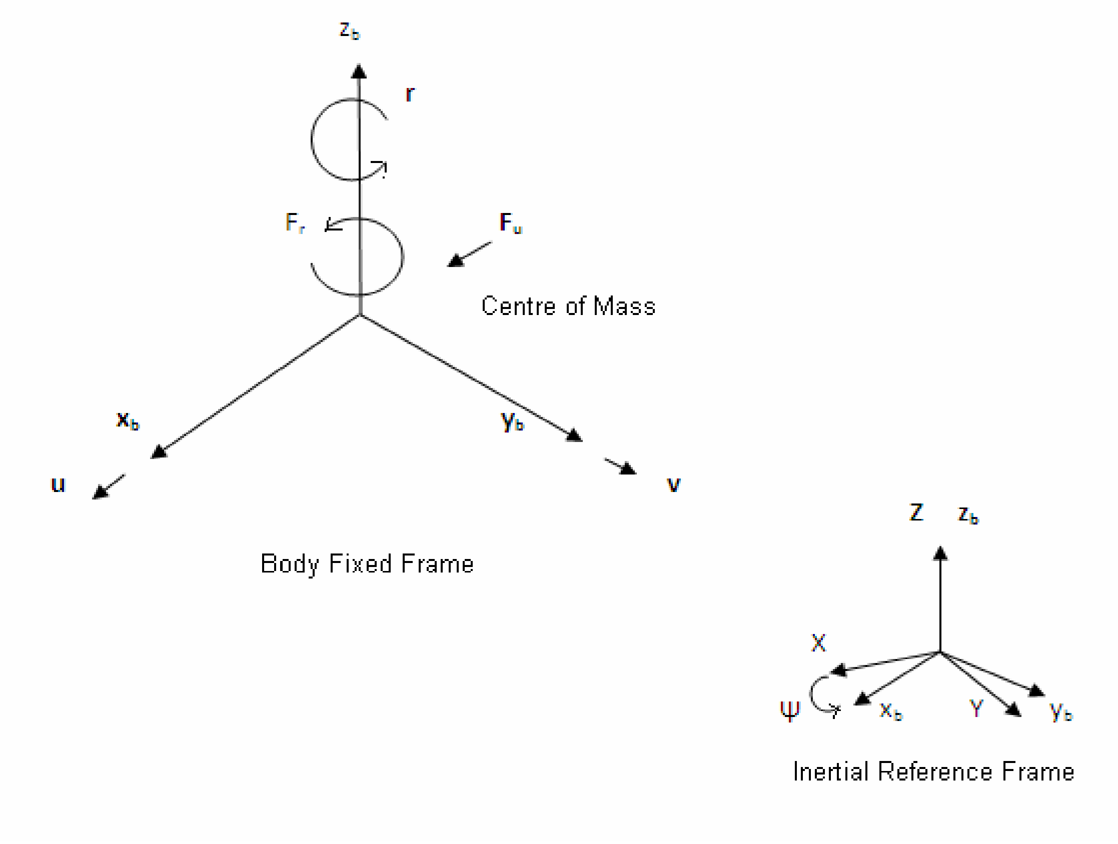

Fig.1. The underactuated AUV model in plane motion

2. Kinematics and Dynamics of the AUV

The kinematic and dynamic equations of the motion of an AUV moving on the horizontal (yaw) plane [12] are reviewed. Fig.1 shows an inertial reference frame $\{I\}$ and a body-fixed frame $\{B\}$ for planar motion. The kinematic equations of motion for an AUV on the horizontal X–Y plane can be written as [12]

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{\psi}\end{bmatrix}=\begin{bmatrix}\cos\psi & -\sin\psi & 0\\ \sin\psi & \cos\psi & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}u\\ v\\ r\end{bmatrix} \tag{1}$$

where x and y denotes the inertial coordinates of the centre of mass of the AUV. u and v are the (linear) surge (forward) and sway (side) velocities, respectively. The dynamics for a neutrally buoyant AUV with three planes of symmetry can be expressed as [12]

$$\dot{u}=\frac{m_{22}}{m_{11}}vr-\frac{X_u}{m_{11}}u-\frac{X_{u|u|}}{m_{11}}u|u|+\frac{1}{m_{11}}F_u \tag{2a}$$

$$\dot{v}=-\frac{m_{11}}{m_{22}}ur-\frac{Y_v}{m_{22}}v-\frac{Y_{v|v|}}{m_{22}}v|v| \tag{2b}$$

$$\dot{r}=\frac{m_{11}-m_{22}}{m_{33}}uv-\frac{N_r}{m_{33}}r-\frac{N_{r|r|}}{m_{33}}+\frac{1}{m_{33}}F_r \tag{2c}$$

Where F_u denotes the control force along the surge motion of the AUV and F_r denotes the control torque that is applied in order to produce angular motion around the z_b axis of the body-fixed frame. The constants m_{11} and m_{22} are the combined rigid-body and added mass terms, and m_{33} is the combined rigid-body and added moment of inertia about the z_b axis. X_u, $X_{u|u|}$, Y_v, $Y_{v|v|}$, N_r, and $N_{r|r|}$ are the linear and quadratic drag terms coefficients as described in Table 1.

3. Kinematic Description of the Reference Path

Reference path to be tracked by CM (Centre of Mass) of AUV is specified as a time function of two variables $x_R(t)$ and $y_R(t)$,which are with respect to inertial coordinate system.The magnitude of the velocity vector $\mathbf{v_p}$ of a reference path point P at time t is given by [12]

$$v_P = \|\mathbf{v_p}\| = \left\|\begin{bmatrix}\dot{x}_R & \dot{y}_R\end{bmatrix}^T\right\| = \sqrt{\left(\dot{x}_R^2+\dot{y}_R^2\right)} \tag{3}$$

The direction of vector $\mathbf{v}_P$ is described by the angle β as shown in Fig.2:

$$\beta=\tan^{-1}\left(\frac{\dot{y}_R}{\dot{x}_R}\right) \tag{4}$$

The first and second derivatives of this angle are given by [12]

$$\omega=\dot{\beta}=\frac{\dot{x}_R\ddot{y}_R-\dot{y}_R\ddot{x}_R}{\dot{x}_R^2+\dot{y}_R^2} \tag{5}$$

$$\Rightarrow \dot{\omega}=f_\omega(\dot{x}_R,\ddot{x}_R,\dddot{x}_R,\dot{y}_R,\ddot{y}_R,\dddot{y}_R) \tag{6}$$

where f_ω indicates that $\dot{\omega}$ is a function of the derivatives of the inertial variables up to the third order. Also, the first and second derivatives of the velocity v_P are

$$\dot{v}_P = \frac{\dot{x}_R \ddot{x}_R + \dot{y}_R \ddot{y}_R}{\sqrt{\left(\dot{x}_R^2 + \dot{y}_R^2\right)}} \quad (7)$$

$$\Rightarrow \ddot{v}_P = f_v(\dot{x}_R, \ddot{x}_R, \dddot{x}_R, \dot{y}_R, \ddot{y}_R, \dddot{y}_R) \quad (8)$$

where f_v indicates that $\ddot{v}_P$ is a function of the derivatives of x_R and y_R.

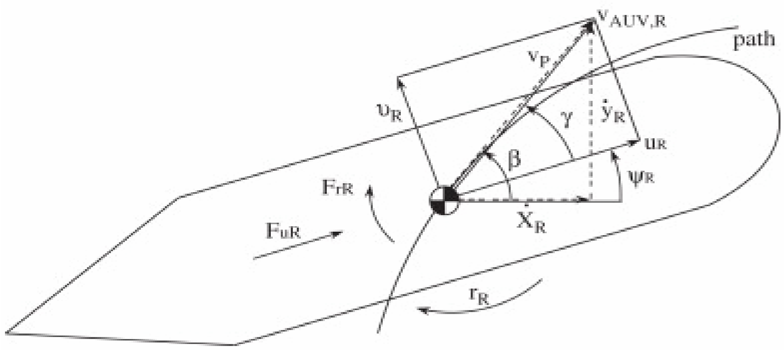

Fig. 2. The AUV moving on a planar path [12]

4. Equations of AUV Dynamics on the path

The magnitude of the linear velocity vector of the CM in {B} frame is given by [12]. Observing the Fig.2.

$$v_{\text{AUV, R}} = \|\mathbf{v}_{\text{AUV, R}}\| = \left\| [u_R, v_R]^T \right\| = \sqrt{\left(u_R^2 + v_R^2\right)} \quad (9)$$

Where u_R, v_R are body-fixed reference velocities. Here we observe coupling term in Eq.(2b),- $-(\frac{m_{11}}{m_{22}})u_R v_R$ gives rise to sway motion v_R.

The appearance of an angle γ (angle u_R and $\mathbf{v}_{\text{AUV, R}}$) is given by

$$\gamma = \tan^{-1}(\frac{v_R}{u_R}) \quad (10)$$

The magnitude of the tangent vector to the reference path $\mathbf{v}_P$ must equal the magnitude of the vehicle's velocity vector $\mathbf{v}_{\text{AUV, R}}$ [12] and is expressed as

$$v_P = v_{\text{AUV, R}} \Rightarrow \sqrt{\left(\dot{x}_R^2 + \dot{y}_R^2\right)} = \sqrt{\left(u_R^2 + v_R^2\right)} \quad (11)$$

The relation between the various angles is given by (12)

$$\psi_R = \beta - \gamma$$

$$\Rightarrow \psi_R = \tan^{-1}(\frac{\dot{y}_R}{\dot{x}_R}) - \tan^{-1}(\frac{v_R}{u_R}) \quad (12)$$

where ψ_R represents angle between inertial X-axis and body- x_b axis.

Assuming that the vehicle moves forward, i.e., $u_R \rangle 0$. The reference sway velocity is

$$v_R = \pm \sqrt{\left(\dot{x}_R^2 + \dot{y}_R^2 - u_R^2\right)} = \pm \sqrt{\left(v_p^2 - u_R^2\right)} \quad (13)$$

where "±"indicates that v_R may be positive or negative depending on path curvature. For example, if $u_R > 0$ and, $r_R > 0$ then $v_R < 0$ .Differentiating Eq. (13) one gets

$$v_R = \frac{v_P \dot{v}_P - u_R \dot{u}_R}{\sqrt{\left(v_P^2 - u_R^2\right)}} \tag{14}$$

From Eq. (13), $-v_P \leq u_R \leq v_P$ (15)

where the equality holds in the case of straight line tracking or when a change in the sign of the curvature occurs (then $v_R = r_R = 0$). The reference angular velocity and acceleration that correspond to the path are obtained by differentiating twice ψ_R in Eq. (12) and taking into account Eqs.(5),(7) and(13), to yield, r_R (reference angular velocity).

$$r_R = \dot{\psi}_R = \frac{d}{dt}\left[\tan^{-1}\left(\frac{\dot{y}_R}{\dot{x}_R}\right) - \tan^{-1}\left(\frac{\sqrt{\left(v_P^2 - u_R^2\right)}}{u_R}\right)\right]$$

$$= \omega - \frac{u_R \dot{v}_P - \dot{u}_R v_P}{v_P \sqrt{\left(v_P^2 - u_R^2\right)}}$$

(16)$\Rightarrow \dot{r}_R = f_r(\omega, \dot{\omega}, u_R, \dot{u}_R, \ddot{u}_R, v_p, \dot{v}_p, \ddot{v}_p)$

(17)

The reference surge velocity is expressed as follows

$$\dot{u}_R = \left[(m_{11} - m_{22})u_R v_P^3\right]^{-1}\left\{\begin{array}{l} -v_P^3\left(m_{22} v_P \dot{v}_P + Y_v\left(v_P^2 - u_R^2\right)\right) \\ + m_{11}[u_R \dot{v}_P v_P^2 + v_P \sqrt{\left(v_P^2 - u_R^2 (\ddot{x}_R \dot{y}_R - \dot{x}_R \ddot{y}_R\right)}] \\ - Y_{v|v|}(v_P^2 - u_R^2) v_P^3 \left|\sqrt{\left(v_P^2 - u_R^2\right)}\right| \end{array}\right\} \tag{18}$$

For a CCW rotation (CCW-type path), $v_R \langle 0$ and $r_R > 0$ thus, we have,

$$\dot{u}_R = \left[(m_{11} - m_{22})u_R v_p^3\right]^{-1}\left\{\begin{array}{l} -v_p^3\left(m_{22} v_P \dot{v}_P + Y_v\left(v_P^2 - u_R^2\right)\right) + \\ m_{11} u_R\left[u_R \dot{v}_P v_P^2 + v_P \sqrt{\left(v_P^2 - u_R^2\left(\dot{x}_R \ddot{y}_R - \ddot{x}_R \dot{y}_R\right)\right)}\right] - \\ Y_{v|v|}\left(v_P^2 - u_R^2\right) v_P^3 \left|\sqrt{\left(v_P^2 - u_R^2\right)}\right| \end{array}\right\} \tag{19}$$

For a general motion in which the curvature of the path changes with time, we switch between the above differential equations. The initial condition $u_{R,0} = u_R(t = 0)$ is used with the appropriate differential equation. As the integration progresses and switches between the two equations, the initial condition for the current equation is the last value obtained from the integration of the previous equation. Numerical integration of equations (18) and (19) yields $u_R(t)$. Using the computed u_R along with v_R, $\dot{x}_R$, $\dot{y}_R$, and Eq. (12), we compute the AUV reference orientation, $\psi_R(t)$.

Thus, the feasible trajectory including the feasible orientation is completely known. Also, all the feasible body-fixed velocities and accelerations, required for tracking, have been computed. Using the AUV dynamic model, it is straightforward now to compute the corresponding open-loop control efforts. Indeed, from Eqs. (2a) and (2c), these controls are given by

$$F_{uR} = m_{11}\dot{u}_R - m_{22}v_R r_R + X_u u_R + X_{u|u|}u_R^2 \qquad (20)$$

$$F_{rR} = m_{33}\dot{r}_R + (m_{22} - m_{11})u_R v_R + N_r r_R - N_{r|r|}r_R \qquad (21)$$

The presented trajectory planning algorithm and the open-loop control efforts given by Eqs.(20) and (21) can be incorporated in a two-step closed-loop trajectory-tracking controller design. Forward feeding the actuators of an AUV with the above open-loop controls, consistent with the vehicle dynamics, allows one to design a feedback controller for taking care of the small tracking errors due to model parameter uncertainty, external disturbances, etc. It is expected that such a controller will not require large gains and, hence, will exhibit an improved performance.

In the next subsection characteristics of a circular planar path is analyzed. The various system parameters for a typical AUV, used for simulation purposes, are given in Table 1.

Also,

$$m_{11} = m - X_{\dot{u}} = 215 \text{ kg}$$

$$m_{22} = m - Y_{\dot{v}} = 265 \text{ kg} \qquad (22)$$

$$m_{33} = I_z - N_{\dot{r}} = 80 \text{ kg-m}^2$$

where, m_{ii} ($i = 1,2,3$) are constants representing the combined inertia and added mass terms.

Table 1. Rigid body and hydrodynamic parameters of the AUV [7]

Parameter	***Symbol***	***Value***	***Unit***
Mass	m	185	kg
Rotational Inertia	I_z	50	kg-m^2
Added mass	$X_{\dot{u}}$	-30	kg
Added mass	$Y_{\dot{u}}$	-80	kg
Added mass	$N_{\dot{r}}$	-30	kg m^2
Surge linear drag	X_u	70	kg/s
Surge quadratic drag	$X_{u\|u\|}$	100	kg/s
Sway linear drag	Y_u	100	kg/s
Sway quadratic drag	$Y_{u\|u\|}$	200	kg/m
Yaw linear drag	N_r	50	kg m^2/s
Quadratic yaw drag	$N_{r\|r\|}$	100	kg m^2

Circular Reference Path

A reference circular inertial planar trajectory [12] is considered here and described as follows:

$x_R(t) = 10\sin(0.01t)$

(23b) $y_R(t) = 10\cos(0.01t)$

(23c)

Here, the derivatives up to the *second order* are needed, since tracking the above circle is obtained by the vehicle with constant linear velocities u_R and v_R and angular velocity r_R. Using Eqs. (3) and (7), we find that $v_P = 0.3$ m/s and $\dot{v}_P = 0$. Also from Eq. (5), $\omega = -0.02$ rad/s meaning that a constant CW-type path. Forward tracking of this circle requires the reference body-fixed velocities of the AUV signed as $u_R > 0$, $v_R > 0$, and $r_R < 0$. Their values are computed as follows: in Eq. (18), we substitute v_P, and $\dot{x}_R(t)$, $\dot{y}_R(t)$, $\ddot{x}_R(t)$, $\ddot{y}_R(t)$ for their previously computed values. We also substitute the constant terms for their numerical values from Table 1.

Solving numerically the differential equation that corresponds to the CW sense of the circle with $u_R(0) = 10^{-3}$, we obtain $u_R = 0.993$ m/s and $\dot{u}_R = 0$. Using Eqs. (13), (14) and (16) with the appropriate substitutions it is easy now to obtain $v_R = 0.0019$ m/s, $\dot{v}_R = 0$, $r_R = -0.02$ rad/s, and . $\dot{r}_R = 0$ .Additionally, from Eq. (10) it is found that $\gamma = 1.23$ rad, which means that during tracking the body-x_b axis must point towards the inside of the circle. From Eqs. (20) and (21), the constant open-loop control force and torque are computed as F_{uR} =8.004 N and $F_{rR} =$ -0.6 Nm.

5. Inverse Dynamics Control Strategy

Development of Error Dynamics:

The closed loop error dynamics of the AUV path tracking control using the inverses dynamics control is described as follows. The tracking errors are defined as follows:

$$u_e = u - u_R, v_e = v - v_R, r_e = r - r_R, x_e = x - x_R, y_e = y - y_R, \psi_e = \psi - \psi_R \tag{25}$$

Then substituting u, v, r, x, y, ψ in equations (1), (2a), (2b) and (2c), the error dynamics is obtained as follows.

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u_e \\ v_e \end{bmatrix} + \begin{bmatrix} \cos\psi - \cos\psi_R & -\sin\psi + \sin\psi_R \\ \sin\psi - \sin\psi_R & \cos\psi - \cos\psi_R \end{bmatrix} \begin{bmatrix} u_R \\ v_R \end{bmatrix} \text{m} \tag{26a}$$

$$\Rightarrow \dot{\mathbf{x}}_e = \mathbf{R}\mathbf{u}_e + \mathbf{R}_{\psi,\psi_R}\mathbf{u}_R \tag{26b}$$

$$\Rightarrow \dot{\mathbf{x}}_e = \mathbf{R}\mathbf{u}_e + \boldsymbol{\delta},$$

$$\dot{\psi}_e = r_e \tag{26c}$$

where

$$\mathbf{u}_e \triangleq [u_e, v_e]^T, \quad \dot{\mathbf{x}}_e \triangleq [\dot{x}_e, \dot{y}_e]^T,$$

$$\mathbf{R} \triangleq \mathbf{R}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix},$$

$$\mathbf{R}_{\psi,\psi R} \triangleq \mathbf{R}(\psi)\text{-}\mathbf{R}(\psi_R).$$

$$\mathbf{u}_R \triangleq [u_R, v_R]^T, \quad \boldsymbol{\delta} \triangleq \mathbf{R}_{\psi,\psi R} u_R.$$

Using Eqs. (2a), (2b), (2c) and (25) the error dynamics is derived as,

$$\dot{u}_e = \frac{m_{22}}{m_{11}}(v_e r_e + v_e r_R + v_R r_e) - \frac{X_e}{m_{11}} u_e - \dot{u}_R + \frac{m_{22}}{m_{11}} v_R r_R - \frac{X_u}{m_{11}} u_R - \frac{X_{u|u|}}{m_{11}}(u_e + u_R)\,|\,u_e + u_R\,| + \frac{F_u}{m_{11}} \quad (27a)$$

$$\dot{v}_e = -\frac{m_{11}}{m_{22}}(u_e r_e + u_e r_R + u_R r_e) - \frac{Y_v}{m_{22}} v_e - \dot{v}_R - \frac{m_{11}}{m_{22}} u_R r_R - \frac{Y_v}{m_{22}} v_R - \frac{Y_{v|v|}}{m_{22}}(v_e + v_R)\,|\,v_e + v_R\,| \quad (27b)$$

$$\dot{r}_e = \frac{m_{11} - m_{22}}{m_{33}}(u_e v_e + u_e v_R + u_R v_e) - \frac{N_r}{m_{33}} r_e - \dot{r}_R + \frac{m_{11} - m_{22}}{m_{33}} u_R v_R - \frac{N_R}{m_{33}} r_R - \frac{N_{r|r|}}{m_{33}}(r_e + r_R)\,|\,r_e + r_R\,| + \frac{F_r}{m_{33}} \quad (27c)$$

In the above equations, when the error variables are zero, we observe that the remaining dynamical system reduces to the reference one with F_u=F_{uR} and F_r=F_{rR}, as computed in Section-III.

Inverse Dynamics Control:

The inverse dynamics control law is given by

$$[y] = -[K_p][q] - [K_D][\dot{q}] + [r_{ref}] \quad (28)$$

where, $[q] = \begin{bmatrix} x \\ y \\ \psi \end{bmatrix}, [y] = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{\psi} \end{bmatrix}, [r_{ref}] = \begin{bmatrix} x_R \\ y_R \\ \psi_R \end{bmatrix}$

The objective here is make the error terms (x_e, y_e, ψ_e) zero so that vehicle follows the circular reference. The gains K_p and K_D are so chosen to make system asymptotically stable. $[K_p]$ and $[K_D]$ are assumed positive definite and diagonal matrices. It is described as follows. From equation (28), the second order equation is obtained as

$$[\ddot{q}]+[K_D][\dot{q}]+[K_P][q]=[r_{ref}] \quad (29)$$

For inverse dynamics $[r_{ref}]$ is chosen for a desired trajectory $q_d(t)=\begin{bmatrix} x_d \\ y_d \\ \psi_d \end{bmatrix}$

$$[r_{ref}]=[\ddot{q}_d]+[K_D][\dot{q}_d]+[K_P][q_d] \quad (30)$$

Putting the value of (30) in (29), a homogeneous second order differential equation is obtained as

$$\ddot{\tilde{q}}+[K_D][\dot{\tilde{q}}]+[K_P][\tilde{q}]=0 \quad (31)$$

Which expresses the dynamics of position error in (26 a) and (26b), while tracking the given trajectory. Such error occurs only if $[\tilde{q}(0)]$ and $/[\dot{\tilde{q}}]$ are different from zero and converges to zero with a speed depending on matrices $[K_P]$ and $[K_D]$ chosen.

$[K_p]$ and $[K_D]$ are assumed positive definite matrices. We choose them as diagonal matrices as follows: $[K_P]=25[I]$ and $[K_D]=5[I]$,where $[I]$ is identity matrix of order 3x3.

6. Simulation Studies

Simulation results under the above designed controller are shown below. The converging characteristics of different error variables are shown for a specified span of time. The following simulations concern the trajectory planning and tracking control design for a circular path in parameters. Here, the AUV starts from rest, resulting in different initial errors as follows:

u_e =0.1 m/s

v_e =0.0021 m/s

r_e =-0.01 rad/sec

x_e =0.5m

y_e =1m

$\psi_e = 1$ radian

The velocity errors with respect to body-fixed frame are shown in Fig.5,Fig.6 and Fig.7.The control force (Fu) required are shown in Fig.3 and the control torque (Fr) needed for tracking is shown in Fig.4 for a period of 40 second in each case. Nature of these control inputs indicate, these are of zero value after 40 second also. The position errors with respect to inertial frame are shown in Fig.8, Fig.9 and Fig.10. After a short period of time—needed for the errors to converge to a small neighborhood of zero—they converge smoothly. Plot of tracking errors v_e and u_e

are shown in Fig.11 from where we find time required for full circle tracking is around 600 second.

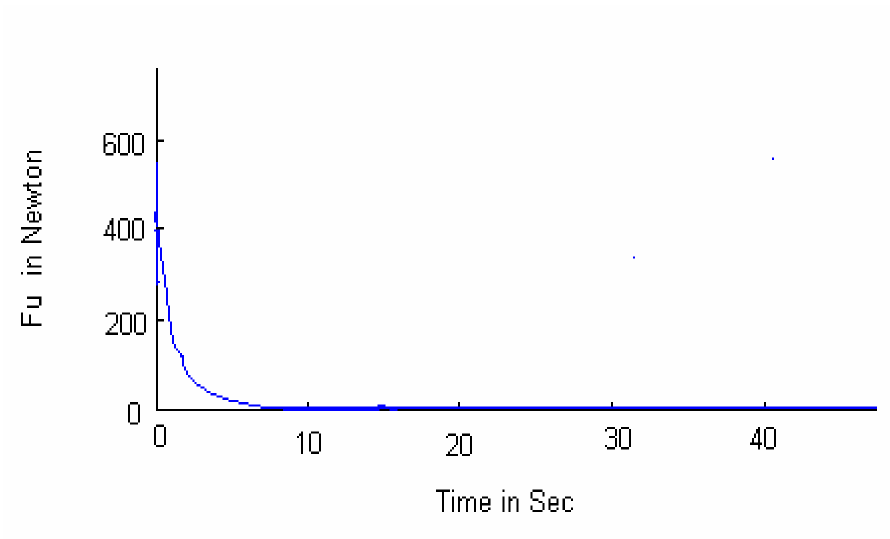

Fig.3. Control Force Fu needed for tracking

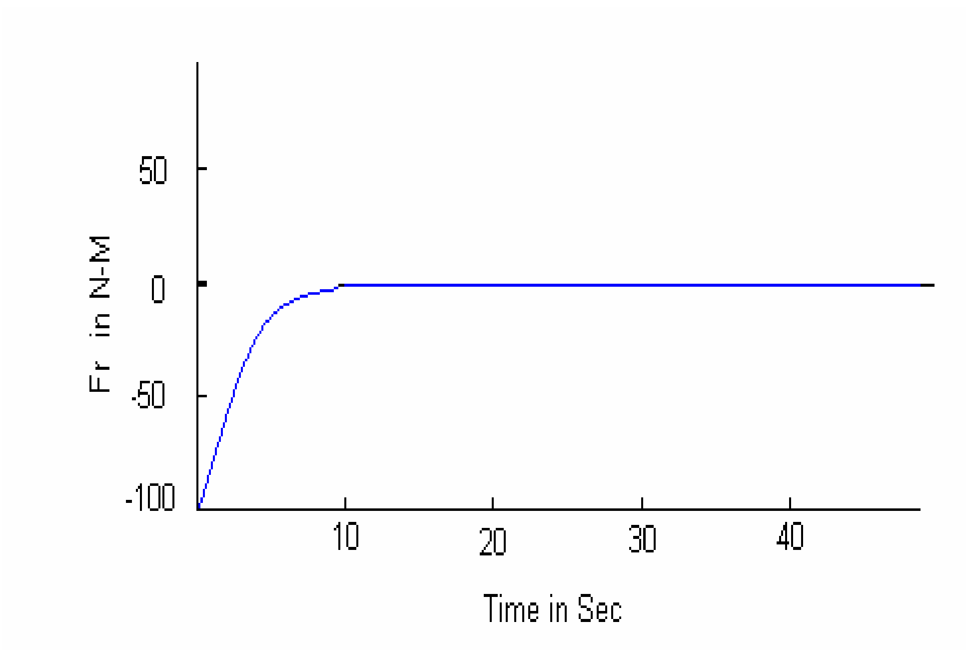

Fig.4. Control Torque F_r needed for tracking

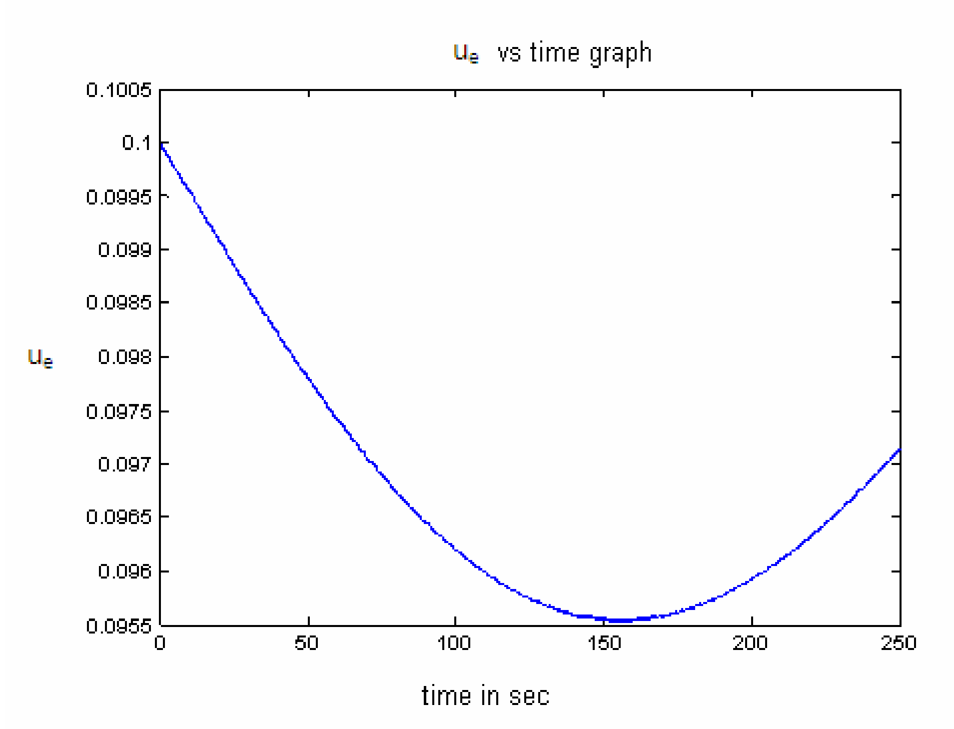

Fig 5. Linear velocity tracking in x-direction

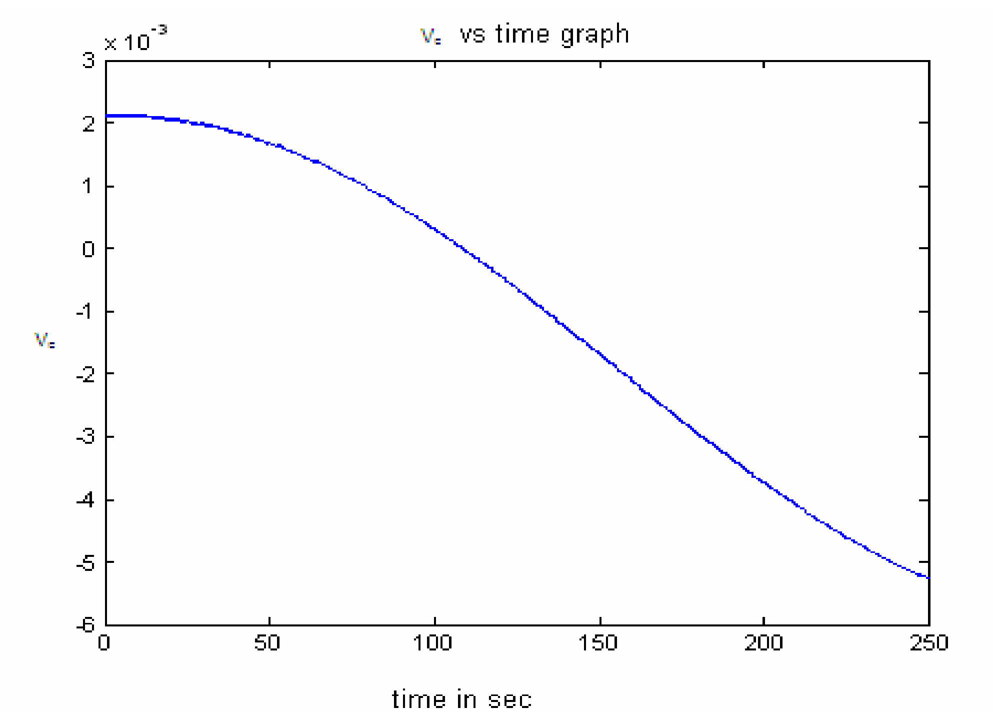

Fig 6. Linear velocity tracking in y direction

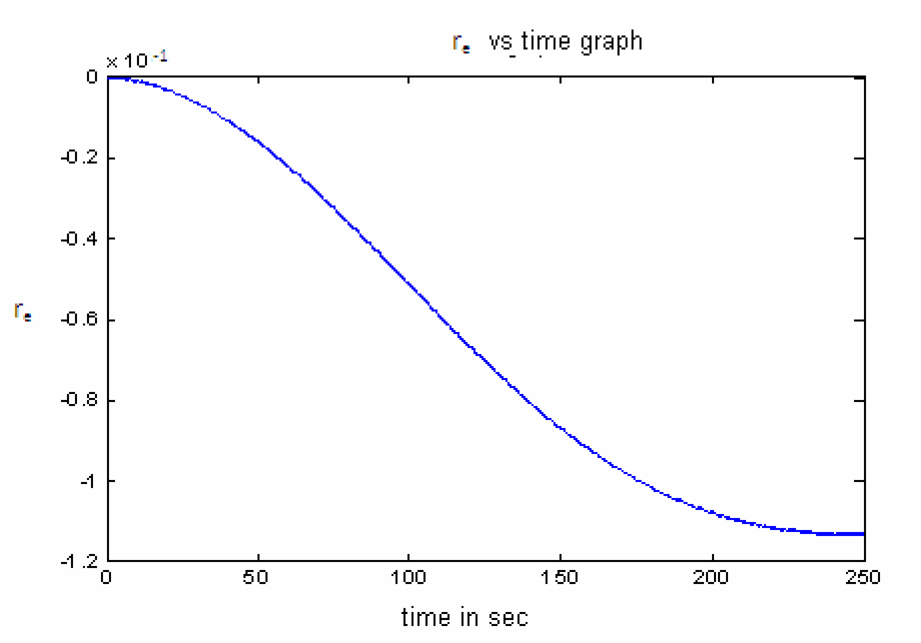

Fig.7 .Angular velocity tracking

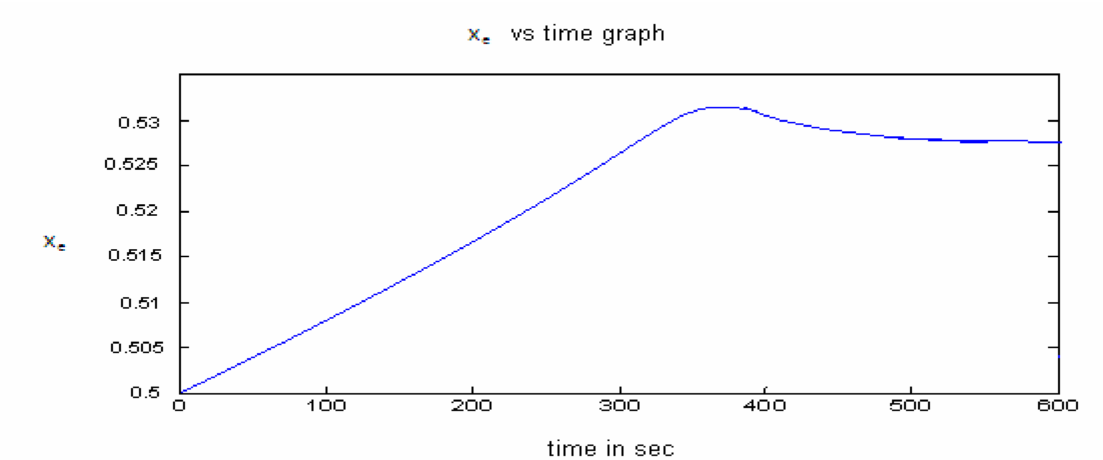

Fig 8. Position error tracking in x-axis

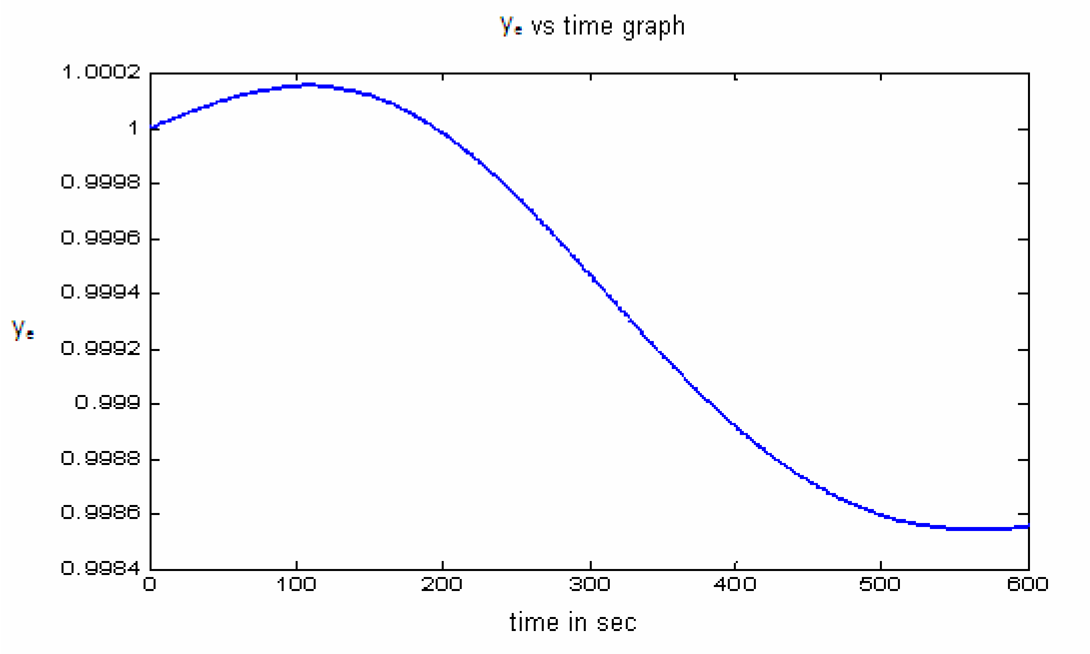

Fig 9. Position tracking error in y axis

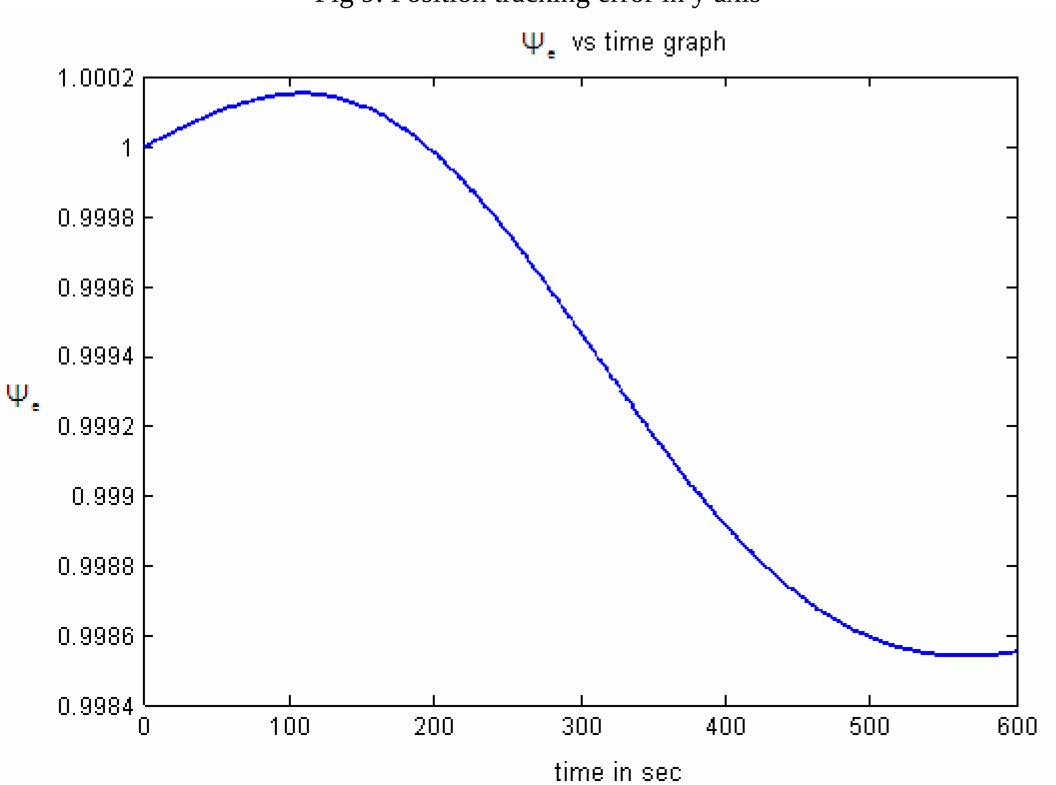

Fig 10. Orientation tracking error

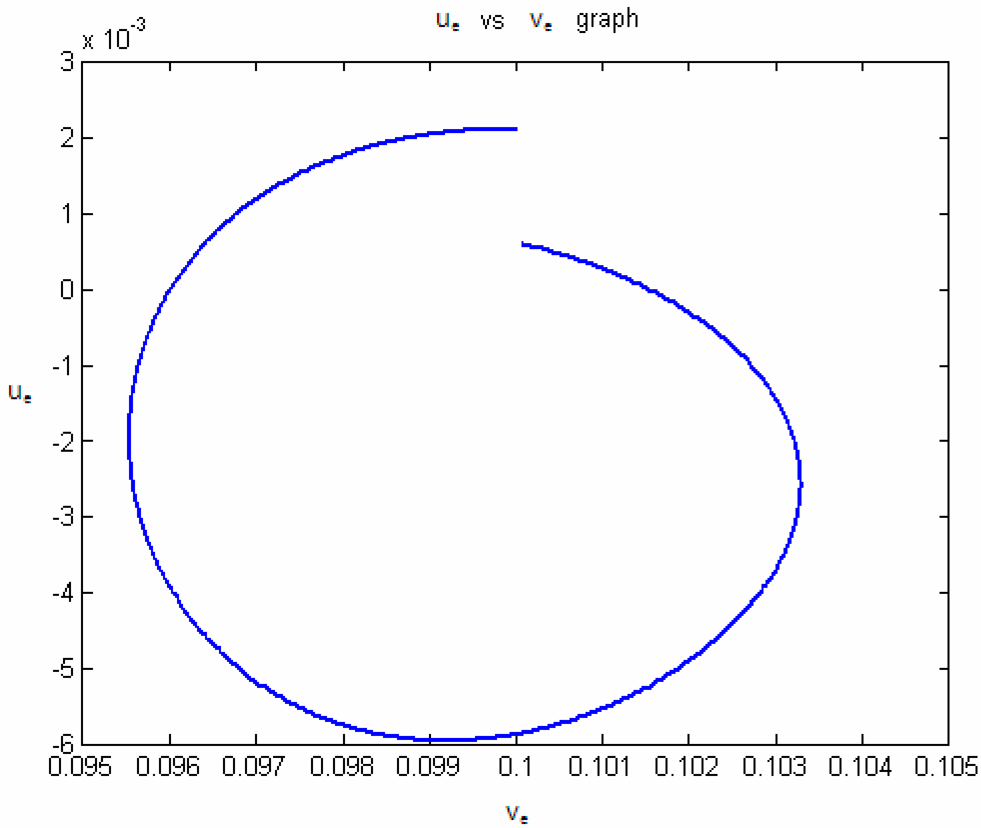

Fig 11. Error trajectory tracking

7. Conclusion

This research work develops a path tracking control scheme for an underactuated AUV moving on an horizontal plane. Path considered here is a 2D circular path and controller chosen is inverse dynamics control. A set of simulation results for proper tracking of circular path is shown in previous section. The result shows the tracking is satisfactory. The future work may be, to propose an adaptive inverse dynamics control for tracking of specified path.

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