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Comparative Performance Evaluation of Multiobjective Optimization Algorithm For Portfolio Management

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Abstract*— The moto of portfolio optimization is to find an optimal set of assets to invest on, as well as the optimal investment for each asset. This optimal selection and weighting of assets is a multi-objective problem where total profit of investment has to be maximized and total risk is to be minimized. In this paper the Portfolio optimization is solved using three different multi-objective algorithms and their performance have been compared in terms of pareto fronts, the delta, C and S metrics. Exhaustive simulation study of various portfolios clearly demonstrates the superior portfolio management capability of NSGA II based method compared to other two methods.

Index Terms*— Multi-objective optimization, Pareto-optimal solutions, global optimization, Crowding distance, Pareto front.

I.INTRODUCTION

Massive investment to different products like pension funds, banking insurance policies, stock exchange and other series of financial assets is one of the complex problems in financial management. The choice of an appropriate investment portfolio is an important task for a portfolio manager.

Optimal selection of stock exchange assets as well as the optimal investment for each asset is a well known portfolio optimization problem. Portfolio optimization is a complex task as it depends on various factors like assets interrelationships, preference of the decision makers and resource allocation. When investing money in a set of stock exchange assets, the investors are interested in obtaining the maximum profit of an investment and minimum risk simultaneously. This optimization problem has many constraints like (i) the number of assets a portfolio can contain is fixed and finite (ii) the minimum and maximum amount of possible investments for each chosen assets. In the present research, the financial assets are modeled by probability distribution. The portfolio profit is measured by average of individual profits of all assets and the risk involved is described by variance.

Choosing an optimal portfolio weighting of assets, when their future rate of return is uncertain can be viewed as a problem of minimizing the uncertainty for a given level of the portfolio expected return. The risk involved in an investment is less important as its contribution to total portfolio risk. By suitably combining a risky investment with a safer one is possible to reduce the total risk associated to that portfolio. Thus the portfolio selection task can be formulated as a multi-objective optimization problem. Some research work has been reported in this area. However there is no comparative study between these algorithms to assess the relative merits and demerits between these algorithms. Such an investigation will guide the investor for optimum portfolio selection. In this paper we consider a multi-objective portfolio assets selection and optimal weighting of assets where the total profit is maximized while total risk is minimized simultaneously. The present study employs PAES, APAES and NSGA II for modeling the Pareto front and for optimizing the portfolio performance. The results obtained with these three algorithms are finally compared by performing different numerical experiments.

Section II outlines the multi-objective optimization formulation of portfolio management. In Section III some of the multi-objective evolutionary techniques used in this paper are dealt. Simulation studies based on several numerical experiments are carried out in Section IV. The results in terms pareto fronts between risk and return are shown in Section V. Conclusions and further research work directions are discussed in the Section VI.

II. MULTI-OBJECTIVE FORMULATION OF PORTFOLIO

A portfolio p consists of N assets. Selection of optimal weighting of assets (with specific volumes for each asset given by weights (w_i) is to be found.

$$\rho_p{}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \sigma_{ij} \tag{1}$$

$$\alpha_p = \sum_{i}^{N} w_i \mu_i \tag{2}$$

$$\sum_{i}^{N} w_i = 1 \tag{3}$$

$$0 \le w_i \le 1 \text{ and } i = 1,2...,N \tag{4}$$

Where N is the number of assets available, μ_i is the expected return of asset i, σ_{ij} is the covariance between assets i and j, and w_i is the decision variables which provides the composition of the portfolio. ρ_p is the standard deviation of portfolio and α_p is the expected return of portfolio.

The multi-objective portfolio optimization problem involves two competing objectives (i) minimize the total variance, denoting the risk associated with the portfolio expressed in (1) (ii) maximize the return of the portfolio shown in (2). The problem is thus to find portfolios amongst the N assets that satisfy these two objectives simultaneously. Equation (3) provides the budget constraint for a feasible portfolio.

III. MULTIOBJECTIVE OPTIMIZATION

In a single-objective optimization problem, an optimal solution is the one which optimizes the objective with certain associated constraints. It is not possible to find a single solution for a multiobjective problem and due to the contradictory objectives a set of solutions is obtained. The general multi-objective minimization problem involves minimization of n objective functions:

$$\left\{ f_1\left(\bar{x}\right), f_2\left(\bar{x}\right)........., f_n\left(\bar{x}\right) \right\} \text{ where } n \geq 2 \quad (5)$$

The solution to this problem is more complex than the single-objective case, and the idea of Pareto-dominance is used to explain it. Consider first an objective function $\bar{F}\left(\bar{x}\right)$, where

$$\bar{F}\left(\bar{x}\right) = \left\{ f_1\left(\bar{x}\right), f_2\left(\bar{x}\right)........., f_n\left(\bar{x}\right) \right\} \quad (6)$$

A point $\bar{x}_1$ with an objective function vector $\bar{F}_1$, is said to dominate point $\bar{x}_2$, with an objective function vector $\bar{F}_2$, if no component of $\bar{F}_1$ is greater than its corresponding component in $\bar{F}_2$, and at least one component is smaller. Similarly, $\bar{x}_1$ is said to be Pareto-equivalent to $\bar{x}_2$ if some components of $\bar{F}_1$ are greater than $\bar{F}_2$ and some are smaller. Pareto-equivalent points represent a trade-off between the objective functions, and it is impossible to infer that one point is better than another Pareto equivalent point without introducing preferences or relative weighting of the objectives.

Therefore the solution to a multi-objective optimization problem is a set of vectors which are not dominated by any other vector, and which are Pareto-equivalent to each other. This set is known as the Pareto-optimal set. Grouping these Pareto optimal set generates a plot, often discontinuous known as the Pareto front or Pareto border. Its name refers to Vilfredo Pareto [4], who generalized these concepts in 1896.

IV. MULTIOBJECTIVE EVOLUTIONARY ALGORITHMS

The classical optimization techniques are ineffective for solving constrained optimization problem such as portfolio management. This short coming has motivated researchers to develop multi-objective optimization using evolutionary techniques. Based on basic concepts from the biological model of evolution, the search dynamic of multi-objective evolution algorithm (MOEA) is guided by biologically inspired evolutionary operators like selection, crossover and mutation. The crossover and mutation operator change and create potential solutions while the selection operator provides the convergence property. When MOEA is applied for portfolio optimization, issues like representation, variation operator and constraint handling techniques are considered. MOEA maintains a population of chromosome, where each of them represents a potential solution to the portfolio optimization problem. One chromosome represented by a weight vector, provides the composition of the portfolio. The weights are normalized to one to satisfy the budget constraint given in (3)

Let

$$s = \sum_{i=1}^{n} w_i \quad (7)$$

Then the new values for each element of weight vector are normalized.

$$w'_i = \frac{w_i}{s} \quad (8)$$

The normalized chromosomes then became

$$x' = \left(x'_1, x'_2, ..., x'_n\right) \quad (9)$$

In this paper we compared the portfolio optimization performance achieved by of three recently developed multi-objective evolutionary algorithms. These are Pareto Archived Evolution Strategy (PAES), Adaptive Pareto Archived Evolution Strategy (APAES) algorithm and Non dominated Sorting Genetic Algorithm II(NSGA II) for selection and optimal weighting of assets in portfolio optimization problem.

Knowles and Corne [1] have suggested a simple evolutionary algorithm called Pareto Archived Evolution Strategy (PAES). In this algorithm one parent generates one offspring by mutation. The offspring is compared with the parent. If the offspring dominates the parent, the offspring is accepted as the next parent and the iteration continues. If the parent dominates the offspring, the offspring is discarded and the new mutated solution is generated which becomes a new offspring. If the offspring and the parent do not dominate each other, a comparison set of previously non dominated individuals is used. For maintaining population diversity along Pareto front, an archive of non dominated solutions is considered. A new generated offspring is compared with the archive to verify if it dominates any member of the archive. If yes, then the offspring enters the archive and is accepted as a new parent. The dominated solutions are eliminated from the archive. If the offspring does not dominate any member of the archive, both parent and offspring are checked for their nearness with the solution of the archive. If the offspring resides in the least crowded region in the parameter space among the members of the archive, it is accepted as a parent and a copy is added to the archive.

The APAES proposed by Abraham and Grosan [2] can be considered as an adaptive representation of the standard PAES. When the current solution dominates the mutated solution for a consecutive fixed number of times it means that the representation of current solution has no potential for exploring the search space from the place where it belongs. Therefore the representation of the current solution must be changed in order to ensure a better exploration.

Dev and Pratab [3] have proposed NSGA II where selection criteria are based on the crowding comparison operator. Here the pool of individuals is split into different fronts and each front has assigned a specific rank. All individuals from a front F_i are ordered according to a crowding measure which is equal to the sum of distance to the two closest individuals along each objective. The environmental selection is processed based on these ranks. The archive will be formed by the non dominated individuals from each front and it begins with the best ranking front. Here the new population obtained after environmental selection is used for selection crossover and mutation to create a new population. It uses a binary tournament selection operator. These algorithms are dealt in sequel.

A. PAES algorithm

repeat
  Generate initial random solution *c* and add it to
  archive Mutate *c* to produce *m* and evaluate *m*
if *c* dominates *m*
  discard *m*
else
  if *m* dominates *c*
  then replace *c* with *m* and add *m* to the archive
else
  if *m* is dominated by any member of the archive
  discard *m*
else apply test (*c*, *m*, archive) to determine which
  becomes the new current solution and whether to
  add *m* to the archive
endif
endif
endif
until a termination criterion has been reached

B.APAES algorithm

repeat
  Generate initial random solution *c* and add it to
  archive
  k = 0
  Mutate *c* to produce *m* and evaluate *m*
If *c* dominates *m*
then *k* = *k* + 1;
if *k* = Maximum number of harmful mutations
then change the representation for the current
  solution (i.e. mutate the alphabet over 2 which
  the current solution is represented);
  k = 0
elseif *m* dominates *c*
then replace *c* with *m* and add *m* to the archive
elseif *m* is dominated by any member of the archive
then discard *m*
else apply test (*c*, *m*, archive) to determine which
  becomes the new current solution and
  whether to add m to the archive
endif
endif
endif
until a termination criterion has been reached

C. NSGA II Algorithm

1. Initialize population
2. Generate random parent population p_0 of size N
3. Evaluate objective Values
4. Assign fitness (or rank) equal to its non dominated level
5. Generate offspring Population Q_0 of size N with binary tournament selection, recombination and mutation.
6. For $t=1$ to Number of Generations
6.1 Combine Parent and Offspring Populations

6.2 Assign Rank (level) based on Pareto Dominance.
6.3 Generate sets of non-dominated fronts
6.4 until the parent population is filled do
6.4.1 Determine Crowding distance between points on each front F_i
6.4.2 Include the ith non dominated front in the next parent population (P_{t+1})
6.4.3 check the next front for inclusion
6.5 Sort the front in descending order using Crowded comparison operator
6.6 Choose the first N - card (P_{t+1}) elements from front and include them in the next parent population (P_{t+1})
6.7Using binary tournament selection, recombination and mutation create next generation
7. Return to 6

IV. SIMULATION STUDIES

In this section we present the simulation results obtained when searching the general efficient frontier that resolves the problem formulated in equation 1 and 2. The efficient frontier is computed using different MOEA like PAES, APAES and NSGA II.

All the computational experiments have been computed with a set of benchmark data available online and obtained from OR-Library being maintained by Prof. Beasley. Five data sets port1 to port5 represent the portfolio problem. Each data set corresponds to a different stock market of the world. The test data comprises of weekly prices from March 1992 to September 1997 from the following indices: Hang Seng in Hong Kong, DAX 100 in Germany, FTSE 100 in UK, S&P 100 in USA and Nikkei in Japan. For each set of test data, the numbers of different assets are 31,85,89,98 and 225. In the paper we have used the first data set which corresponds to Hang Seng stock having 31 assets. The data can be found at http://people.brunel.ac.uk/~mastjjb/jeb/orlib /portinfo.html.

The PAES and APAES use a population size of 100 each and number of gene equals to number of assets, number of generations as 100 and mutation rate 0.05. The NSGA II has population size of 100, number of generations 100, crossover rate 0.8 and mutation rate 0.05. The number of real-coded variables is equal to number of assets and the selection strategy used is tournament selection.

1. S metric

The S metric proposed in [7] indicates the extent of objective space dominated by a given nondominated set A. If the S metric of a non dominated front f_1 is less than another front f_2 then f_1 is better than f_2. It has been proposed by Zitzler [7].

2. Δ metric

This metric called as spacing metric (Δ) measures how evenly the points in the approximation set are distributed in the objective space. This formulation introduced by K. Deb [3] is given by

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{N-1}\left|d_i - \bar{d}\right|}{d_f + d_l + (N-1)\bar{d}} \tag{6}$$

Where d_i be the Euclidean distance between consecutive solutions in the obtained nondominated set of solutions. $\bar{d}$ is the average of these distances. d_f and d_l are the Euclidean distance between the extreme solutions and the boundary solutions of the obtained non dominated set and N is the number of solutions from nondominated set. The low value for Δ indicate a better diversity and hence better is the algorithm.

3. C metric

Two sets of non dominated solutions are compared using C metric. The definition of C metric given in [7] for convergence of two sets A and B is given by:

$$C(A,B) = \frac{\left|\{b \in B \mid \exists a \in A : a \succ b\}\right|}{|B|} \tag{7}$$

TABLE I
THE RESULTS OBTAINED FOR S AND Δ METRICS

Algorithm	PAES	APAES	NSGA II
Metric S	0.000404236	0.0000057372	0.000000574
Metric Δ	0.892482853	0.7862596192	0.5967844252

Table I shows the S and Δ metrics obtained using all the three algorithms. It may be observed from the Table I that NSGA II performs better as its S and Δ metric values are less than those obtained by other two algorithms.

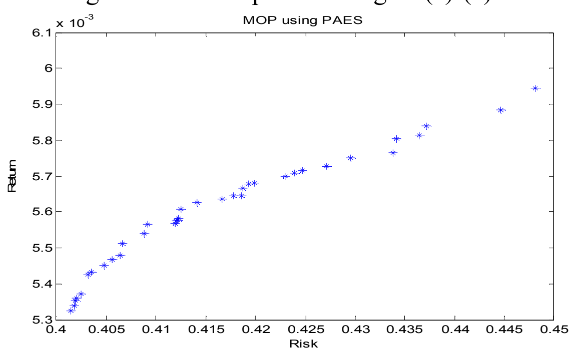
TABLE II
THE RESULTS OBTAINED FOR C METRIC

	PAES	APAES	NSGA II
PAES	—	0.0000	0.0000
APAES	0.95990	—	0.2653
NSGA II	0.96627	.07534	—

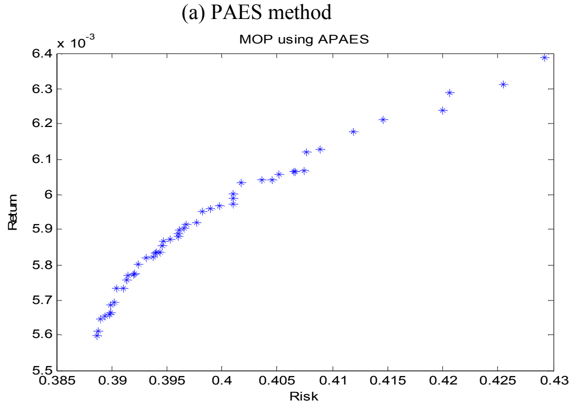
Table II demonstrates the results of C metric. A magnitude of 0.96627 on the third line, first column signifies that almost all solutions from final populations obtained by NSGA II dominate the solutions obtained by PAES. The value 0 on first row means that no solution from the nondominated population obtained by APAES and by NSGA II is dominated by solutions from final populations obtained by PAES. The performance of the two algorithms NSGA II and APAES are almost same but closely analyzing the value of C it can be concluded that performance of NSGA II is better than APAES.

V.THE PARETO FRONTS OF DIFFERENT METHODS

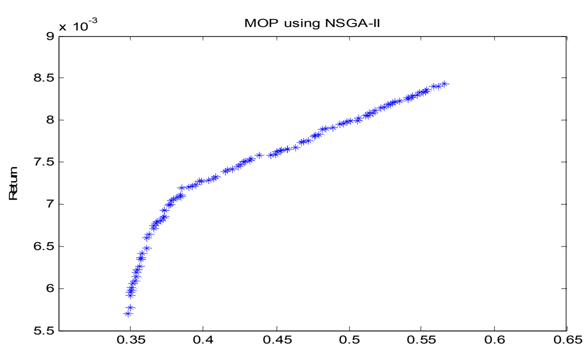
The Pareto fronts (between risk and return) obtained by three algorithms are depicted in Figs. 3(a)-(c)



(a) PAES method



(b) APAES method



(c) NSGA II method

Fig. 3 Plots of Pareto fronts achieved by three methods

VI.CONCLUSION

The paper makes a comparative study of three multi-objective approaches PAES, APAES and NSGA II. Experimental results reveal that the NSGA II algorithm outperforms other two MOEA algorithms in different experiments conducted. Future work include introduction of different operators for local search in the existing models which allow better exploration and exploitation of the search space when applied to portfolio optimization problem.

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