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Design of a Robust Neuro-Controller for Complex Dynamic Systems

Ki-Young Song and Madan M. Gupta*
Division of Biomedical Engineering
University of Saskatchewan
Saskatoon, Canada

Debashisha Jena and Bidyadhar Subudhi
Department of Electrical Engineering
National Institute of Technology
Rourkela, India

*Corresponding author: madan.gupta@usask.ca

Abstract- **Design of neuro-controller for complex dynamic systems is a big challenge faced by the researchers. In this paper we present a design of a robust neuro-controller for a dynamic system to make the system response fast with no overshoot. Here the control action decided by the controller completely depends on the value of the error at that point of time. The position feedback which controls the bandwidth of the system as well as the dynamic response is a function of the system error. For large error the position feedback is made large increasing the bandwidth of the system, and for small errors the position feedback value is small. Thus, during the dynamic response of the system the bandwidth of the system is controlled by the system error. Similarly, the velocity feedback which controls the damping in the system is kept very small for large errors, and large for small errors. Thus, in the proposed neuro-controller the position feedback $K_p(e,t)$, and velocity feedback $K_v(e,t)$ are made as a function of error which yields a very fast response with no or very little overshoot.**

Keywords – robust neuro-controller; position feedback; velocity feedback; Neural Networks

I. INTRODUCTION

Neural networks (NNs) are now being used as a function to identify linear and nonlinear dynamic systems in engineering. The NNs have established their usefulness in such fields as function mapping, pattern recognition, and image processing. NNs have potential of developing attributes such as parallelism, adaptability, robustness, and the inherent ability to handle nonlinearity [1]. Recently, higher-order artificial neural networks have been studied in order to emulate and present a superior performance of the nonlinear functions of the biological neural networks [2-7]. The artificial higher-order neural units consist of a combination of synaptic operation and somatic operation which are fundamental processes referred to as static or feedforward structures [4, 8]. The mathematical representation of the neural units with m^{th} order synaptic operation can be written as

$$v=\sum_{i_1=0}^{n}\sum_{i_2=i_1}^{n}\cdots\sum_{i_m=i_{m-1}}^{n} w_{i_1 i_2 \dots i_m} x_{i_1} x_{i_2} \dots x_{i_m} \qquad (1)$$

where $x_{i_j}, i_j = 0,1,\dots,n$ are the neural inputs and $w_{i_1 i_2 \dots i_m}$, $i_j = 0,1,\dots,n$ are the synaptic weights. The output is given by the somatic operation,

$$y_N = \Phi[v] \in \Re^1 \qquad (2)$$

where y_N is an output scalar, and $\Phi[\bullet]$ is a sigmoidal activation function [2]. If the error is reduced to an infinitesimally small value as the number of learning iterations increases, the learning scheme is said to be convergent [9]. However, biological neural systems are composed of recurrent connections and more sophisticated [10-12]. Thus, the limited nonlinear combination of a mathematical neural structure is not fully able to describe the complexity of the biological neuron. We expand the conventional mathematical synaptic operation with functionalized nonlinear combinations such as exponential, absolute and sinusoidal functions. The functionalized nonlinearity of a neural structure can be selected depending on the applications. In this paper, we apply the functionalized nonlinearity in the synaptic operation and present that a suitable nonlinearity of a novel neuro-controller can make the control system response faster, more stable and robust. For a simulation purpose, a second-order system, satellite model, is applied.

II. DESIGN OF A ROBUST NEURO- CONTROLLER

Now we present the design of a robust neuro-controller using some of the observations made in the above sections for a second order dynamic system.

A. Dynamic Response of a second-order system: An example

Generally the dynamic behavior of a second-order system can be described in terms of two parameters, undamped natural frequency (ω_n) and damping ratio (ζ). Those are a function of position feedback (K_p) and mainly a function of velocity feedback (K_v) respectively as Fig. 1 and Eqn.(3).

$$\dot{x}_1 = x_2$$

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$$\dot{x}_2 = u = r - (K_v x_2 + K_p x_1)$$
$$y = K_p x_1 \tag{3}$$

where r is the reference input and the initial conditions are $x_1(0) = 0$ and $x_2(0) = 0$. The transfer function of the system is derived as

$$\frac{Y(s)}{R(s)} = G(s) = \frac{K_p}{s^2 + K_v s + K_p} \tag{4}$$

The characteristic equation of $G(s)$ can be compared with that of the standard form of the second-order system. Thus,

$$s^2 + K_v s + K_p = s^2 + 2\zeta\omega_n s + \omega_n^2$$
$$\omega_n = \sqrt{K_q}$$
$$\zeta = \frac{K_v}{2\sqrt{K_p}}$$
$$\omega_d = \omega_n\sqrt{1-\zeta^2} \tag{5}$$

where ω_n is undamped natural frequency, ζ is damping ratio and ω_d is damped natural frequency [15]. The definition of the parameters is illustrated in Fig. 2.

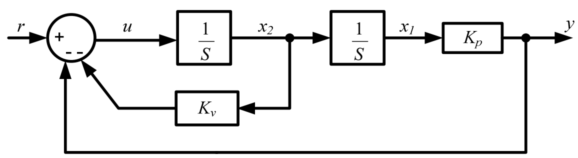


Figure 1. A typical second-order system with velocity feedback (K_v) and position feedback (K_p)

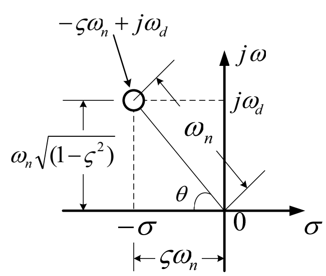


Figure 2. Definition of the parameters

The transient response of a practical control system often exhibits damped oscillations before reaching steady state. In specifying the transient-response characteristics of a control system to a unit-step input (Fig. 3), it is common to specify the following [13-15]:

- Delay time, *Td*
- Rise time, *Tr*
- Peak time, *Tp*
- Maximum overshoot, *Mp*
- Settling time, *Ts*

Since the rise time (*Tr*) and settling time (*Ts*) are defined as

$$Tr = \frac{\pi - \theta}{\omega_d} = \frac{\pi - \theta}{\omega_n\sqrt{1-\zeta^2}} \text{ and,}$$

$$Ts = \frac{4}{\zeta\omega_n} \text{ (2\% criterion)} \tag{6}$$

it is important to notice that *Tr* and *Ts* are dependent on undamped natural frequency (ω_n) and damping ratio (ζ) which decide the positions of poles of a system as shown in Fig. 2.

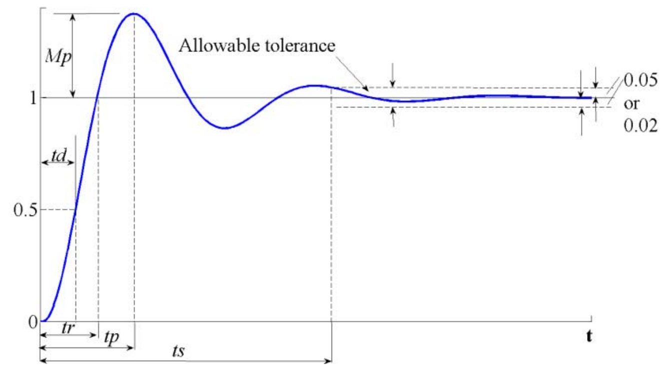


Figure 3. A typical system response to a unit-step input

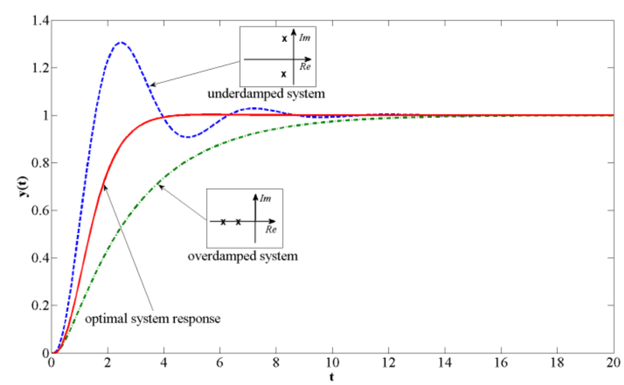


Figure 4. System responses to a unit-step input with different pole positions which lead underdamped case and overdamped case. The initial conditions of the systems are zeros. The optimal system response can be considered with optimal pole positions.

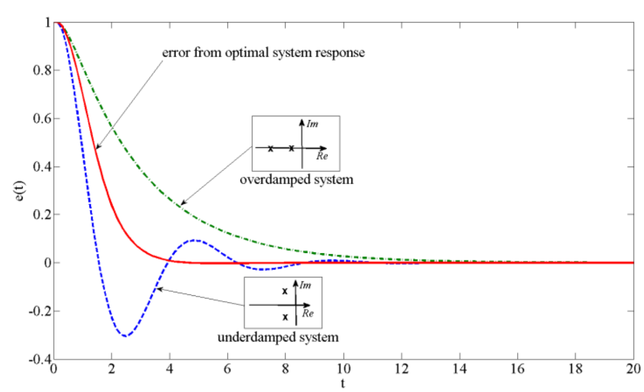


Figure 5. Errors from the different system responses

The system responses and the errors to a unit-step input with different pole positions are shown in Fig. 4 and Fig. 5. The transient response of the underdamped system yields larger *Mp* and slower *Ts*, but faster *Tr*. However, in the case of the overdamped system, the transient response presents no *Mp*, but slower *Tr* and *Ts* are acheived. Regarding the transient responses from the two systems, an optimal system response can be proposed considering ω_n and ζ.

As observed in the previous section, for large errors the small ζ and large ω_n will yield a very fast response with a very small rise time, $Tr = \frac{\pi - \cos^{-1}(\zeta)}{\omega_n\sqrt{1-\zeta^2}}$, and for small errors a large ζ and small ω_n will inhibit any overshoot. Thus, if we make $K_p(=\omega_n^2)$ and $K_v(=2\zeta\omega_n)$ as a function of system error $e(t) = r(t) - y(t)$ then we can achieve a very fast dynamic response with no overshoot, small rise time and fast settling time. Thus, we propose the following design parameters. In this example, there are two design parameters:

- Position feedback: K_p which controls the natural frequency of the system ω_n i.e. $K_p = \omega_n^2$
- Velocity feedback: K_v which controls the damping ratio ζ i.e. $K_v = 2\zeta\omega_n$

Thus, the controller design criteria are:

1. *When the system error is large, ζ should be very small with large ω_n*
2. *When the system error is small, ζ should be large with small ω_n*

Thus we can make K_p and K_v as a function of error such that ($K_p = \omega_n^2$) change from a large value to a small value with decreasing error and ($K_v = 2\zeta\omega_n$) change from a very small value to a large value with decreasing error.

One can develop many K_p and K_v functions which satisfy the above two criteria. One such proposed design for K_p and K_v is

$$K_p = K_{p0}(1 + \alpha e^2) \tag{7}$$

$$K_v = K_{v0}\mathrm{Exp}(-\beta e^2) \tag{8}$$

where α and β are some gain constants, K_p and K_v are equivalent to the time varying synaptic weights w_1 and w_2, and K_{p0} and K_{v0} are initial values of K_p and K_v. The other possible functions for $K_p(e,t)$ and $K_v(e,t)$ are given in Table 1.

III. SIMULATION STUDIES

As discussed, an optimal transient response of a system yields faster settling time, faster rise time and smaller overshoot. In order to meet the proposed status, when the system starts, lower damping ratio is applied and makes the system response faster, but larger damping ratio is applied to make the system stable when the system response is near the target value.

TABLE 1. VARIOUS FUNCTIONS FOR POSITION AND VELOCITY FEEDBACK GAINS

$K_p(e,t)$	$K_v(e,t)$
$K_{p0}(1+\alpha\lvert e\rvert)$	$K_{v0}\frac{1}{1+\beta\lvert e\rvert}$
$K_{p0}(1+\alpha e^2)$	$K_{v0}\frac{1}{1+\beta e^2}$
$K_{p0}(1+\alpha e^2)$	$K_{v0}\mathrm{Exp}(-\beta e^2)$

Note: Many other functions can be derived for $K_p(e,t)$ and $K_v(e,t)$ for example taking the hyperbolic tangent (tanh(•)) and cosine (cos(•)) functions.

Since the natural frequency determines the rise time and settling time as well as expressed in Eqn.(6), we design a neuro-controller considering both damping ratio and natural frequency. A satellite model is used as an example for the simulation study.

A. Satellite positioning control: An example

Satellite usually needs a decent control to adjust its antenna to the station on the earth by rotation as shown in Fig. 6. The rotation angle θ of the satellite is determined by the force of gas jet on the satellite. The motion of the satellite is derived by basic Newton's law as

$$2RF(t) = J\ddot{\theta}(t), \tag{9}$$

and the rotation angle θ of the satellite. Thus, the transfer function of the satellite system is

$$\theta(s) = \frac{2RF(s)}{Js^2} \tag{10}$$

From Eqn.(10), it is clear that the transfer function of the satellite has two integrators and two states x_1 and x_2 which represent position and velocity of the satellite. A controller is implemented to control the satellite. In order to control the satellite stable, for the zero steady state error, the closed loop poles need to be positioned on the real axis, which yields an overdamped system. Thus, we allocate two poles at the position of -1 and -3, which makes the value of velocity feedback (K_v) and position feedback (K_p) 4 and 3 respectively (Fig. 1). The system response and error curves with the given value of K_v and K_p is shown in Fig. 7.

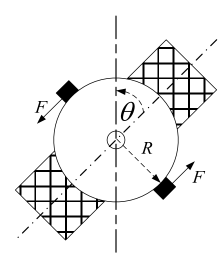


Figure 6. Schematic satellite positioning by jetting gas to rotate

The result explains that this overdamped system is very stable and does not generate overshoot (*Mp*). The rise time (*Tr*) of the system is 2.67 seconds. However, the high damping ratio (ζ) renders the system sluggish to reach the settling time (*Ts*). In order to develop the system response as the way that the response yields faster *Tr* and lower *Mp*, we design and implement a neuro-controller in the satellite model. Since the overdamped system is very stable, the object of the neuro-controller is to adapt the stable system with faster *Tr* as the design criteria. That is to say, the system is underdamped at the starting point and overdamped when the system is near the target.

B. Nonlinear neuro-controller

An optimal damping would make a system response very quickly with minimal overshooting. The nonlinearity of the controller can generate an optimal damping. A neuro-controller can be applied to meet the nonlinearity of a desired controller, which refers the cross-relation of position and velocity.

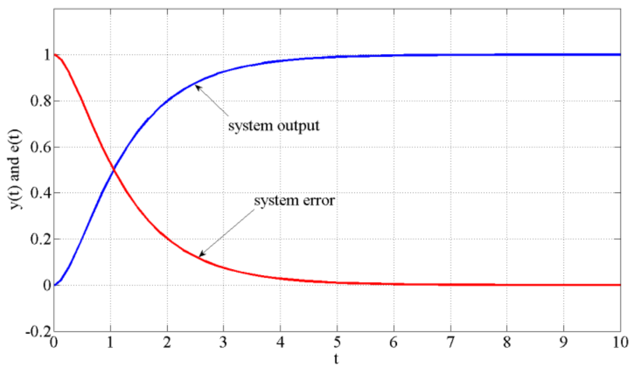


Figure 7. The system output and error of the satellite position control with overdamping to a unit-step input. *Mp = 0* and *Tr = 2.67 (sec)*.

A proposed neuro-controller for the satellite is shown in Fig. 8. The velocity feedback (K_v) and position feedback (K_p) are adapted by the system error. The synaptic operation of a neural structure is derived with given two inputs, position (x_1) and velocity (x_2), and generates the control signal u as

$$u = r - \left(K_v x_2 + K_p x_1\right) \tag{11}$$

where r is the reference input. Thus, K_p and K_v are derived with respect to the error as,

$$K_p = K_{p0}(1 + \alpha e^2) \tag{12}$$

$$K_v = K_{v0}\text{Exp}(-\beta e^2) \tag{13}$$

$$e = r - y \tag{14}$$

where K_{p0} and K_{v0} are the initial values of K_p and K_v, α and β are the gain constants, Exp(●) is the exponential function, e is the system error, and y is the system output. For this satellite positioning control, K_p is proportional to the undamped natural frequency (ω_n), and K_v is proportional to damping ratio (ζ) from Eqn.(5). By the proposed criteria, ω_n should decrease and ζ should increase as the error decreases.

Now, we design the neuro-controller with the selected functions and the proposed criteria. Considering pole positions, we arrange the initial poles at 0.1±*j*2 and final poles at -1 and -3 on the real-imaginary plane. The initial poles of the closed-loop system lead fast *Tr*. However, if the position of the poles is firm, the system becomes unstable.

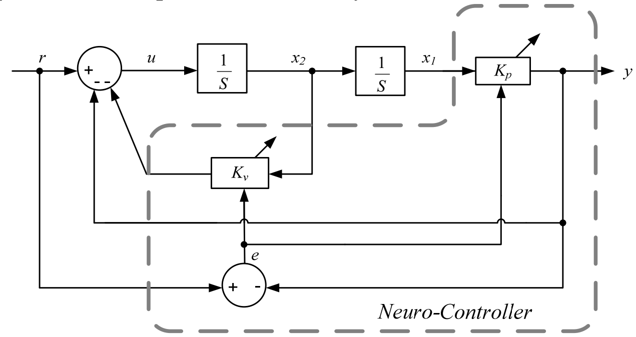


Figure 8. Robust neuro-controller for the satellite positioning: $K_p(e,t) = K_{p0}\Phi_1(\text{e})$ and $K_v(e,t) = K_{v0}\Phi_2(\text{e})$

Thus, the poles move to the desired position as the error becomes zero. Regarding the error of the system which responses to a unit-step input, $e = 1$ at $t = 0$ and $e = 0$ at $t = \infty$ as shown in Fig. 7. From the initial and final positions of the poles and the value of the error, the values of K_{p0}, K_{v0}, α and β are calculated as 4, 3, 0.3367 and 2.9957 respectively. From Eqns.(5), (12), (13) and (14), the desired value of ω_n and ζ at time t are derived as

$$\omega_n(t) = \sqrt{K_{p0}(1 + \alpha(r(t) - y(t))^2)} \tag{15}$$

$$\zeta(t) = \frac{K_{v0}\text{Exp}(-\beta(r(t)-y(t))^2)}{2\sqrt{K_{p0}(1+\alpha(r(t)-y(t))^2)}} \tag{16}$$

The system response and the error curves are shown in Fig. 9. The results compare the response curves of the previous result from overdamped system and neuro-control system.

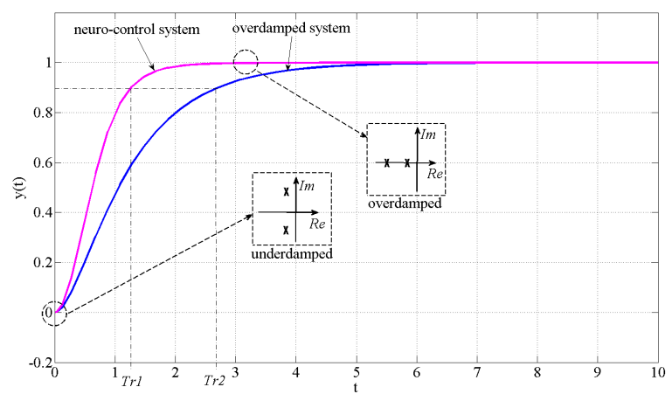
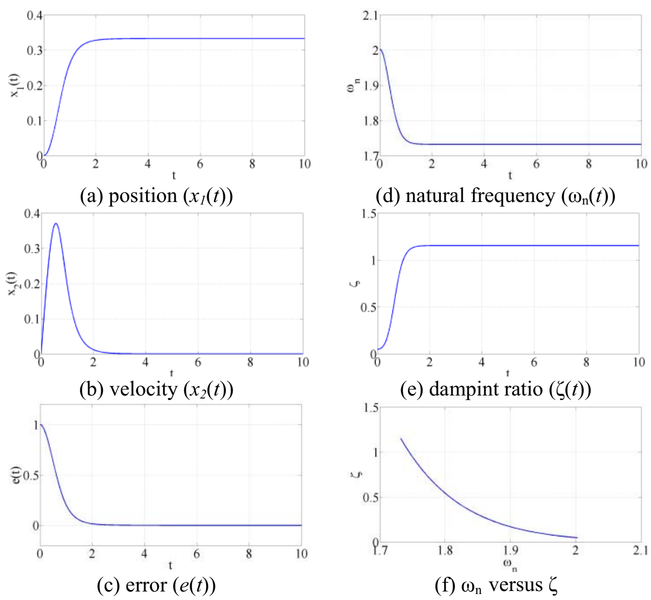


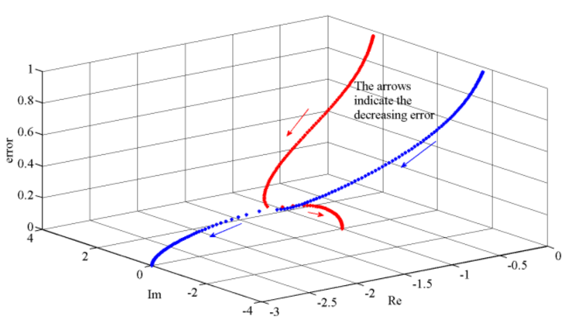
Figure 9. The system response curves of two systems. The neuro-control system reaches to the target faster and stable with *Mp = 0* and *Tr1 = 1.26 (sec)*. The initial poles of the closed-loop neuro-control system are positioned at 0.1±*j*2 for fast response and moved to -1 and -3 for settling down at the end as shown in pole positions in small windows. *Tr2 = 2.67(sec)* for the overdamped system.

As shown, about 53% of rise time is compensated without overshoot with the robust neuro-controller. In Fig. 10, the dynamic response curves of the system with the neuro-controller are plotted. At the beginning of the response, ζ is small and ω_n is high, which makes very small *Tr* with large system error. However, as the system reaches near the target, ζ becomes high and ω_n is small, which drives the system stable with small error. Further, Figure 11 presents the locus of the poles with respect to the error which is named as error based dynamic pole locus. It is clear from the figures that as the error decreases, the poles are approaching faster to their final position without yielding small or no overshoot.

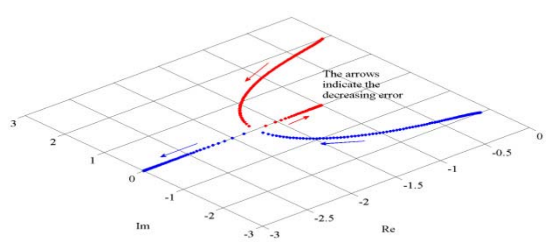


(a) position ($x_1(t)$) (d) natural frequency ($\omega_n(t)$)
(b) velocity ($x_2(t)$) (e) dampint ratio ($\zeta(t)$)
(c) error ($e(t)$) (f) ω_n versus ζ

Figure 10. Dynamic responses of the satellite system



(a) Error-based dynamic pole locus with respect to the system error. As the error decreases, the poles moves to the final positions.



(b) The poles of the system moves from from $0.1\pm j2$ to -1 and -3

Figure 11. Pole placement with respect to the system error. The arrows indicates the decreasing error. The initial poles are positioned at $0.1\pm j2$ and settles at -1 and -3 at the end.

IV. Conclusion

In this paper we have suggested an error based neuro-controller for controlling the dynamic response of a complex dynamic system. The design of the neuro-controller is conceptually error based and using a satellite control problem. It is shown that the response is very fast yielding a very small rise time with no overshoot. Further work is under way to extend this neuro-controller design philosophy for higher order complex dynamic systems.

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