Buckling and Parametric Instability Behavior of Functionally Graded Shells

S. Pradyumna* and J.N. Bandyopadhyay†
*Assistant Professor, Department of Civil Engineering, National Institute of Technology Rourkela, Orissa – 769 008, email: spradyumna@nitrkl.ac.in
†Professor, Department of Civil Engineering, Indian Institute of Technology Kharagpur, West Bengal – 721 302, email: jnb@civil.iitkgp.ernet.in

Functionally graded (FG) materials are a class of composites that have a continuous variation of material properties from one surface to another and thus eliminate interface problems found in laminated composites. FG materials are typically manufactured from isotropic components such as metals and ceramics since they are used as thermal barrier structures in environments with severe thermal gradients. FG materials have the advantage of heat and corrosion resistance typical of ceramics and mechanical strength and toughness typical of metals. Buckling and parametric instability behavior of functionally graded shells subjected to in-plane static and pulsating loads are carried out in the present paper. The shell forms considered here are cylindrical (CYL), spherical (SPH) and hypar (HYP). Temperature change through the thickness is not uniform, and is governed by one-dimensional Fourier equation of heat conduction. Finite element formulation based on a higher order shear deformation theory is used to carry out the analyses. The parametric instability problem is solved using the Bolotin's approach.

The displacement fields are assumed in the form as [1],

$$\begin{aligned} u(x,y,z) &= u_0(x,y) + z\theta_y + z^2 u_0^*(x,y) + z^3\theta_y^*(x,y) \\ v(x,y,z) &= v_0(x,y) - z\theta_x + z^2 v_0^*(x,y) - z^3\theta_x^*(x,y) \\ w(x,y,z) &= w_0(x,y) \end{aligned} \quad \text{.....(1)}$$

where *u, v* and *w* are the displacements of a general point (*x,y,z*) in an element along *x, y* and *z* directions, respectively. The parameters u_0, v_0 and w_0 are the displacements of mid-plane along *x, y* and *z* axes, respectively. The symbols θ_x and θ_y are rotations of the mid-plane about *x* and *y* axes, respectively, while u_0^*, v_0^*, θ_x^* and θ_y^* are the higher-order terms in Taylor's series expansion and represent higher-order transverse cross sectional deformation modes.

The effective material property *P* (such as Young's modulus *E*, Poisson's ratio ν, mass density ρ, coefficient of thermal expansion α, thermal conductivity *K* etc.) can be expressed as

$$P(z) = (P_c - P_m)\left(\frac{2z+h}{2h}\right)^n + P_m \quad \text{.....(2)}$$

where P_c and P_m refer to the corresponding properties of the ceramic and metal constituents, respectively and *n* represents the volume fraction index.

For a FG panel, the temperature change through the thickness is not uniform, and is governed by one-dimensional Fourier equation of heat conduction.

$$-\frac{d}{dz}\left[K(z)\frac{dT}{dz}\right] = 0 \quad \text{.....(3)}$$

subjected to conditions, $T = T_c$ (at $z = h/2$), $T = T_m$ (at $z = -h/2$)

where T_c and T_m denote the temperature changes at the ceramic and metal sides, respectively.

The governing differential equation of motion of the undamped system is written as [2],

$$\mathbf{M}\{\ddot{d}_s\} - \left[(\mathbf{K_e} + \mathbf{K_T}) + P(t)\mathbf{K_g}\right]\{d_s\} = \{0\} \quad \text{.....(4)}$$

where, $\mathbf{K_T}$ is the initial stress stiffness matrix due to temperature field, $\mathbf{K_e}$, $\mathbf{M}$ and $\mathbf{K_g}$ are the elastic stiffness, mass and geometric stiffness matrices, respectively, and $\{d_s\}$ and $\{\ddot{d}_s\}$ are the structural displacement and acceleration vectors, respectively.

In Eq. (4), the in-plane load factor $P(t)$ is periodic and can be expressed in the following form

$$P(t) = P_s + P_t \cos\theta t \quad \text{.....(5)}$$

in which, P_s is the static portion of the load, P_t is the amplitude of the dynamic portion and θ is the frequency of excitation. The quantities P_s and P_t are expressed in terms of static elastic buckling load P_{cr} of panel as,

$$P_s = \alpha P_{cr}, \qquad P_t = \beta P_{cr} \quad \text{.....(6)}$$

where α and β are static and dynamic load factors, respectively.

For a static buckling problem with in-plane load, equation (4) becomes:

$$\left[\left(\mathbf{K_e}+\mathbf{K_T}\right)+P_{cr}\mathbf{K_g}\right]\{d_s\}=0 \qquad (7)$$

Eigenvalues of the above governing equation are the buckling loads for different modes. The parametric instability regions are obtained by solving the eigenvalue problem

$$\left[(\mathbf{K_e}+\mathbf{K_T})-\alpha P_{cr}\mathbf{K_g}\pm\frac{\beta}{2}P_{cr}\mathbf{K_g}-\frac{\theta^2}{4}\mathbf{M}\right]\{d_s\}=0 \qquad (8)$$

The validity of the present approach is established by comparing the results obtained by the present study with those available in the literature (Table 1).

Table 1 Comparison of $\gamma = P_{cr}b^2/\left(\pi^2 D_c\right)$ of a clamped plate made of Aluminum-Zirconia

	a/*h* = 20		*a*/*h* = 40	
	Present	Ref. [3]	Present	Ref. [3]
n = 0.0	9.2760	9.3922	9.7985	9.6938
n = 2.0	5.9325	6.0544	6.2757	6.2517
n = 5.0	5.4752	5.6770	5.8111	5.8829
n = ∞	4.5892	4.3540	4.8573	4.4938

Figure 1 shows the effect of volume fraction index *n* on the dynamic instability regions of CYL (Fig. 1a), SPH (Fig. 1b) and HYP (Fig. 1c) shells for five different values of *n*. For HYP shells, the rise to length ratio *c/a* is taken as 0.2.It is observed that the origin of instability regions for pure ceramic shells (*n* = 0) occurs at higher excitation frequencies than the same for pure metallic shells (*n* = ∞) of all the shell forms. Also it is seen that with the increase in the value of *n*, the origin of instability shifts to lower excitation frequency and width of the instability regions is reduced.

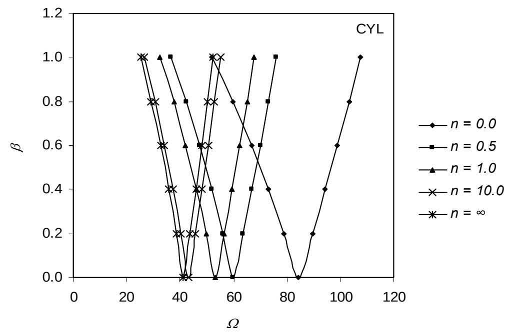

Fig. 1 (a) SPH shell (*a*/R_x = 10)

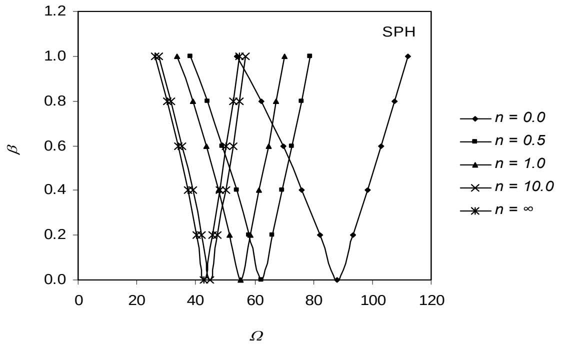

Fig. 1 (b) SPH shell (*a*/R_x = 10)

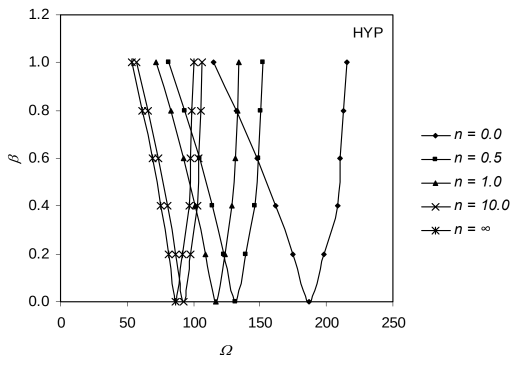

Fig. 1 (c) HYP shell (*c*/*a* = 0.2)

Fig. 1. Effect of volume fraction index on dynamic instability regions of FG shells

References:

1. Kant T., Khare RK., 2003, “A higher-order facet quadrilateral composite shell element.” *Int. J. for Num. Meth.in Engg.*, 40, 4477-4499.
2. Pradyumna S, Bandyopadhyay JN, (In press), “Dynamic stability of functionally graded shells using a higher order theory”, *ASCE Journal of Engineering Mechanics*.
3. Liew KM, Yang J, Kitipornchai S, 2003, “Postbuckling of piezoelectric FGM plates subject to thermo-electro-mechanical loading”, *International Journal of Solids and Structures* 40, 3869-3892.