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COMPARISON OF INTACT ROCK FAILURE CRITERIA USING VARIOUS STATISTICAL METHODS

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ABSTRACT

This paper describes comparison of four different rock failure criteria based on triaxial test data of ten different rock strength data using various statistical methods. Least square, least median square and re-weighted least square techniques are used to find out the best fit parameters utilizing the experimental data that describes the failure state for each criterion. The least median square method could identify the scattered data and these scattered data points are observed at higher confining stress. It was observed that the fitting of failure criteria to different rock strength data depends upon the statistical methods used. The prediction of unconfined compressive strength and failure strength for different rock, estimated using various statistical methods are discussed in terms of different statistical performances of the prediction.

Key words: Rock failure criteria; parameter estimation; least square method; least median square method; re-weighted least square method.

Symbols:

σ_1 Major principal stresses

σ_3 Minor principal stresses

C_0 Uniaxial compressive strength of intact rock for Hoek-Brown failure criteria

m, s The material parameters for Hoek-Brown failure criteria

$\hat{\sigma}$ Standard deviation for Least Median Square method

r_i Residuals from Least Median Square fit

C_0, b and α Material parameters for Yudhbir et al. failure criterion

C_0, σ_t and β Material parameters for Sheorey failure criterion

$\sigma_{1predicted}$ Predicted major principal stresses

$\sigma_{1Experiment}$ Experimental major principal stresses

μ Mean value

σ Standard deviation

1. INTRODUCTION

For constructing engineering structures through rocks, it is essential to know the stress-strain-time behavior and the strength characteristics of the concerned rock mass. However, for stability problems like deep excavations and excavation by tunneling, the primary goal is to avoid failure state, defining the ultimate stress condition influenced by the induced stress and strength of intact rock which can be predicted by using some failure criteria. The strength criteria used in rock mechanics are either empirical or based on mechanics (Hoek and Brown 1980). There are nearly twenty rock failure criteria (Yu 2002) to predict the strength of rocks, though each one is suitable for a particular type of rock depending upon its physical and mineralogical characteristics.

All such predictive models involve some material/ empirical parameters which need to be determined from the available experimental data. The predictive capability of the strength theories are then compared with the laboratory test data based on the derived parameters. Sometimes estimation of such parameters from actual measurement is difficult and can be derived by trial and error procedure (Carter et al.

1991). But as the failure envelopes are drawn using the above parameters, it should be predicted accurately. For nonlinear strength models, more systematic approach is used by minimizing an error (objective) function constructed with the purpose to find the error in estimation from the observed values.

The least square (LS) (Shah and Hoek 1992; Li et al. 2000) and weighted least square (Desai 2001) techniques are widely used to frame the objective function and different mathematical optimization methods available in literature are used to solve the objective function. Colmenares and Zoback (2002) used the grid search method to find out the parameters using minimization of the mean standard deviation misfit to the test data. They discussed the applicability of different rock failure criteria for different rock strength data and observed that applicability of strength theories depends upon the type of laboratory rock strength data. Abdullah and Dhawan (2004) have also drawn similar conclusions and emphasized the need for re-examination of semi-empirical correlations.

But in general, scatter-ness of experimental data is almost unavoidable due to material nonhomogeneities, equipment/procedural error, and random testing effects. The least square error method has problems with such scattered data (known as outliers in statistics). A single outlier, if severe, can significantly affect the results. In case of geomaterials, the parameters are very often found from limited data due to the cost involved and the data are scattered. Rousseeuw (1998) suggested the use of the least median square (LMS) method, which is more robust and stable against perturbations due to outliers. The method also helps in identifying the outliers, which may be either corrected or deleted and the revised LS method is known as re-weighted least square (RLS) method (Rousseeuw 1998).

With the above in view, studies have been made to find out the model parameters of different failure criteria applicable to different rock strength data based on LS, LMS and RLS methods. The accuracy of different failure criteria are compared in terms of errors associated with the prediction of unconfined compressive strength value. The failure envelopes as obtained from above parameters and comparison of predicted data for different failure theories using different statistical methods is discussed. The strength, material parameters comparison and suitability of failure criteria to a particular rock is as per the laboratory test data and should not be extrapolated for field rock masses. The strength of rock mass may be reduced due to structural discontinuity, in which case above failure criteria may not hold good (Pariseau 2007).

2.0 ANALYSIS

2.1 Adopted failure criteria

The failure criteria due to Hoek and Brown (1980), Yudhbir et al. (1983), Ramamurthy et al. (1985) and Shereoy (1997) has been considered for the present study and are presented in Equ 1, 2, 3 and 4 respectively as follows.

$$(\sigma_1 - \sigma_3) - (m\sigma_3 C_0 + sC_0^2)^\alpha = 0 \tag{1}$$

where σ_1, σ_3 are major and minor principal stresses respectively, C_0 the uniaxial compressive strength of intact rock, m and s are the material parameters. The value of s is unity for intact rock.

$$\frac{\sigma_1}{C_0} = a + b\left(\frac{\sigma_3}{C_0}\right)^\alpha \tag{2}$$

For intact rock a = 1.0, the parameters need to be determined are C_0, b and α.

$$\frac{\sigma_1 - \sigma_3}{\sigma_3 + \sigma_t} = B\left(\frac{C_0}{\sigma_3 + \sigma_t}\right)^b \tag{3a}$$

Ramamurthy (1994) also observed that for most of the rock the value of b = 2/3 and $B = 1.3 \times \left(\frac{C_0}{\sigma_t}\right)^{1/3}$.

So equation (3a) can be rewritten as

$$\sigma_1 = \sigma_3 + 1.3 C_0 \left(1 + \frac{\sigma_3}{\sigma_t}\right)^{1/3} \tag{3b}$$

The parameters to be determined are C_0 and uniaxial tensile strength (σ_t).

$$\sigma_1 = C_0 \left(1 + \frac{\sigma_3}{\sigma_t}\right)^{\beta} \tag{4}$$

The parameters need to be determined are C_0, σ_t and β.

All the above are triaxial nonlinear failure criteria ignoring the effect of intermediate stress and the advantage of nonlinear failure criteria over linear failure criteria is well recognized (Mostyn and Douglas, 2000; Pariseau 2007)

2.2 Adopted statistical methods and the error function

The error function for the above failure criteria, while using LS method, with n number of experimental data can be written as (Shah and Hoek 1992; Li et al. 2000):

$$\left[ERR(f)\right] = \sum_{j=1}^{j=n} (\sigma_{1Experimental} - \sigma_{1predicted})^2 \tag{5}$$

When LMS method is used the objective is to minimize the median of square of error instead of the conventional sum of square of errors as in LS method. Thus the objective function can be written as (Rousseeuw 1998)

$$\left[ERR(f)\right] = median\ (\sigma_{1Experimental_i} - \sigma_{1predicted_i})^2 \quad for\ i = 1, \dots, n \tag{6}$$

Estimation of nonlinear failure criterion parameters using LMS is difficult (Rousseeuw 1998). However, the LMS method has the advantage of identifying scattered data. The material parameter appearing in the above mentioned failure criterion for different rock strength data are found by minimizing their respective error functions using traditional nonlinear optimization techniques (Press et al. 2000) for LS method and evolutionary algorithm i.e. genetic algorithm (Deb 2001) for LMS method.

2.3 Identification of scattered data and re-weighted least square method

Rousseeuw (1998) proposed the method of identifying the regression outlier and reweighted least squares (RLS) concept for the estimation of the regression parameters. First the parameter corresponding to LMS parameter is estimated and the errors from this regression. The error in each observation is expressed in terms of standardized residuals as $\left|\frac{r_i}{\hat{\sigma}}\right|$ where $\hat{\sigma}$ is a consistent estimator of standard deviation and defined as $\hat{\sigma} = 1.483\sqrt{median\, r_i^2}$ i = 1,…,n and r_i are the residuals from LMS fit. The data that are having large errors (standardized residual ≥2.5) are termed as regression outliers and LS regression applied deleting the outlier is known as RLS method.

2.3 Rock strength data

The ten rock type investigated are Tennessee marble, basalt rock mass (Shah and Hoek,1992), Carrara marble, Pennant sandstone, Blackingstone quarry granite (granite) and Darley Dale sandstone (Dale sandstone) (Franklin and Hoek 1970), dunite, Apache leaf tuff stone, South Africa sand stone (SA sandstone), norite, quartzite, mudstone and silt stone (Mostyn and Douglas, 2000). The triaxial rock strength data of the above rocks obtained from published work as described above.

3.0 RESULTS AND DISCUSSION

Results obtained using the above rock strength data for different failure criteria based on different statistical (LS, LMS and RLS) methods and the parameters so obtained are critically discussed. Finally, a comparative study is made regarding the prediction of unconfined compressive strength and failure envelope using different rock failure criteria for different rocks.

3.1 Comparative study of rock failure criteria using LS method

The rock failure theories should have three independent characteristics, the opening angle, the curvature and the tensile strength (Mostyn and Douglas 2000). Hence, if the exponent α in Hoek-Brown failure criterion is varied, instead of fixed value of 0.5 (Shah and Hoek 1992; Li et al. 2000), it can include an accurate estimation of uniaxial tensile strength of rock (Mostyn and Douglas 2000). Mostyn and Douglas (2000) observed that *m* value with $\alpha = 0.5$ is misleading and does not truly represent as material characteristic. Table 1 shows the predicted model parameters for Hoek-Brown failure criterion for different rocks using a fixed α value (0.5) and using it as a variable. The α value varies from 0.36 (granite) to 0.91(SA sandstone) and there is considerable variation in *m* and C_0 values. The maximum variation in *m* value was observed for dunite (50 and 6.71). As *m* is a measure of C_0/σ_t (Mostyn and Douglas 2000), hence, there may be large variation in prediction of uniaxial tensile strength.

The unconfined compressive strength (C_0) is important for design analysis of underground excavation. Figure 1 shows the variation of predicted C_0 values for different rocks using different failure criteria. The predicted C_0 values by Ramamurthy et al. criterion are 16 to 40% less in comparison to observed values. The variation in C_0 values between observed and predicted is -11.2 % (Dale sandstone) to 15 %(dunite) and -7.0% (Dale sandstone) to 27% (dunite) for Yudhbir et al. failure criterion and Sheorey failure criterion respectively.

The accuracy of different failure criteria, in predicting the C_0 value is compared in terms of (i) root mean square error (RMSE), (ii) maximum absolute error (MAE), and (iii) absolute average error (AAE). Pariseau (2007) compared different failure criteria in terms of Euclidean norm, which is nothing but the root square error. The correlation coefficient (R^2) is a biased estimate and not a reliable index (Pariseau 2007), hence not considered in this study. The RMSE gives an indication of how accurate the approximation was overall, while the maximum error can reveal the presence of regional areas of poor approximation. Figure 2 shows the variation of errors for different rocks. The Yudhbir et al. failure criterion is found to be 'best' followed by Sheorey failure criterion.

3.2 Comparative study of rock failure criteria using LMS method

As in LMS method, only half of the data points close to the fitted equation are considered, the parameters obtained by using this method are different from that obtained by LS method. However, the α value found to vary from 0.36 to 1.0, which is similar to that of LS method. For Hoek-Brown failure criterion the C_0 value predicted by LS method for dunite, quartzite, SA sandstone, siltstone, tuff and Penant sandstone is more than that predicted by LMS method, where as for other rocks opposite trend is observed. Maximum variation was obtained for Pennant sandstone followed by dunite. In case of Ramamurthy et al. failure criterion the prediction of C_0 by LS method is more for dunite, mudstone, norite and Dale sandstone and for other rocks it is less than that predicted by LMS method. Similar differences are also observed for Yudhbir et al. and Sheorey failure criteria. Figure 3 shows the variation of RMSE, MAE, and AAE between predicted and observed C_0 values for different rocks. Like LS method, Yudhbir et al. failure criterion is found to be 'best' followed by Sheorey failure criterion. The RMSE values found to be less than that of LS method, except Sheorey failure criterion.

3.2.1 Identification of outliers

The LMS method can identify the outliers (scattered data) which may be due to experimental error or the data points belong to different group (Rousseeuw 1998). For both LS and LMS methods, Figures 4a, 4b show the variation of the standardized residuals with principal stress values (σ_1) for granite and tuff stone for Hoek-Brown and Ramamurthy et al. failure criteria, respectively. These figures reveal the presence of several outliers in the data set when LMS method is used. But when LS method is used the presence of such outliers could not be revealed. This indicates the inadequacy of LS method to handle the scattered data points. Figure 5a and 5b show the standardized residuals for granite and tuff for Yudhbir et al. and Sheorey failure criteria, respectively. The number of scattered data points depends upon the applicability/adequacy of the failure criterion or the natural variability in the rock sample. As the same data points are identified as scattered data using Hoek-Brown, Yudhbir et al. and Sheorey failure criteria, it may be due to natural variability in the rock sample. Hence, such a study will help in finding out the natural variability in the rock strength data. Table 2 shows the number of outliers in different rock strength data as per different failure criteria and found to depend upon the rock type and failure criterion. Least number of scattered data was observed for norite.

3.3 Comparative study of rock failure criteria using RLS method

Once the outliers are identified by using LMS method, the process of parameter estimation can be further extended by adopting RLS method. The predicted strength envelopes using LS, LMS and RLS methods for granite and tuff stone are shown in

Figures 6 and 7, respectively as per Hoek-Brown and Yudhbir et al. failure criterion. The scattered data as obtained using LMS method are also shown along with other data points. More numbers of scattered data are observed at higher confining stress. This may be due to transition from brittle to ductile failure occurs with increases in confining pressure (Schwartz 1964). The scattered data points for granite and tuff stone respectively, showed strain softening and strain hardening characteristics. The strength envelope as per LS method passes through the average the points, LMS method considered only half the data points close to the fitted equation, but RLS method use LS method deleting the scattered data. The close proximity of LMS and RLS strength envelop indicates the necessity of deleting scattered data while fitting the failure criterion.

The errors (RMSE, AAE, MAE) associated with predicting C_0 values for different failure criteria using RLS method is shown in Figure 8. Yudhbir et al. failure criterion, found to be better than all other failure criteria, based on the error values. There is also decrease in error values compared to that of LS methods.

The predictive performances of different failure criteria are also compared in terms of coefficient of variation (COV) of the ratio of predicted major principal stress (σ_{1p}) and measured major principal stress (σ_{1m}). The coefficient of variation is defined as the ratio of standard deviation (σ) to the mean value (μ) expressed in percentage. As an accurate and precise method gives the mean value 1.0 and standard deviation of 0.0, the COV as 0.0 is an ideal condition and the method with small COV value is better. Table 3 shows the variation of COV values of the ratio (σ_{1p}/σ_{1m}) for different rocks using both LS and RLS method for the four failure criteria considered in this study. In general, the prediction accuracy increased using RLS method compared to LS method signified by decrease in COV values. The maximum difference was observed for tuff stone with COV value for LS method found to 3.5 times more than that of RLS method. Yudhbir et al. failure criteria is found to better performances for eight rock strength data followed by two for Sheorey failure criteria, while using LS method. But using RLS method, Yudhbir et al. and Hoek-Brown failure criteria are found to be more accurate for four rock strength data followed by two for Sheorey failure criteria. However, the following observations are as per laboratory test data and should not be extrapolated for field rock masses, in which case above failure criteria may not hold good due to structural discontinuity (Pariseau 2007).

4.0 CONCLUSIONS

Based on the above studies the following general conclusions can be drawn:

1. The parameters obtained using LS, LMS methods are found to be different for the rock strength data, and the LMS method could identify the scattered data.
2. As the same data points are identified as scattered data using Hoek-Brown, Yudhbir et al. and Sheorey failure criteria, it may be due to natural variability in the rock sample.
3. More numbers of scattered data are observed at higher confining stress. This may be due to transition from brittle to ductile failure occurs with increases in confining pressure. The scattered data points for granite and tuff stone respectively, showed strain softening and strain hardening characteristics.
4. Based on the root mean square error, maximum absolute error and absolute average error in predicting the unconfined compressive strength (C_0) the Yudhbir et al. failure criterion is found to be ‘best’ followed by Sheorey failure criterion.

5. Based coefficient of variation (COV) values, Yudhbir et al. failure criteria is found to better performances for eight rock strength data followed by two for Sheorey failure criteria, while using LS method. However, using RLS method, Yudhbir et al. and Hoek-Brown failure criteria are found to be more accurate for four rock strength data followed by two for Sheorey failure criteria. However, the following observations are as per laboratory test data and should not be extrapolated for field rock masses

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Table 1 Model parameters of Hoek-Brown failure criteria using LS method

Rock type	m	C_0	α	RMSE	C_0(Observed)
Dunite	50.00	65.36	0.5	112.70	70.00
	6.71	85.36	0.75	35.23	
Mudstone	6.27	59.27	0.5	6.36	50.00
	8.92	57.66	0.42	6.34	
Norite	20.50	346.49	0.5	22.50	313.40
	35.69	326.54	0.4	21.29	
Quarzite	16.42	204.55	0.5	37.31	188.40
	28.32	188.52	0.41	37.05	
SA sandstone	15.35	61.27	0.5	8.88	64.50
	4.62	70.11	0.91	8.14	
Siltstone	7.80	53.17	0.5	10.62	50.00
	6.22	54.15	0.56	5.94	
Tuff	19.88	137.65	0.5	17.72	147.30
	8.28	151.86	0.74	16.02	
Granite	19.87	216.57	0.5	27.78	179.60
	49.81	191.12	0.36	26.55	
Dale sandstone	16.23	76.8	0.5	6.61	80.10
	10.77	63.81	0.53	6.38	
Penant sandstone	12.3	206.5	0.5	10.81	197.00
	19.81	287.12	0.47	10.41	

Table 2 Number of outliers as per LMS method for different rock strength data and failure criteria

Rock strength data	Total data point	Number of outliers			
		Hoek-Brown(1980)	Yudhbir et al. (1983)	Ramamurthy et al.(1985)	Sheorey (1997)
Dunite	20	3	1	4	3
Mudstone	48	2	3	2	2
Norite	16	0	2	2	2
Quarzite	48	2	2	2	2
Sandstone	54	1	7	12	9
Siltstone	64	3	3	7	3
Tuff	16	5	5	4	5
Granite	39	7	7	2	7
Dale Sand	23	5	6	2	6
Penant	29	5	3	3	3

Table 3 The Coefficient of variation (COV) of the ration (σ_{1p}/σ_{1m})

Rock strength data	LS method				RLS Method			
	Hoek-Brown (1980)	Yudhbir et al. (1983)	Ramamurthy et al.(1985)	Shereoy (1997)	Hoek-Brown (1980)	Yudhbir et al. (1983)	Ramamurthy et al.(1985)	Shereoy (1997)
Dunite	7.70	7.18	36.04	8.53	5.79	6.87	6.08	7.40
Mudstone	6.95	6.78	6.84	7.01	6.65	6.10	6.26	6.65
Norite	3.86	3.56	3.90	3.90	3.86	2.76	3.14	2.87
Quarzite	11.47	11.56	11.89	11.45	9.97	10.84	10.33	10.81
Sandstone	6.29	6.29	8.69	6.30	6.24	5.98	6.66	5.96
Siltstone	6.22	6.20	6.45	6.22	5.62	5.58	5.75	5.63
Tuff	5.83	5.76	9.26	5.83	1.66	1.71	5.29	1.66
Granite	7.10	7.01	7.20	7.10	5.05	4.67	5.72	5.07
Dale Sandstone	4.24	4.98	8.40	4.15	2.57	2.82	5.39	2.91
Penant sandstone	3.50	3.39	3.13	6.52	2.42	2.21	2.19	2.22

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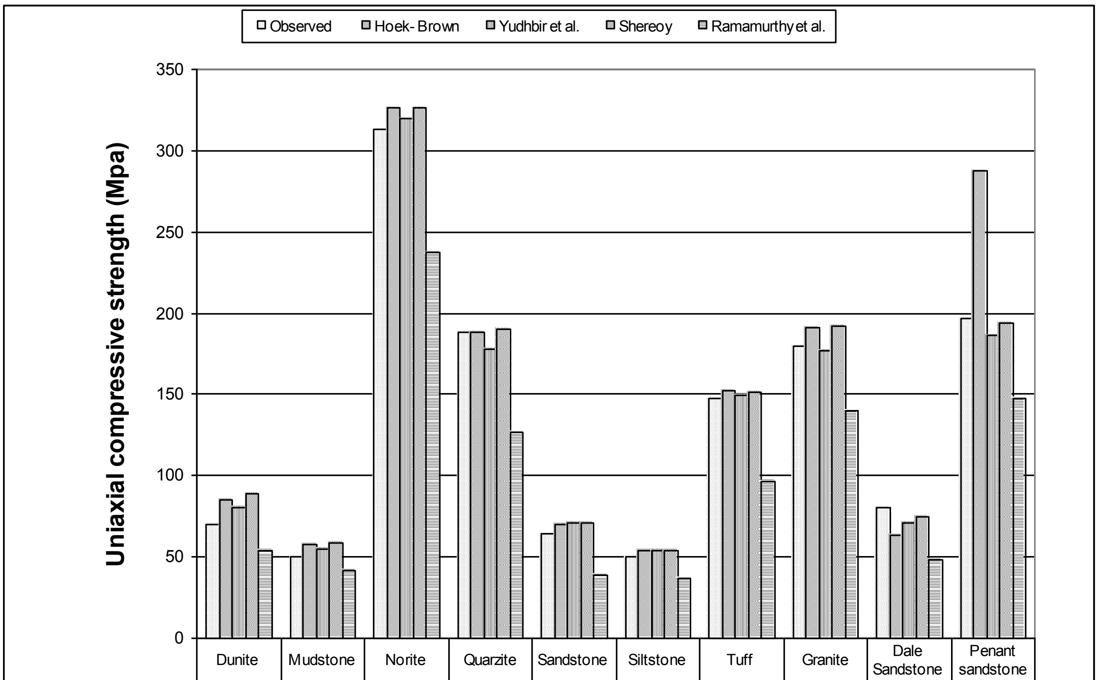
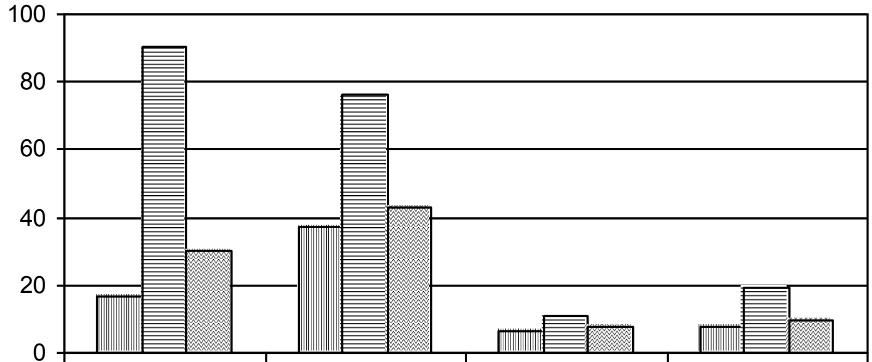


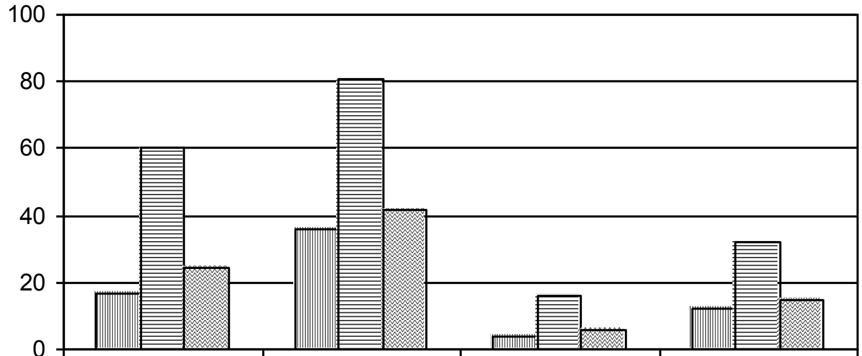
	Dunite	Mudstone	Norite	Quarzite	Sandstone	Siltstone	Tuff	Granite	Dale Sandstone	Penant sandstone
Observed	70	50	313.4	188.4	64.5	50	147.3	179.6	80.1	197
Hoek- Brown	85.36	57.66	326.54	188.52	70.11	54.15	151.86	191.12	63.81	287.12
Yudhbir et al.	80.48	55.25	319.55	177.89	70.71	53.48	149.52	176.61	71.12	186.33
Shereoy	89.2	58.39	326.5	190.2	71.18	54.13	151.7	192.3	74.3	193.7
Ramamurthy et al.	54.02	41.98	237.4	126.6	38.34	37.16	96.93	139.9	47.92	147.1

Figure 1 Variation of Predicted and Observed Co value for different rock strength data using LS method.



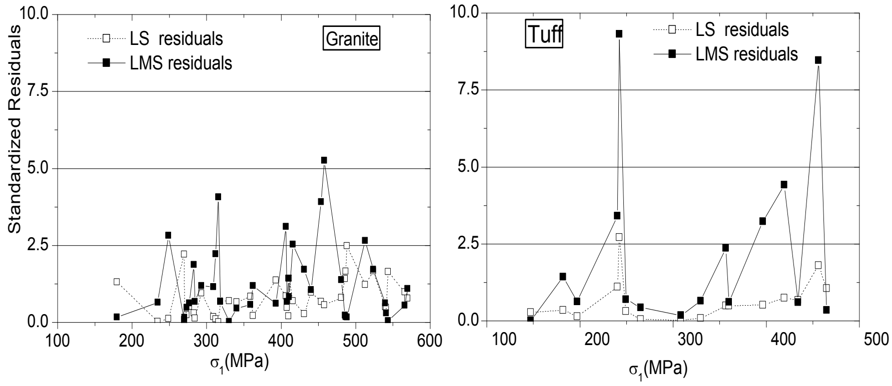
	Hoek-Brown	Ramamurthy et al.	Yudhbir et al.	Sheorey
AAE	16.85	37.3	6.69	7.95
MAE	90.12	76.00	10.67	19.2
RMSE	30.09	42.90	7.38	9.49

Figure 2 Comparison of errors between different failure criteria for predicting C_0 value using LS method.



	Hoek-Brown	Ramamurthy et al.	Yudhbir et al.	Sheorey
AAE	16.95	35.83	3.90	11.93
MAE	59.98	80.50	16.07	32.30
RMSE	24.30	41.51	5.82	14.97

Figure 3 Comparison of errors between different failure criteria for predicting C_0 value using LMS method.



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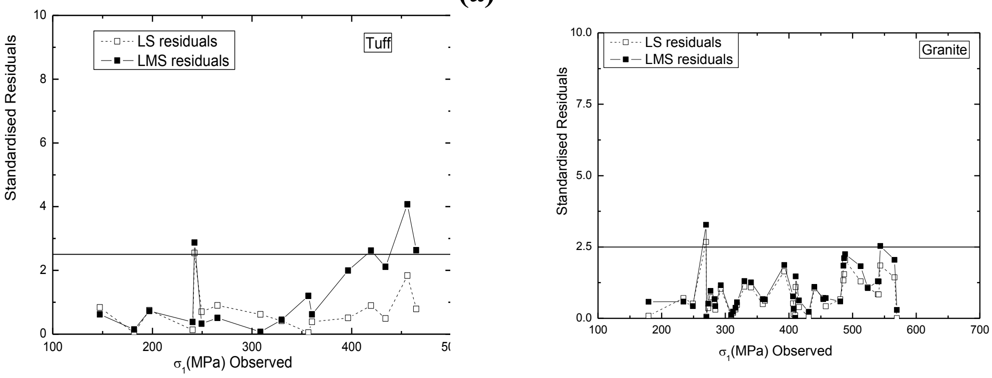
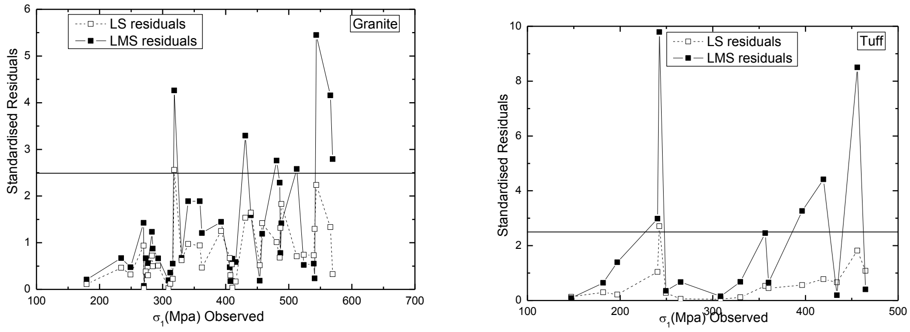
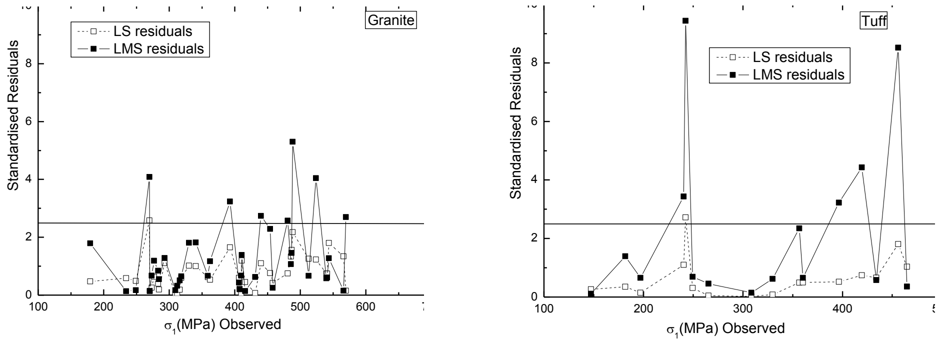


(b)

Figure 4 Standardized residuals for granite and tuff using (a) Hoek- Brown, (b) Ramamurthy et al. failure criterion.

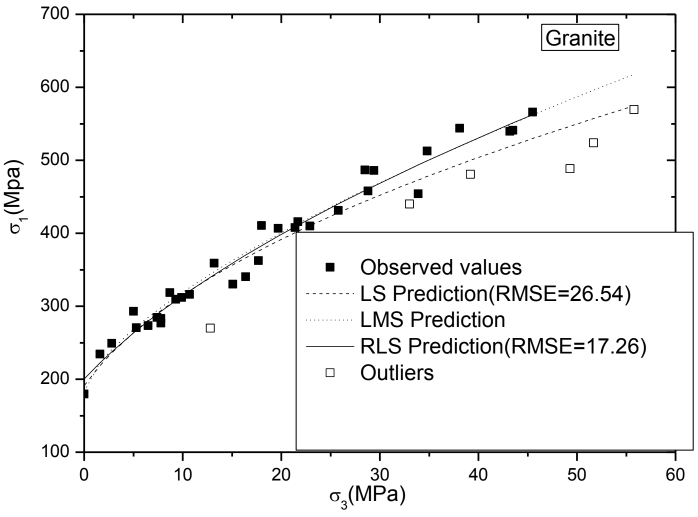


(a)

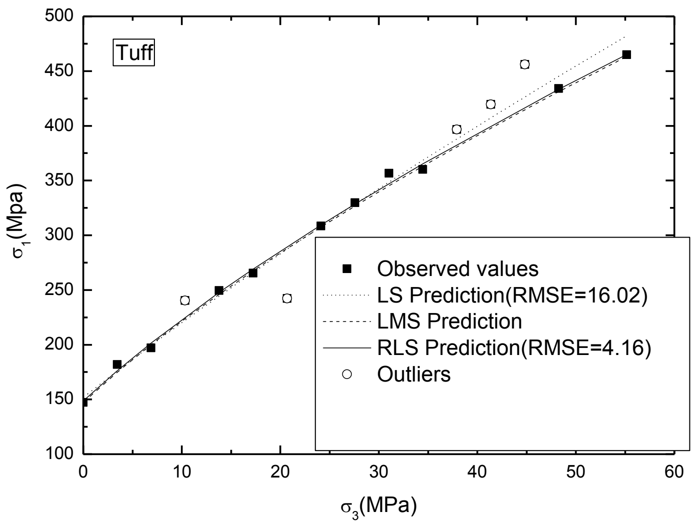


(b)

Figure 5 Standardized residuals for granite and tuff using (a) Yudhbir et al. and (b) Sheorey failure criterion.

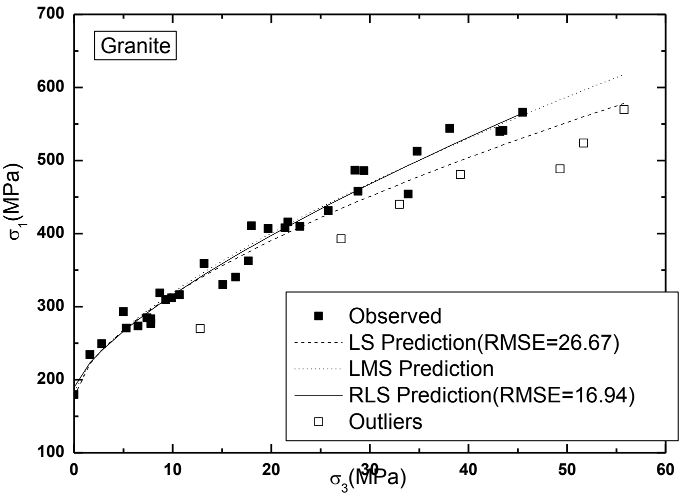


(a)

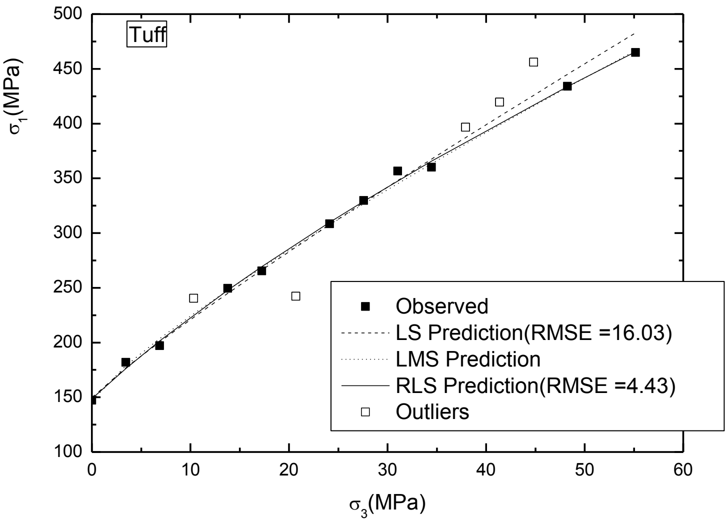


(b)

Figure 6 Triaxial strength envelopes for (a) granite (b) tuff and corresponding to Hoek- Brown failure criterion using LS, LMS and RLS method.

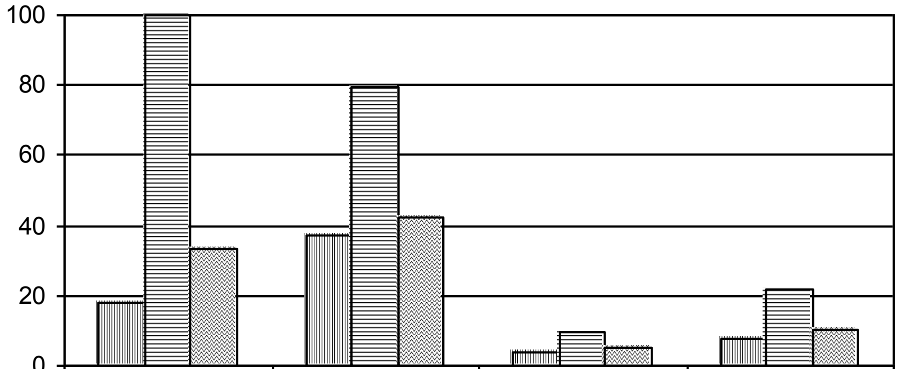


(a)



(b)

Figure 7 Triaxial strength envelopes for (a) granite (b) tuff and corresponding to Yudhbir et al. failure criterion using LS, LMS and RLS method



	Hoek-Brow n	Ramamurthy et al.	Yudhbir et al.	Sheorey
AAE	18.11	36.86	4.00	7.71
MAE	100.26	79.22	9.70	21.56
RMSE	33.33	42.50	4.98	9.96

Figure 8 Comparison of errors between different failure criteria for predicting C_0 value using RLS method.