Presented in International Conference on Extrusion and Benchmark, Dortmund, Germany, 16-17 September, 2009

COMPUTER-AIDED SIMULATION OF METAL FLOW THROUGH CURVED DIE FOR EXTRUSION OF SQUARE SECTION FROM SQUARE BILLET

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Publication on: Scientific/Industrial journal

Abstract

Extrusion through mathematically contoured die plays a critical role in improvement of surface integrity of extruded product. There is gradual deformation which results in the uniform microstructure. In the present investigation non-dimensional extrusion pressure and optimum die length for cosine die profile has been obtained by three dimensional upper bound method using dual stream fuction method for different reductions. The theoretical modeling has been validated with experiments. The experimental results are found to be compatible with the theory.
Keywords: *Extrusion, cosine die, Dual stream function,, Upper-Bound,Square section*

1.0 Introduction

The geometry of die profile plays a significant role for smooth flow of material resulting in the evolution of uniform microstructure in extruded product with improved mechanical properties and reduction in extrusion pressure. Recently the process design involving mathematically contoured die profile has drawn the attention of various researchers to improve not only dimensional accuracy but also quality of products. Richmond and Devenpeck [1] first conceptualized an ideal die of perfect efficiency and developed the governing equations for metal flow. Later, Richmond and Morrision [2] used the above concept to design stream-lined wire-drawing dies of minimum length with sigmoidal die shape. The total work of deformation is equal to that for homogeneous compression with redundant work at entry and exit equal to zero. However, due to difficulty in the manufacture of sigmoidal dies, those have found very little application in actual extrusion/drawing of metals. On the other hand alternative die profiles such as cosine, parabolic, hyperbolic etc. have been preferred because of ease of manufacture. A number of investigations have been carried out in the past to predict the deformation load for extrusion through curved die. Slipline field solution for extrusion through cosine shaped die has been postulated by Samanta [3]. Upper bound solutions for extrusion through axisymmetric curved dies have been proposed by Chen and Ling [4] and Chang and Choi [5]. Upper bound solutions with experimental flow studies through different axisymmetric curved dies have also been reported by Frisch and Mata-Pietri [6] [7]. Narayansamy et. al [8] carried out an upper bound solution to extrusion of circular billet to circular shape through cosine dies. But adequate literature is not available for three-dimensional extrusion through mathematically contoured dies because of difficulty in predicting kinematically admissible velocity fields. Maity et.al [8] investigated three dimensional upper bound methods for extrusion of square sections from square billet using general dual stream function method to determine kinematically admissible velocity field. Ponalagusamy et al. [9] investigated computer aided metal flow in stream lined extrusion dies. Lee et. al [10] designed an optimal Bezier shaped die profile that yielded more uniform microstructure. Rani Kainew et al. [11] carried out finite element analysis of copper extrusion process in both flat faced and converging dies. In the present investigation a three-dimensional upper bound analysis using dual stream function has been carried out for extrusion of square section from square billet using cosine die profile. The metal flows tangentially at the exit and entry to the die-wall resulting the redundant work at entry and exit equal to zero. The mode of deformation is very gradual resulting uniform microstructure. The kinematically admissible velocity field has been obtained from dual stream function method. Because of symmetricity of the deformation zone, only one quadrant of deformation zone has been chosen. The two stream functions are chosen in such a way that automatically satisfies the boundary condition. The velocity and strain rate components determined from stream functions have been used to determine the internal work and the work against the velocity discontinuities. This has been achieved with the help of a 5-point Gauss-Legendre quadrature alogarithm for volume and surface integral. The constant friction factor is assumed at the die-billet interface with constant flow stress of rigid plastic material. A FORTRAN main programme with a number of sub-programs and subroutine has been developed to compute total power of deformation and extrusion pressure. The total power of deformation has been minimized with respect to die length to determine optimum die length and extrusion pressure from upper bound method. The velocity and strain rate distributions have been determined in the deformation zone. The curved die with optimum cosine die profile was manufactured. The experimental investigation has been carried for hot-work operation using telerium-lead for extruding square section from square billet. The extrusion pressure computed from the upper bound theory was found to be compatible with the experiment using the above boundary condition.

2.0 Upper bound method

The upper bound theorem states that the power estimated from Kinematically-admissible velocity fields (KAVF) is always higher than the actual one. Amongst all kinematically-admissible velocity fields the actual one minimizes the expression:

$$J = \left(\frac{2\sigma_0}{\sqrt{3}}\right)\iint_V \sqrt{(\varepsilon_{ij}\varepsilon_{ij})}\,dV + \left(\frac{\sigma_0}{\sqrt{3}}\right)\iint_{S_i} |\Delta V|_{S_i}\,dS_i + \left(\frac{m\sigma_0}{\sqrt{3}}\right)\iint_{SD_i} |\Delta V|_{SDi}\,dS_{SD_i} \tag{1}$$

Where J is the power of dissipation rate, σ_0 is the flow stress; ε_{ij} is the derived strain-rate tensor, $|\Delta V|_{S_i}$ is the velocity discontinuity at the entry and exit surfaces S_i, $|\Delta V|_{SD_i}$ is the velocity discontinuity at the die-metal interfaces SD_i and m is the friction factor.

The kinematically admissible velocity field has been obtained using dual stream function method. According to Yih [13], the velocity components can be derived from these stream functions using the equations.

$$V_x = \left(\frac{\partial \psi_2}{\partial y}\right)\left(\frac{\partial \psi_1}{\partial z}\right) - \left(\frac{\partial \psi_1}{\partial y}\right)\left(\frac{\partial \psi_2}{\partial z}\right) \tag{2a}$$

$$V_y = \left(\frac{\partial \psi_2}{\partial z}\right)\left(\frac{\partial \psi_1}{\partial x}\right) - \left(\frac{\partial \psi_1}{\partial z}\right)\left(\frac{\partial \psi_2}{\partial x}\right) \tag{2b}$$

$$Vz = \left(\frac{\partial \psi_2}{\partial x}\right)\left(\frac{\partial \psi_1}{\partial y}\right) - \left(\frac{\partial \psi_1}{\partial x}\right)\left(\frac{\partial \psi_2}{\partial y}\right) \tag{2c}$$

To derive the dual stream functions for the present problem, the geometry shown in Fig.1 with prescribed reference system is considered. Because of symmetry about two mutually perpendicular axes, only one quadrant of the actual deformation zone is considered, F(z) is the die-profile function such that the die faces in the x-z and y-z planes are represented by x=F(z) and y=F(z) respectively. The function F (z) must satisfy the conditions that F (z)=W at z=0 and F(z)=A at x=L, where W and A are the semi width of billet and product respectively and L is the die length.

The dual stream functions ψ_1 and ψ_2 are chosen as shown below

$$\psi_1 = -\frac{x}{F(z)} \tag{3a}$$

$$\psi_2 = \frac{W^2 V_b\ y}{F(z)} \tag{3b}$$

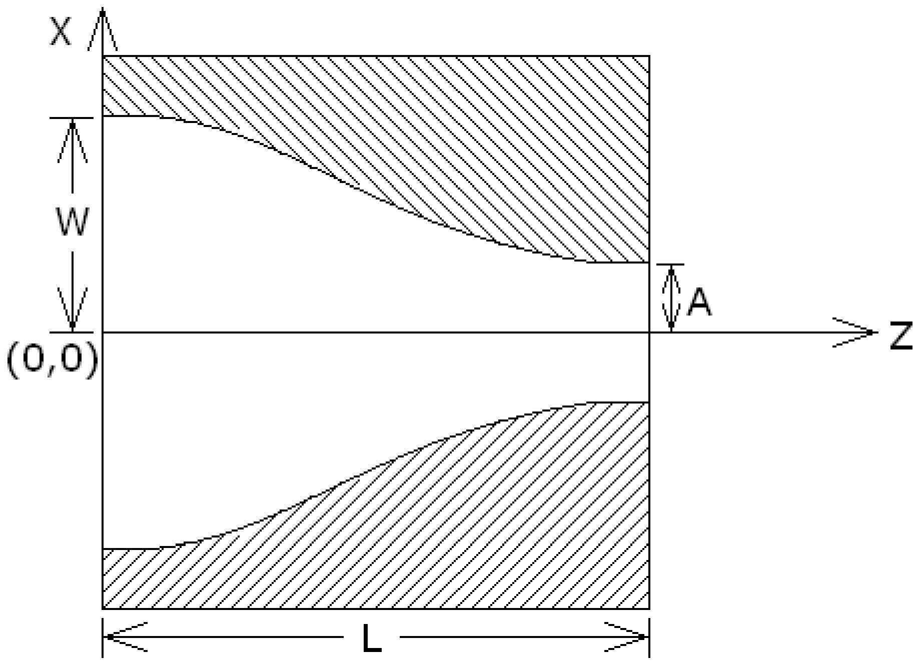

Figure 1 (a) Profile of a curved die with the axes of reference.

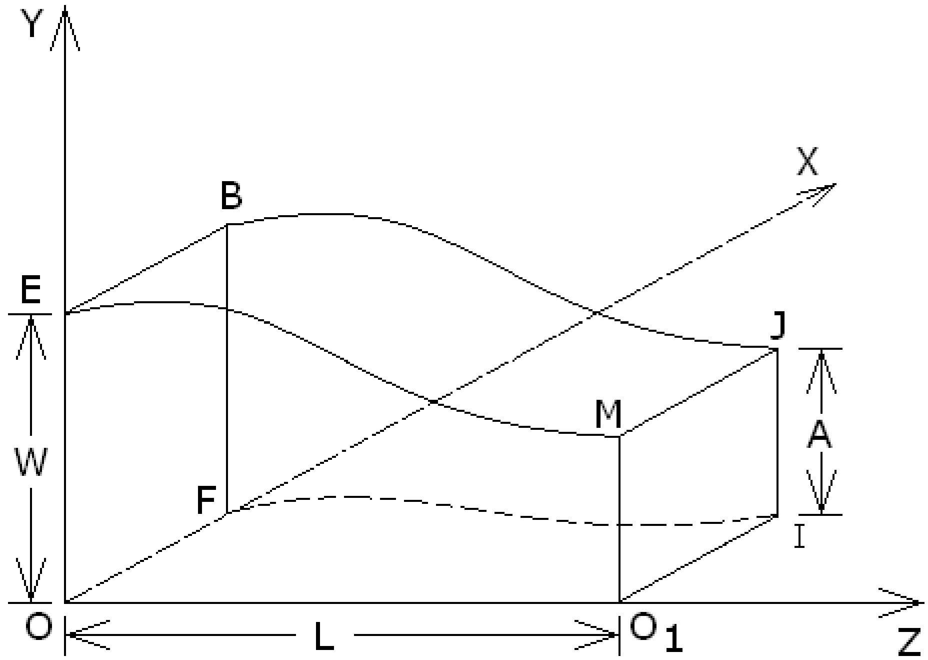

Figure 1 (b) One quadrant of the deformation zone.

where V_b is the billet velocity. It can be easily verified that (i) $\psi_1 = 0$ on the plane x=0 and $\psi_1 = -1$ on the die surface plane x=F (z); and (ii) $\psi_2 = 0$ on the plane y=0 and $\psi_2 = W^2 V_b$ (which is a constant) on the die surface y=F (z)). Such constant values ensure that surfaces x=0, x=F (z), y=0 and y=F (z) are stream surfaces, and as such velocity components normal to those surface vanish. Thus, ψ_1 *and* ψ_2 defined in the above-mentioned manner satisfy all velocity boundary conditions. Hence they are valid boundary conditions. Hence, they are valid stream functions to generate a Kinematically-admissible velocity field. Substituting Eq. (3) into Eq. (2) and simplifying, the velocity components in the deformation region are:

$$V_x = \frac{W^2 V_b \, xF'}{F^3} \tag{4a}$$

$$V_y = \frac{W^2 V_b \, yF'}{F^3} \tag{4b}$$

$$V_z = \frac{W^2 V_b}{F^2} \tag{4c}$$

Where F = F (z) and F' = dF/dz

The strain-rate components ε_{ij} are derived from the velocity components using the relationship:

$$\varepsilon_{ij} = \frac{1}{2}\left[\left(\frac{\partial V_j}{\partial x_i}\right) + \left(\frac{\partial V_i}{\partial x_j}\right)\right] \tag{5}$$

The strain-rate components for the proposed flow field are as follows by substituting Eq. (4) in Eq. (5)

$$\varepsilon_{xx} = \frac{\left(W^2 V_b F'\right)}{F^3}$$

$$\varepsilon_{yy} = \frac{\left(W^2 V_b F'\right)}{F^3}$$

$$\varepsilon_{zz} = \frac{\left(-2W^2 V_b F'\right)}{F^3}$$

$$\varepsilon_{xy} = \varepsilon_{yx} = 0$$

$$\varepsilon_{yz} = \varepsilon_{zy} = \left(\frac{1}{2}\right) W^2 V_b y \left[\left(\frac{F''}{F^3}\right) - \left(\frac{3\left(F'\right)^2}{F^4}\right)\right]$$

$$\varepsilon_{zx} = \varepsilon_{xz} = \left(\frac{1}{2}\right) W^2 V_b x \left[\left(\frac{F''}{F^3}\right) - \left(\frac{3\left(F'\right)^2}{F^4}\right)\right] \tag{6}$$

where $F'' = d^2F / dz^2$

From equation (6), it is evident that velocity field satisfy incompressibility condition. Using Eq. (6), J can be evaluated from Eq. (1) when the die-profile function, F, is known. For any reduction and friction factor m, J then can be minimized with respect to appropriate parameters to yield the best upper bound.

3.0 Die-profile function

The die geometry examined in the present investigation is shown in Fig. (1). Referring to this figure, it may be seen that the die profile function F (z) is similar in both x- and y- direction. The die profile function is given as:

$$x = y = F(z) = \frac{W+A}{2} + \frac{W-A}{2}\cos\left(\frac{\pi z}{L}\right) \tag{7}$$

The die profile function satisfies the boundary condition such that at z=0, x=W and z=L, x=A. The exit and entry angles are non-zero angles. The velocity-discontinuity surfaces are normal to the axial flow directions.

4.0 Computation

An integrated FORTRAN code was developed to compute the upper bound extrusion load using Eq. (1). For any reduction R and friction factor m, the program first calculates the velocity components and the strain rate components using Eq. (4) and (5) respectively and then evaluates the upper-bound on power Eq.(1) by numerical integration using the 5-point Gauss-Legendre quadrature algorithm. The total power of deformation has been minimized using a multivariable optimization technique [12].

5.0 Results and Discussion

The variation of non-dimensional extrusion pressure with respect to reductions are shown in Fig.2 for different friction factor (m=0.0 to 1.0). It is observed that there is an increase in extrusion pressure with increase in reduction and friction factor. At low reduction, R=30% the extrusion pressure varies from $0.5\sigma_0$ to $1.5\sigma_0$ whereas at high reduction R=90%, it varies from $2.0\sigma_0$ to $6.2\sigma_0$. It is interesting to note that the effect of friction is more significant at higher reduction compared to lower reduction. Hence, the role of lubricant at larger reduction is more effective in reducing the extrusion pressure.

The present modeling is validated with experiments. The extrusion was carried with lead specimens using set up as shown in Fig.3. The dry and wet friction conditions at the die-billet interface corresponds to constant friction factor m=0.75 and m=0.38 respectively, which was determined by ring compression test. The flow stress of the work material determined using uni-axial compression test and found to be 52.5 N/mm^2. On lubricated condition, the variation of extrusion loads with ram travel as obtained from the experiments for 30% and 60% reductions are shown in Fig.5. The computed results for different area reductions are shown in Table 1. The non-dimensional extrusion pressure as obtained from experiment is compared with that of theoretical solution in Fig.6. It seems that the present modeling agrees well with the experimental investigations.

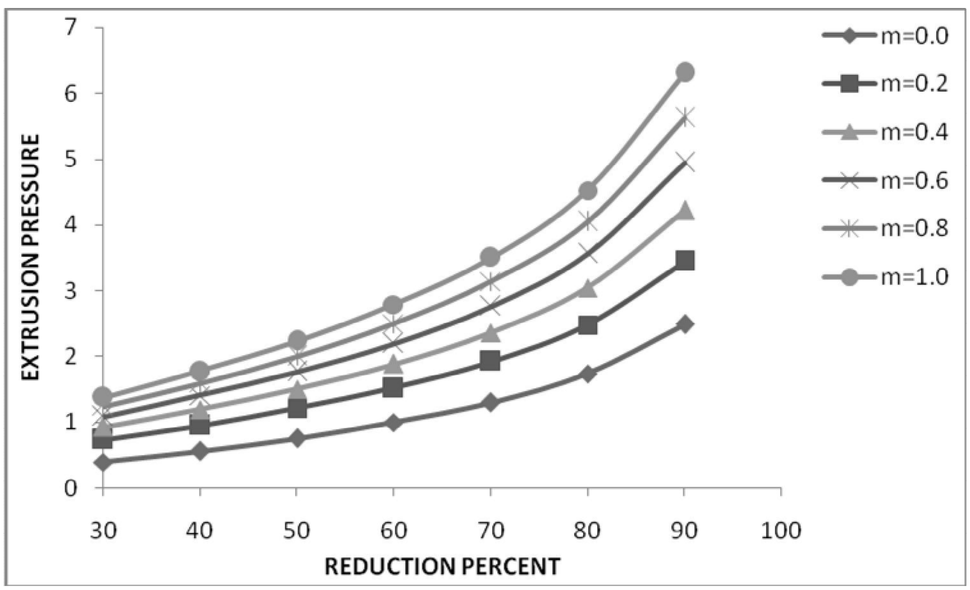

Figure 2. Variation of the non-dimensional extrusion pressure with percentage reduction.

Fig.3. The experimental equipment with assembled set-up.

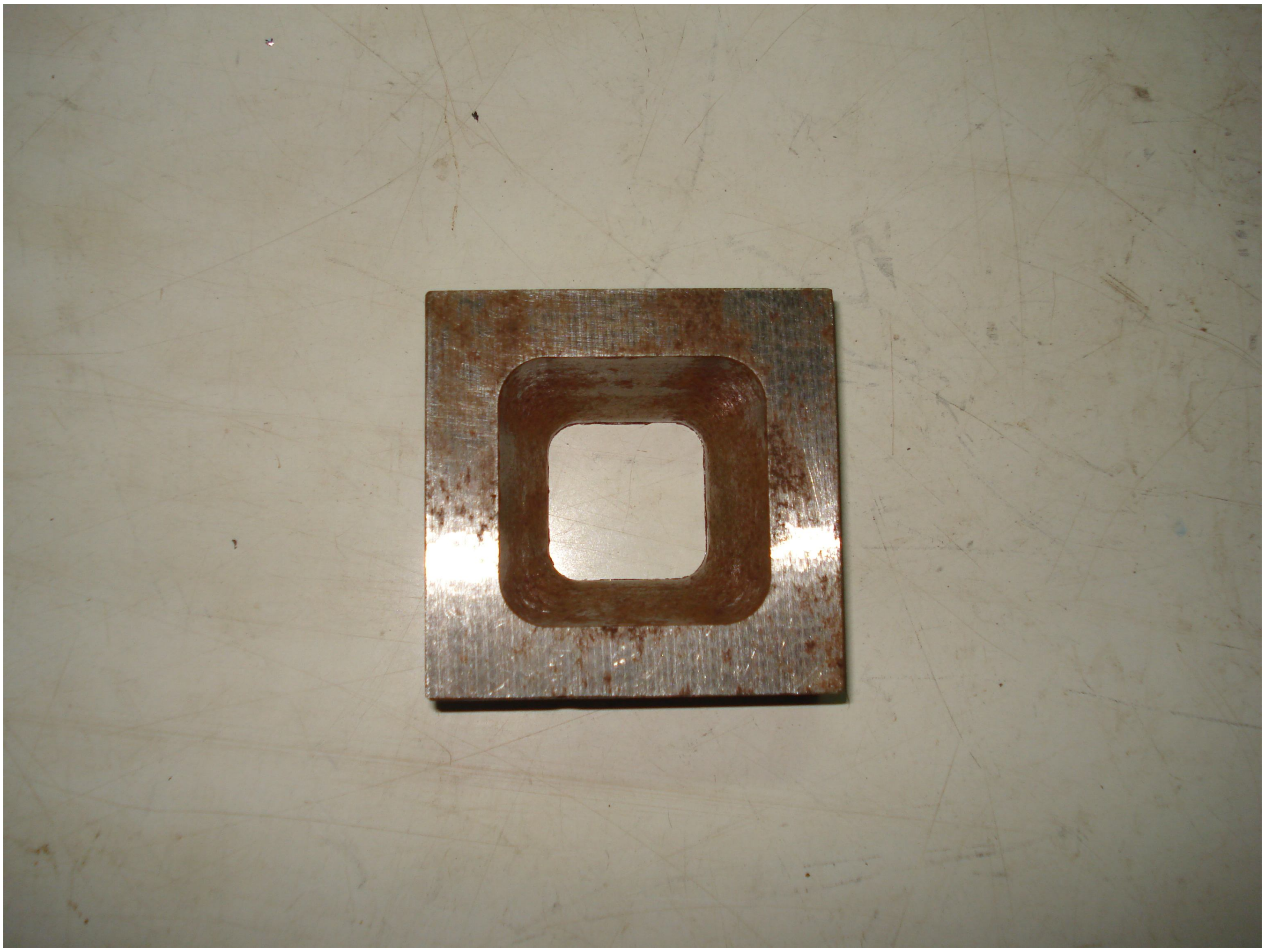

Fig. 4. Extrusion Die

Table 1
Comparison of experimental and computed results[a] (COSINE DIE)

Reduction (%)	Punch Load (N)	P_{av} (N/mm^2)	σ_0 (N/mm^2)	P_{av}/σ_0 Exp.	P_{av}/σ_0 Computed	Error (%)
Dry (m=0.38)						
30	70×10^3	43.75	52.5	0.8333	0.903	7.718
60	150×10^3	93.75	52.5	1.785	1.830	2.459
90			52.5		4.060	

[a] Area of the extrusion chamber = 16.00 cm^2 = 1600 mm^2

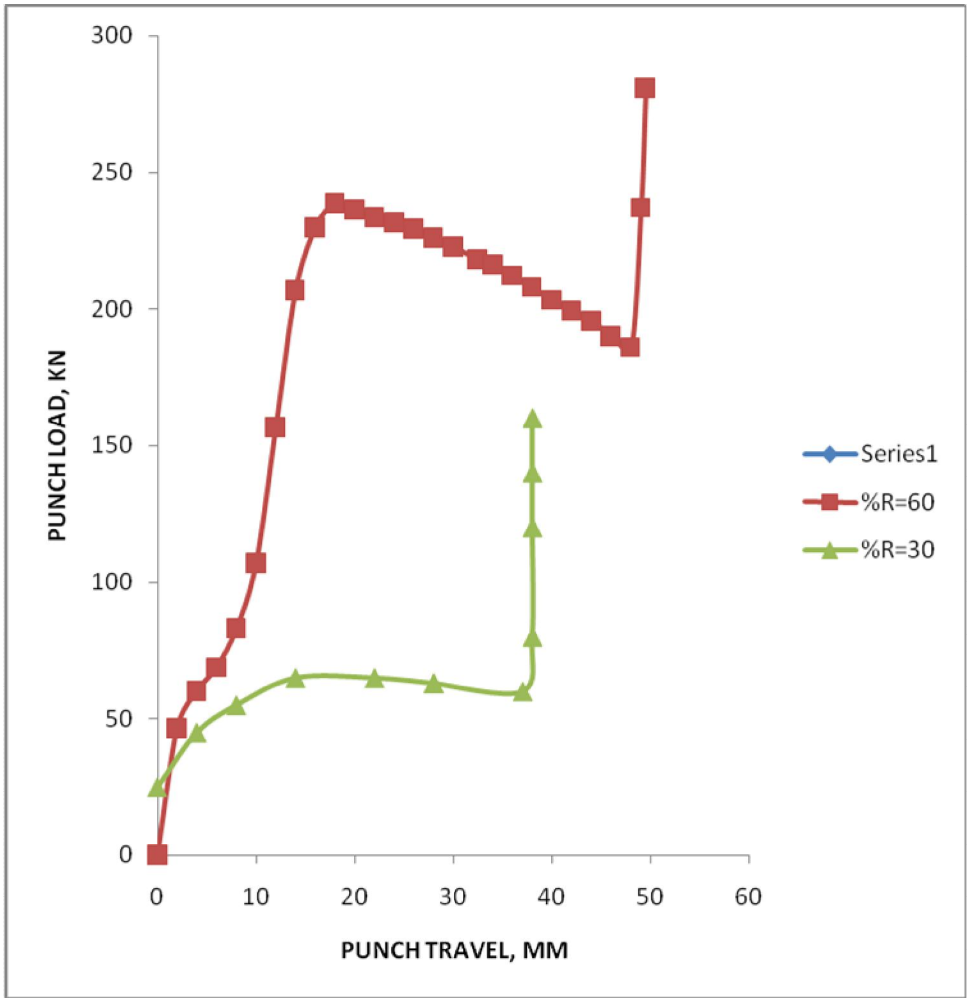

Fig. 5. Punch load vs. punch travel

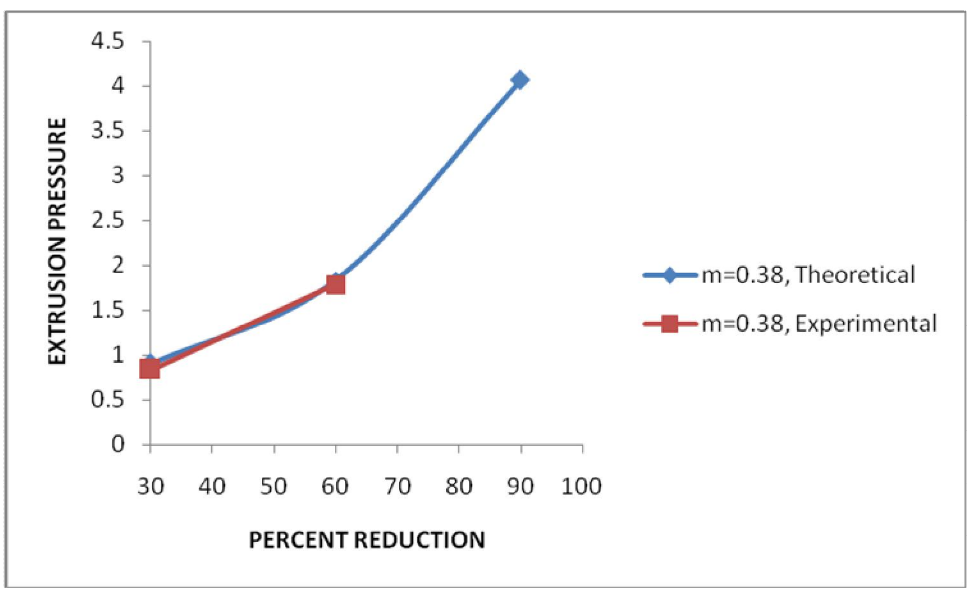

Fig. 6. Comparison of Theory with experiments

6.0 Conclusion

A three dimensional upper bound method has been used to metal deformation process for the extrusion of square section from square billet using a cosine die profile. The dual stream function method has been used to determine kinematically admissible velocity field. The theoretically solution agrees with the experimental.

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