# ON-LINE TRACKING OF TIME VARYING HARMONICS USING AN INTEGRATED EXTENDED COMPLEX KALMAN FILTER AND FOURIERLINEAR COMBINER

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ABSTRACT: The paper presents an integral approach for the estimation of harmonic components of a power system based on the use of Fourier linear combiner (FLC) and extended complex Kalman filter (ECKF). The ECKF estimates the accurate frequency of the signal to update the nominal frequency of the input vector to the FLC. The FLC tracks the the Fourier coefficients of the signal data corrupted with noise very accurately. Once signal is modeled properly, the time varying harmonics of a power system can be estimated accurately using this new approach. Several numerical tests have been conducted to highlight the effectiveness of the technique even in the presence of frequency jump, amplitude variations, noise etc.

Key Words- Harmonic estimation, Fourier Linear Combiner, Kalman Filter

### **I.INTRODUCTION**

Most electronically switched industrial loads found in mining, refining and melting processes, paper mills, etc. are dynamic in nature. In normal operation, repeated stop / start and braking / acceleration cycles tend to generate significant speed variations resulting in time-varying current amplitudes having substantial amount of non stationary harmonics. An increasing number of high voltage transmission systems have static VAR compensators placed at strategic locations which can inject time varying harmonics to the systems. The advent of FACTS devices suggests their use in future power transmission and distribution systems. This gives rise to thr possibility of generation of non stationary harmonic voltages and currents in the power system. Moreover, despite the amount of harmonic filtering they usually have, HVDC systems can still generate current and voltage harmonics at meaningful levels, under constraining operating conditions. The series compensation can also produce low frequency oscillations that interact with SVCs to produce amplitude-modulated harmonics. Several disturbances further complicate this phenomenon by modulating the fundamental frequency, which ultimately yields harmonics with changing amplitudes and frequency.

Thus accurate measurement of harmonic levels is essential for designing harmonic filters, monitoring the stress to which the power system devices are subjected due to harmonics and specifying digital filtering techniques for phasor measurements for relaying. Several numerical techniques have been presented for the estimation of power system harmonics during the last decade. Some of them include Discrete Fourier and Fast Fourier Transforms, Spectral observer, Hartley transform, Kalman filtering, Neural Networks [1-9], etc. As in non stationary harmonics, both frequency and amplitudes are time varying most of the above formulations yield inaccurate estimates, the estimation error could be as high as 10% in the presence of random noise and fundamental frequency deviations.

Amongst the techniques researched in this area, the Kalman filter [8] and the Fourier linear combiner [10] are very effective means for on-line tracking of power system harmonics. It is well known that the Kalman filter produces accurate estimates of the amplitude variation and phase jumps when the fundamental frequency is fixed. During frequency changes it cah not automatically retune itself to the new incoming frequency. In a similar way it is seen that the Fourier linear combiner using a single layer neural network can produce accurate harmonic estimates, if the incoming frequency is stationary. During frequency changes the accurate tracking time becomes much larger and the errors creep into the estimation.

The authors, therefore, propose a hybrid combination of a Kalman filter and Fourier linear combiner for time varying harmonics in the presence of frequency changes. The Kalman filter is of an extended complex type (ECKF) proposed by the authors [11] that has the capability of modeling the frequency as a state vector and the Fourier linear combiner comprises a single layer neural network (also proposed by Dash et al). Once the frequency is estimated accurately by the ECKF, the FLC computes the time varying amplitudes and phase jumps of the stressed power system signal accurately within a shorter time window of 1 to 2 cycles. Further the FLC is easy to implement in real time in comparison to the fixed frequency Kalman filter as it involves large computational overhead.

### **II. PROBLEM FORMULATION**

It is reported in [10] that the FLC can be used for harmonic estimation when the fundamental frequency is fixed. However, although the analyzer can track time variations, its capability in this respect is restricted to amplitude variation and phase jumps. During frequency changes, it will fail because it can not automatically retune itself to the new frequency. Therefore a scheme as shown in Fig. 1 is proposed where the actual system frequency is fed to the FLC with an ECKF based frequency estimator.



Fig. 1. The proposed harmonic estimation scheme

### A. Amplitude Estimation

As shown in Fig. 2, the collective fundamental  $V_1(t)$  and harmonic amplitudes  $(V_2(t),...,V_m(t))$  can be estimated using an adaptive FLC algorithm (Dash et al., 1996). This algorithm, however, suffers from poor convergence in the presence of noise and harmonics and hence a new LMS weight updation technique is presented below to speed up the convergence to the true value and exhibit a strong noise rejection property.



Fig. 2. Fourier linear combiner to estimate fundamental and harmonic components

The discrete form of power system voltage waveform is

$$V(k) = \sum_{i=1}^{N} V_i \cos(n\omega k \Delta T + \phi_i) + \eta(k)$$
(1)

In equation (1)  $\eta(k)$  is a zero mean random noise, N is the highest order of the harmonic in the signal,  $\omega$  is the fundamental angular velocity ( $\omega = 2\pi f_n f_r$  = frequency) and obtained from the ECKF,  $V_i$  is the magnitude of the ith harmonic and  $\phi_i$  its phase angle and  $\Delta T$  = sampling interval, k being the time step. The voltage signal (1) can be expressed as  $V(k) = s(k) + \eta(k)$ 

$$= \sum_{i=1}^{N} (a_i \cos i\omega k \Delta T + b_i \sin i\omega k \Delta T) + \eta (k)$$
(2)

The parameters  $a_i$  and  $b_i$  of the signal corrupted by noise  $\eta(k)$  are obtained by minimizing the error e(k) between the desired signal V(k) and the estimated signal  $\hat{V}(k)$  by a weight adjustment algorithm in the following way:  $e(k) = V(k) - \hat{V}(k)$ 

$$\hat{V}(k) = \sum_{i=1}^{N} \left( a_i \cos i\omega k \Delta T + b_i \sin i\omega k \Delta T \right)$$
(3)

$$W_{i}(k+1) = W_{i} + \frac{\mu_{i} sign(e(k))e^{p-1}(k)x_{i}(k)}{\lambda + x_{i}^{T}(k)x_{i}}$$
(4)

$$i = 1, 2, \dots, N \text{ for odd } p, \text{ where the variable } x_i \text{ is given by}$$
$$x_i(k) = \begin{bmatrix} \cos i\omega k\Delta T & \sin i\omega k\Delta T \end{bmatrix}^T$$
(5)

$$W_i(k) = \left[a_i(k) \ b_i(k)\right]^T \tag{6}$$

T = Transpose of the quantity and  $\mu_i$  is the step size of parameter for the i-th harmonic component and sgn(.) is the sign function and  $\lambda$  is a small constant (usually  $\lambda = .01$ ).

The value of p = 1 refers to the standard LMS algorithm. For best results, the value of p is found to be 3. Thus for p=3, the parameter W is updated as

$$W_{i}(k+1) = W_{i}(k) + \mu_{i}e^{2}(k)\operatorname{sgn}[e(k)x_{i}(k)]/\left[\lambda + x_{i}^{T}(k)x_{i}(k)\right]^{(7)}$$

The input for the entire Fourier linear combiner is

 $X_1(k) = [\cos k\omega \Delta T \sin k\omega \Delta T, ... \cos ik\omega \Delta T \sin ik\omega \Delta T]$ (8)

...
$$\cos kN\omega\Delta T \quad \sin kN\omega\Delta T$$
]<sup>1</sup>

*i*= 1,2,....N

For improving noise rejection and faster convergence of the above algorithm the step size parameter  $\mu_i$  is adapted as

$$c_{i}(k) = \beta c_{i}(k-1) + (1-\beta)e_{i}(k)e_{i}(k-1) \quad (9)$$
  
$$\mu_{i}(k) = \mu_{i}(k-1) + \gamma c_{i}^{2}(k)$$

and c(k) denotes the error co-variance of the power system signal. The step size  $\mu_i$  will be selected to be large when there is significant divergence between the actual signal and the computed signal. For small divergence, however, the step size  $\mu_i$  is selected to be small. A suitable value of  $\mu_i$  lies between 0.01 and 0.4. The values of  $\beta$  and  $\gamma$  are chosen for optimum performance. The formulation presented in equations 4 to 9 is used to compute the component  $V_1(t)$  of the voltage signal described in equation 2 in the first step.

### **B.** Frequency Estimation

Power system frequency can be accurately estimated by the ECKF [11]. A complex signal y(k) is obtained with the  $\alpha\beta$  transform as in equation 10.

$$\begin{bmatrix} V_{\alpha} & (k) \\ V_{\beta}(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_{\alpha} & (k) \\ V_{b} & (k) \\ V_{c} & (k) \end{bmatrix}$$
(10)

where  $V_{a}(k)$ ,  $V_{b}(k)$  and  $V_{c}(k)$  are the voltage signals of the three phases. A complex voltage  $V_{k}$  is obtained from equation 2 as

$$y(k) = V_{\alpha}(k) + jV_{\beta}(k)$$

$$= Ae^{J(\omega K \Delta T + \phi)} + \xi(k)$$
(11)

Where A is amplitude of the signal and  $\xi(k)$  is the noise component.

The discrete observation signal y(k) can now be modeled in a state-space form as

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & x_1(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$
(12)  
$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \xi(k)$$
(13)

Where the states  $x_1$  and  $x_2$  are

$$x_{1k} = e^{j\omega\Delta T} = \cos k\omega\Delta T + j\sin k\omega\Delta T$$
  

$$x_{2k} = Ae^{j(\omega k\Delta T + \phi)}$$
(14)

and  $\Delta T$  = sampling interval The above linear stochastic filter is also equivalent to the following nonlinear one

 $\begin{aligned} x_{k+1} &= f(x_k) \quad (15) \\ y_k &= H x_k + \eta_k \\ \text{where} \\ x_k &= \begin{bmatrix} x_{1_k} & x_{2_k} \end{bmatrix} \\ f(x_k) &= \begin{bmatrix} x_{1_k} & x_{1_k} x_{2_k} \end{bmatrix}, \text{ a nonlinear function} \\ H &= \begin{bmatrix} 0 & 1 \end{bmatrix} \end{aligned}$ 

### **III. SIMULATION RESULTS**

To evaluate the performance of the proposed scheme in estimating amplitudes of harmonics numerical experiments implemented in MATLAB have been performed. Three phase signals are generated and are utilized by the ECKF to estimate the frequency whereas the FLC considers only phase-a signal. A sampling rate of 64 samples per cycle (50 Hz system) is considered in all calculations.

## A. Estimation with different Harmonic components A sample signal of the type

 $v(t) = 1.0sin(wt + \pi/6) + 0.3sin(3wt + 6\pi/7) + .15sin(5wt + \pi/3) + .1sin(7wt + \pi/18) + .05sin(11wt + \pi/8)$ 

is considered for the purpose.

The results in Fig. 2 show that the estimations converge to the true value within two to three cycles once the FLC is initialized.





Fig. 3. Fundamental and harmonics tracking by the FLC

B. Estimation of time varying harmonics A time varying signal of the form  $v(t) = (1.0+a_1(t)) \sin(wt+\pi/6)$  $+ (0.3+a_3(t)) \sin(3wt+6\pi/7)+(.15+a_5(t))$ 

 $sin(5wt+\pi/3)+\eta(t)$ 

is used to test the robustness of the proposed algorithm. The amplitude modulating parameters  $a_1(t)$ ,  $a_3(t)$  and  $a_5(t)$  are

 $a_1(t) = 0.1 sin(2\pi f_1 t) + 0.03 sin(2\pi f_5 t)$ 

 $a_3(t) = 0.05 \sin(2\pi f_3 t) + 0.02 \sin(2\pi f_3 t)$ 

 $a_1(t) = 0.02 \sin(2\pi f_1 t) + 0.005 \sin(2\pi f_3 t)$ 

where  $f_1=1.0$  Hz,  $f_3=3.0$  Hz and  $f_5=5.0$  Hz. In the above example the random noise  $\eta(t)$  is taken to be 40 dB SNR. The estimation of time varying fundamental, third and fifth harmonics in the presence of random noise is shown in Fig. 4. The results demonstrate that the performance of the FLC and ECKF combination provide excellent estimation even for noisy, time varying amplitude situation.





### C. Effect of frequency change on estimation

Earlier fixed frequency FLC [10] estimation algorithm was effected for any frequency drift. A frequency jump up phenomenon is considered where the system frequency at 50Hz suddenly goes up to 52 Hz at 0.19sec. It is observed that the estimation of higher harmonics take more time to converge to the true value. The frequency tracking by the ECKF is also shown in Fig. 5f.







Fig. 5. Performance of the scheme during frequency jump

### **V. CONCLUSIONS**

The paper presents a new approach for adaptive estimation of harmonic amplitudes in a power system. The technique is based on the hybrid combination of FLC and ECKF to track the time varying amplitudes accurately at different frequencies. The results presented in the paper shows the potential of the scheme in rejecting noise and retuning itself to the incoming frequency automatically.

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