Correlations for the Grindability of the Ball Mill
As a Measure of Its Performance

By
A. Sahoo* and G. K. Roy

Chemical Engineering Department, National Institute of Technology, Rourkela-769008, India

Abstract

Ball mill is vital equipment in industries viz. mineral dressing, ore processing, fertilizers, food and diary, pharmaceuticals and many others. The present work involves a meticulous study of the effect of the various parameters on the performance of a ball mill. The parameters studied in this work are particle size, number of balls, time of grinding, particle density, and speed of the ball mill (rpm). An attempt has been made to develop correlation for the performance of the ball mill by correlating these variables with the grindability on the basis of dimensional analysis approach as well as fractional factorial design method. It is observed that these parameters influence the performance of the ball mill significantly. The performance of a ball mill is measured with reference to the quantity of undersize or fines (amount of grounded material passing through120 mesh screen) per revolution of the mill, collected for variation of each of the individual parameters.

Finally the calculated values of the fines in terms of the grindability of the mill obtained through the correlations by both the above-mentioned methods are compared with the experimental values thereby justifying the analysis with reasonable deviations. Thus the developed correlations can be applied to know the grindability of the Ball Mill over a wide range of parameters.

Keywords
Grindability, Ball mill, Dimensional Analysis approach, and Fractional Factorial Design.

*Corresponding author; E-Mail:abantisahoo@gmail.com
1. Introduction

In almost all the metallurgical processes, crushing/grinding is considered to be an inseparable unit operation which is adopted at the very first stage of a series of other unit operations and/or unit processes. Size reduction of solids is done almost invariably. Ball mill finds its application in many industries like abrasives, animal products, brewing industry, chemical, confectionery, food processing, fuel preparation, metal powder, mineral preparation, paint preparation, paper, pigments for color industry, abrasives for grinding, plastics, printing ink, rubber, textiles, sintering, cement and limestone, powders for the detergent industry, pulverized coal for power generation, refractory materials for investment casting, dry powder opacifiers for ceramics industry, pharmaceuticals, mineral preparation, refractory materials for investment casting, tungsten powder and dry lubricants, carbon black for rubber, charcoal for briquetting and others.

Mainly because of their simple construction and application ball milling is a wide spread milling technology, particularly in mining. Various parameters viz. particle size, number of balls, time of grinding, particle density and speed of the ball mill (rpm) have been considered for the present work to determine the performance of the ball mill.

2. Literature

Ball mills are cylindrical or conical shell rotating about a horizontal axis, partially filled with a grinding medium such as natural flint pebbles, ceramic pellets or metallic balls. The common classification of ball mill is any type of mill in which mild steel or iron balls are used. In most cases the cylinder of such mill is made of alloy steel or some special type of lining. The material to be ground is added so that it is slightly more than fills of the voids between the balls (with a maximum limit of number of balls not exceeding 50% of the mill capacity for proper grinding). The shell is rotated at a speed, which will cause the pellets to cascade, thus reducing particle sizes by impact. It has been proposed that in the plastics industry the term ball mill be reserved for metallic grinding media, and the term pebble mill
for non-metallic grinding media. There are two different methods of comminution: autogeneous comminution where the material is pure and heterogeneous comminution where the material is mixed with heavy spheres from steel of typical diameter of several centimeters to increase efficiency. There exists much experimental knowledge on the operation mechanism of ball mills and on the comminution of granular matter in these mills. To increase efficiency of the mill one would have to tune the rotation velocity so that the average collision velocity becomes maximum. In this context attempt has been made for a meticulous study of the effect of the various system parameters on the performance of ball mills.

2.1 Terms Associated with Ball Mill Operation

2.1.1 Grindability (G):

Grindability is the number of net grams of screen undersized product per revolution \([1]\). The chief purpose of study of the grindability is to evaluate the size and type of mill needed to produce a specified tonnage and the power requirement for grinding. Detailed prediction of grinding rate and product size distribution from mills await the development of a simulation based on physics of fracture.

N. Magdalinovic \([2]\) has stated simplified procedure for a rapid determination of the work index by just two grinding tests. The applicability of the simplified procedure has been proved on samples of Cu ore, andesite and limestone. The result by this method was not more than 7% from the values obtained in the standard Bond test.

T. Yalcin et al. \([3]\) investigated the effect of various parameters on the grindability of pure Sulfur and used the obtained grinding data to establish mathematical models and set up a computer simulation program. The established mathematical model is as shown below.

\[
y = \frac{100}{k} \left[ \frac{1}{1 - e^{-\left( \ln 2 \left( \frac{d}{d_{50}} \right)^{0.8} \right) \text{rpm}}} \right]
\]  
(1)
Where, \( y \) is cumulative percent passing size \( d \), \( d_{50} \) is the 50% passing size, \( n \) is distribution constant, and \( k \) is a correction factor. The \( n \) values ranged from 0.84 to 1.84, and \( k \) values from 0.95 to 1.00.

By using the Bond method of grindability, H. Ipek et al. [4] have observed that less specific energy input is required in separate grinding of ceramic raw materials than grinding them in admixtures. They have stated that the Bond work indices of the admixtures containing softer component are greater than the weighted average of the work indices of the individual components in the mixture.

2.1.2 Critical Speed:

If the peripheral speed of the mill is very high, it begins to act like a centrifuge and the balls do not fall back, but stay on the perimeter of the mill and that point is called the "Critical Speed" (\( n_c \)). This phenomenon is called centrifuging. Ball mills usually operate at 65% to 75% of the critical speed. The critical speed is calculated as under [5].

\[
n_c = \frac{1}{2\pi} \sqrt{\frac{g}{R - r}}
\]  

(2)

2.1.3 Work Index:

Work index is defined as the gross energy required in kilowatt-hours per ton of feed needed to reduce a very large feed to such a size that 80% of the undersize passes through 100-\( \mu \)m screen [5]. The expression for this is as given below

\[
W = 0.3162 \times W_i \left( \frac{1}{d_{p,0.5}} - \frac{1}{d_{F,0.5}} \right)
\]  

(3)

Deniz and Ozdag [6] have investigated the effect of elastic parameters on grinding and examined the relationship between them. The most widely known measure of grindability is Bond’s work index which is defined as the resistance of the material to grinding. The standard equation used by them for the ball mill work index (Bond work index) is as follows.

\[
W_i = 1.1 \times P_i^{0.23} G_{bg}^{0.82} \left[ \frac{44.5}{10/\sqrt{P_{80}}} - \left( \frac{10}{\sqrt{F_{80}}} \right) \right]
\]  

(4)
In designing and optimizing a milling circuit using Bond Ball Mill Work Index[7], the following equations are used (Bond 1961).

\[ W = 10W_i \left( \frac{1}{\sqrt{P_{80}}} - \frac{1}{\sqrt{F_{80}}} \right) \quad \text{And} \quad P = T \times W \quad (5) \]

Based on this equation it is possible to calculate, for example, the specific energy requirement for a given grinding duty, BBMWI, feed size and required product size. It is then possible to determine the size of mill required based on throughput and therefore the motor power.

2.2 Factors Affecting Size of Product from Ball Mill

It is important to fix the point where the charge, as it is carried upward, breaks away from the periphery of the Mill. This is called as “break point” or “angle of break” because it is measured in degrees. It is measured up the periphery of the Mill from the horizontal [8].

There are four factors affecting the angle of break:

1. Speed of Mill
2. Amount of grinding media
3. Amount of material
4. Consistency or viscosity (for wet grinding)

2.3 Fractional Factorial Design [9]:

Full and fractional Factorial Design analysis is common in designed experiments for engineering and scientific applications. In many cases, it is required to consider the factors affecting the production process at two levels. The experimenter would like to determine whether any of these changes affect the results of the production process. The most intuitive approach to study these factors would be to vary the factors of interest in a full factorial design, that is, to try all possible combinations of settings.

In statistics, fractional factorial designs are experimental designs consisting of a carefully chosen subset (fraction) of the experimental runs of a full factorial design. The subset is chosen so as to exploit the sparsity-of-effects principle to expose information about the most
important features of the problem studied, while using a fraction of the effort of a full factorial design in terms of experimental runs and resources.

Fractional designs are expressed using the notation $l^k-p$, where $l$ is the number of levels of each factor investigated, $k$ is the number of factors investigated, and $p$ describes the size of the fraction of the full factorial used. Formally, $p$ is the number of generators, assignments as to which effects or interactions are confounded, i.e., cannot be estimated independently of each other (see below). A design with $p$ such generators is a $1/(l^p)$ fraction of the full factorial design. For example, a $2^{5-2}$ design is $1/4$ of a two level, five factor factorial design. Rather than the 32 runs that would be required for the full $2^5$ factorial experiment, this experiment requires only eight runs.

If four or five factors are involved, the complete factorial might involve more than a practical number of experiments. A $2^5$ factorial would require 32 experiments. By careful selection of the experimental conditions it is possible with only a fraction of the total experiments required for the complete factorial to determine the main effects by aliasing them with the higher order interactions which are usually not significant.

The eight experiments required for a complete three factor, two level factorial can be used to determine the change required in four, five or under ideal conditions, even in seven experimental variables to obtain the maximum change in the response variable. As $(k-p)$ factorial design is set up and the $p$ factors not included in the complete $2^{k-p}$ factorial are aliased with one of the higher order interactions to form a generating contrast.

In practice, one rarely encounters $l > 2$ levels in fractional factorial designs, since response surface methodology is a much more experimentally efficient way to determine the relationship between the experimental response and factors at multiple levels. In addition, the methodology to generate such designs for more than two levels is much more cumbersome. The levels of a factor are commonly coded as $+1$ for the higher level, and $-1$ for the lower level. For a three-level factor, the intermediate value is coded as 0.

With two cube ($2^3$) Factorial Design Analysis, the correlation will be represented in the following form.
3. Experimental

A ball mill of 36.6 cm diameter and 50 cm length has been used in the laboratory for experimentation. The material of construction of the grinding media used is mild steel. The steel balls each of size 5.41 cm in diameter and density 7.85 kg/m$^3$ were selected for the experiments. The mill was made to revolve at different speeds to grind various materials like dolomite, manganese, iron ore, and limestone. Exhaustive study was carried out with the dolomite material. The various system parameters (viz. particle size, material density, speed of the mill, time of grinding and the number of balls) were considered to study their effects on the performance of the ball mill. Scope of the experiment is given in Table 1 and the experimental set up is shown in Fig. 1. The amounts of undersize or fine were found out by sieving with the 120-mesh screen. Each time 1.0 kg of material was taken as feed material for running the ball mill.

4. Results

4.1 Development of the Correlations

In the present work, attempt has been made to develop an expression correlating the grindability of the ball mill with the various system parameters by means of dimensional analysis and the fractional factorial design methods. The correlation plot for the former is shown in Fig. 2 and the developed correlation is given as under.

$$Y_{1pr} = a_0 + a_1A + a_2B + a_3C + a_{12}AB + a_{13}AC + a_{23}BC + a_{123}ABC$$  \hspace{1cm} (6)

$$G = 1.0402 \times \left[ \left( \frac{d_F}{d_b} \right)^{-0.054} \left( \frac{\rho_F}{\rho_b} \right)^{-0.040} \left( \frac{n}{n_c} \right)^{1.897} \left( \frac{t}{t_c} \right)^{0.073} \left( N \right)^{0.059} \right]$$  \hspace{1cm} (7)

In case of the fractional factorial design method, a $2^{5-2}$ fractional factorial design was employed to determine the path of the steepest ascent for five factors with eight experiments. The design involved the calculation of the grindability of the ball mill and the variables with
the upper and lower levels are shown in Table 2(A). The actual experiment design with the
calculation of the mean effects is shown in Table 2(B).

To study the effect of various parameters on the grindability of the mill the following
correlations has been developed using fractional factorial design by $2^{5-2}$ approach.

$$G = 3.0925 - 0.19A + 0.165B - 0.115C - 0.19D - 0.11E - 0.09AC - 0.035BC$$  \hspace{1cm} (8)

5. Discussion

The critical speed of the ball mill was calculated to be 76rpm. It is observed that the
gridability of the ball mill or in other words the amount of fines increase with respect to the
overall effect of increase in number of balls, time of grinding, speed (rpm) of the ball mill.
But with increase in particle size and particle density the grindability of the mill decreases;
whereas it increases with the increase in speed of the ball mill, time of grinding and the
number of balls. Therefore the grinding is not so satisfactory for high density particles like
iron ores. The equations obtained are in accordance with the experimental observations.
From the above expressions it is clear that the factor, which affects the most, is the speed. So
for optimizing the grinding process, as compared to other parameters speed has to be
controlled to a maximum extent. The effect of various system parameters on grindability has
been shown in Fig. 2. The calculated values of the grindability from both the approaches
obtained through the developed correlations have been compared with the respective
experimental values with mean and standard deviations of -4.38 and 18.75 for D.A.-approach
and 3.54 and 11.56 for the F.F.D.-approach respectively as shown in Fig. 3. It is found that
the calculated values agree well with the experimental values.

6. Conclusion

In today’s industrial scenario, ball mill is widely used in multifarious industries as size
reduction process is energy inefficient, it is necessary to optimize the operation so as to
reduce cost to some extent. As it has been explicitly seen that the parameters influencing the
performance of ball mill cannot be ignored, the expression correlating all these variables can
be considerably used to optimize the operation of a ball mill in general over a wide range of parameters. Based on the results one can comfortably determine the ranges of various parameters to be used for a specific process. The future aspect of this work can be extended to bond index calculation where the power consumption will indicate directly about the cost benefit too.

**Nomenclature**

$a_0, a_1, a_2, a_3, a_{12}, a_{13}, a_{23}, a_{123}$ : Mean effects for different factors

$A, B, C, D, E$ : Factors for factorial design

d : Diameter (size) in mm

$F_{80}$ : 80% passing size of feed μm

g : Gravitational constant, 981 gm/cm²

G : Grindability of the mill g/rev

$G_{bg}$ : Bond’s standard ball mill grindability g/rev

n : Speed of ball mill, rpm

N : Number of balls

P : Power draw kW

$P_i$ : screen size for performing the test μm

$P_{80}$ : 80% passing size of product μm

r : radius of grinding balls cm

R : Radius of the ball mill cm

t : Time of grinding min

T : Throughput of new feed t/h

W : Work input kW-hr/t

$W_i$ : Work index of the material kW-hr/t

ρ : Density of material kg/m³

Subscripts:

b : for the grinding balls

c : for critical condition

F : for feed particles

p : for product particles

Abbreviations:

D.A. : Dimensional analysis

F.F.D. : Fractional Factorial Design
References


Figure Caption:

Fig.-1: Experimental set-up

Fig.-2: Correlation plot for grindability against system parameters by D.A. approach

Fig.-3: Comparison plot of calculated values of grindability by both the approaches against the experimental values.
Table 1: Scope of the experiment

<table>
<thead>
<tr>
<th>SL No.</th>
<th>Materials Used</th>
<th>Density, $\rho_F$ (Kg/m$^3$)</th>
<th>Particle size, $d_F$, mm</th>
<th>No. of balls, $N$</th>
<th>Speed of ball mill, $n$, rpm</th>
<th>Time of grinding, $t$, min</th>
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<td>1</td>
<td>Dolomite</td>
<td>2900</td>
<td>4.76</td>
<td>12</td>
<td>58</td>
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<tr>
<td>2</td>
<td>Manganese</td>
<td>4150</td>
<td>4.76</td>
<td>12</td>
<td>58</td>
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<tr>
<td>3</td>
<td>Iron ore</td>
<td>5200</td>
<td>4.76</td>
<td>12</td>
<td>58</td>
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<tr>
<td>4</td>
<td>Lime stone</td>
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<td>12</td>
<td>58</td>
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<td>10</td>
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**Table-2(A):** A $2^{5-2}$ Fractional Factorial Design for the actual experiment with the upper and lower levels of the variables

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<th>Experiment</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D=ABC</th>
<th>-E=AB</th>
<th>AC</th>
<th>BC</th>
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<tbody>
<tr>
<td>1</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>a (de)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>b (de)</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
</tr>
<tr>
<td>ab</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>c (d)</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>ac (e)</td>
<td>+</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
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<td>-</td>
</tr>
<tr>
<td>be (e)</td>
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<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
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<td>abc (d)</td>
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<td>+</td>
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**Table-2(B):** Results and calculation of the mean effects for $2^{5-2}$ Fractional Factorial Design

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<th>Experimental Results</th>
<th>A-effect</th>
<th>B-effect</th>
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<th>D-effect</th>
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<td>-1</td>
<td>+1</td>
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</tr>
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<td>2.85</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td></td>
<td>Mean A-effect -0.19</td>
</tr>
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<td>b (de)</td>
<td>3.15</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td></td>
<td>Mean B-effect 0.165</td>
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<td>+1</td>
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<td>-1</td>
<td>+1</td>
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<td>Mean BC-effect -0.035</td>
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Fig-1: Experimental Set-up

Fig-2: Correlation plot for grindability of the ball mill against system parameters
Fig.-3: Comparison of calculated values of grindability by both the approaches against the experimental values