Fast tracking of transient power system signals using fuzzy LMS algorithm

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The paper presents an adaptive least mean squares (LMS) algorithm for the fast estimation of voltage and current signals in power networks. The new estimator is based on the use of linear combiners. The learning parameter of the proposed algorithm is constrained by two variable parameters which causes an automatic suitable adjustment of the step size using a fuzzy gain scheduling method to provide fast convergence and noise rejection for the tracking of fundamental and harmonic components from distorted signals. Several numerical tests have been conducted for the adaptive estimation of fundamental and harmonic components from simulated waveforms from power networks supplying converter loads and switched capacitors.

Keywords: signal tracking, LMS algorithm, Fuzzy rule base

I. Introduction

The problem of estimating the amplitudes, phase angles and frequencies of sinusoidal signals from noisy and distorted data has received considerable attention recently due to the proliferation of power electronic loads in electric power networks. Further, the estimation of basic parameters of voltage and current signals is a prerequisite for evolving suitable protection and control strategies of power networks. Power quality issues are very much dependent on the RMS amplitudes of these waveforms, and hence suitable analysis methodologies and measurement tools are assuming some importance in the power industry.

With the introduction of microcomputers, the digital monitoring of voltage and current phasors in a power network has become feasible. The discrete Fourier transformation (DFT), least mean squares (LMS), recursive least squares (RLS), and Kalman filter techniques [1–4] are some of the known signal processing techniques used for the estimation of voltage and current phasors. The computational cost of a DFT-based algorithm is very low, but its performance is adversely affected by decaying d.c. components or a low signal-to-noise ratio. Both the LMS and RLS algorithms [4] suffer from inaccuracies in the presence of decaying d.c. components and random noise. On the other hand, Kalman filters are well suited for estimating time-varying signal parameters accurately in the presence of noise and harmonics. However, a common problem with these Kalman filters is the high computational requirements, due to transcendental function evaluation in real time. A Newton-type algorithm has been proposed in Ref. [5] to estimate voltage phasor and local system frequency from a distorted voltage waveform. This algorithm, however, suffers from a heavy computational burden and the choice of initial starting parameters. A few algorithms using the neural network [6,7] approach have been presented to recover fundamental components from signals corrupted by noise and harmonics. However, these algorithms are susceptible to errors due to random noise and involve heavy computational overheads.

The purpose of this paper is to present a new algorithm for the fast tracking of voltage and current phasors using an adaptive linear combiner [8] which is analogous to a one-layer neural network. The structure is based on the early work of Widrow and Lehr in the 1960s [9] and has been widely applied in neural networks, signal processing and many other areas. A generalized weight adaptation algorithm for the adaptive linear combiner is used to arrive at the magnitude and phase of the voltage or current phasor. In this algorithm, the learning parameters are adjusted to force an error between the actual and desired outputs in order to satisfy a stable difference error equation, rather than to minimize an error function. This approach allows one to better control the stability and speed of convergence by an appropriate choice of parameters of the error difference equation [8]. The algorithm presented in this paper is an adaptive one and is based on the assumption that the frequency of the fundamental voltage or current phasor is known *a priori*. A fuzzy logic based learning parameter computation is used to provide fast convergence and noise rejection. Several numerical examples are given in the paper

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to validate its performance in the presence of random noise and harmonics.

II. Adaptive linear combiner

The power system voltage or current waveform is assumed to comprise fundamental and harmonic components, as

$$y(t) = \sum_{i=1}^{N} A_i \sin(\mathrm{i}\omega t + \phi_i) \tag{1}$$

where A_i and ϕ_i are the amplitude and phase of the harmonics, respectively, N is the total number of harmonics, and ω is the angular frequency of the fundamental component of the signal. To obtain a solution for on-line estimation of the harmonics, we propose the use of an adaptive estimator in the form of a linear combiner, shown in Figure 1.

To obtain the input variables for the linear combiner, the signal given in the equation is written in the discrete form as

$$\begin{aligned} y(k) = {} & A_i \cos\phi \sin\theta + A_i \sin\phi \cos\theta \\ & + \ldots + A_N \cos\phi_N \sin N\theta + A_N \sin\phi_N \cos N\theta \end{aligned} \tag{2}$$

where

$$\theta = \frac{2\pi k}{N_\mathrm{s}}$$

In the above equations, N is the order of the highest harmonic present in the signal, k is the sample number or iteration count, and N_s is the sample rate.

Thus, the input vector to the linear combiner is given by

$$x(k) = (\sin\theta \cos\theta \sin 2\theta \cos 2\theta \ldots \sin N\theta \cos N\theta)^T \tag{3}$$

and T is the transpose of the quantity.

If the power system signal contains a decaying d.c. component and the waveform is described by

$$y(t) = A_\mathrm{dc}\, \mathrm{e}^{-\beta t} + \sum_{i=1}^{N} A_i \sin(\mathrm{i}\omega t + \phi) \tag{4}$$

the signal is expressed using a Taylor series expansion (neglecting higher order terms) as

$$y(t) = A_\mathrm{dc} - A_\mathrm{dc}\beta t + \sum_{i=1}^{N} A_i \sin(\mathrm{i}\omega t + \phi) \tag{5}$$

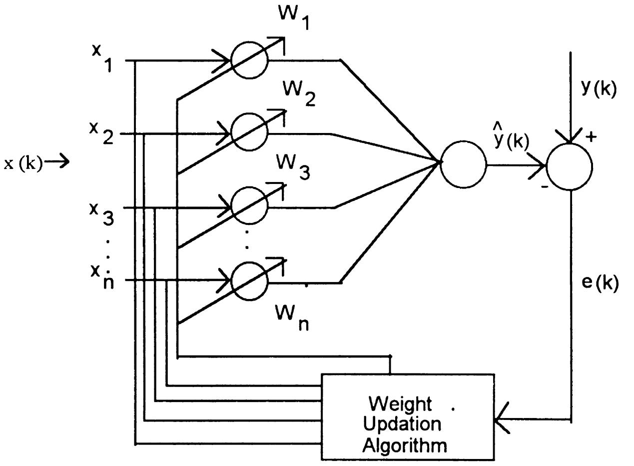

Figure 1. Block diagram of linear combiner

In this the input vector to the adaline is expressed as

$$x(k) = \left(1 \frac{\theta}{\omega} \sin\theta \cos\theta \ldots \sin N\theta \cos N\theta\right)^T \tag{6}$$

The weight vector of the linear combiner is updated using a non-linear weight adaptation algorithm (modification of the Widrow–Hoff delta rule), as

$$(k+1) = W(k) + \frac{\alpha e(k) X(k)}{\lambda + x^T(k) X(k)} \tag{7}$$

where the following hold good at the kth sampling instant:

k is the input vector

$T(k) = [W_1(k)\ W_2(k)\ W_3(k)\ W_4(k) \ldots W_{2N+1}(k)\ W_{2N+2}(k)]$;
$e(k) = y(k) - \hat{y}(k) isthee$
$y(k)$ is the actual signal amplitude;
$\hat{y}(k)$ is the estimated signal amplitude;
α is a learning parameter;
λ is a parameter to be suitably chosen to avoid division by zero.In the above equation the vector X is chosen as

$$\begin{aligned} X(k) = {} & [11 SGN(\sin\theta) SGN(\cos\theta) \\ & \ldots SGN(\sin N\theta) SGN(\cos N\theta)]^T \end{aligned} \tag{8}$$

and the SGN function is given by

$$SGN(x_i) = \begin{cases} +1 \text{ if } x_i > 0 \\ -1 \text{ if } x_i < 0 \end{cases} \tag{9}$$

where $i = 3, \ldots, 2N+2$Instead of using the SGN function, we can use the tanh or arc(tanh) functions for *X(k)*.

The error *e(k)* between the actual signal and the estimated signal is brought down to zero when perfect learning is attained and the weight vector will yield the Fourier coefficients of the signal. If W_0 is the weight vector after the final convergence is reached, the Fourier coefficients are obtained as

$$\begin{aligned} W_0 = {} & [A_\mathrm{dc} \beta A_\mathrm{dc} A_1 \cos\phi_1\ A_1 \sin\phi_1 \\ & \ldots A_N \cos\phi_N\ A_N \sin\phi_N]^T \end{aligned} \tag{10}$$

The amplitude and phase of the Nth harmonic are given by

$$\begin{aligned} A_N = {} & \sqrt{W_0^2(2N+1) + W_0^2(2N+2)} \text{ and } \phi_N = \tan^{-1} \\ & \times \left[\frac{W_0(2N+2)}{W_0(2N+1)}\right] \end{aligned} \tag{11}$$

For tracking three-phase voltage and current phasors, three adaptive linear combiners will be required, as shown in Figure 2.

III. Fixing the learning parameter α

The learning parameter α used in the modified Widrow–Hoff delta rule is an important parameter which controls the convergence and noise rejection property of the adaptive linear combiner. The learning parameter α is adapted recursively in the following way.

$$\alpha(k+1) = \alpha(k) + \mu\, SGN[\nabla_a e^2(k+1)] \Delta\alpha_{k+1} \tag{12}$$

where

$$\nabla_a e^2(k+1) = \frac{\partial e^2(k+1)}{\partial \alpha} = \frac{\partial e^2(k+1)}{\partial W(k+1)} \frac{\partial W(k+1)}{\partial \alpha} \tag{13}$$

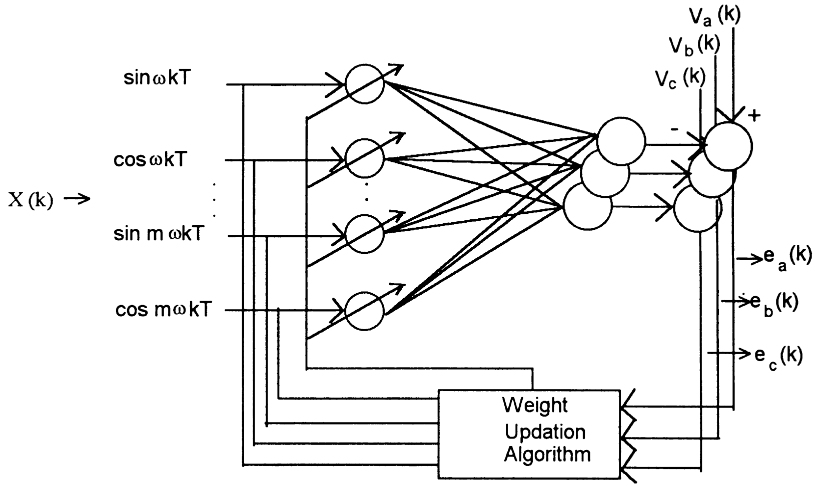

Figure 2. Block diagram of the linear combiner used for tracking three-phase voltages and currents

In the above equation (equation (13)), μ is a small positive constant that controls the adaptive behaviour of the step size. The term $\Delta\alpha_{k+1}$ of the same equation is explained in detail in the next section.

The initial value of α is chosen to be 2.0. The learning parameter α is, however, constrained to lie between the limits

$$\alpha_{\min} \leq \alpha(k+1) \leq \alpha_{\max} \tag{14}$$

The value of $\alpha_{\max}$ is chosen to ensure that the mean square error of the algorithm remains bounded. A sufficient condition to ensure mean squared convergence of the algorithm is

$$\alpha_{\max} = \frac{2}{3\, tr(R)} \tag{15}$$

where R is the autocorrelation matrix of the input vector X given by

$$R = E[X(k+1)X^T(k+1)]$$

The $\alpha_{\min}$ is chosen to provide a minimum level of step size without making system tracking very sluggish. The step size is always positive. For large prediction error, the learning parameter is increased to provide faster tracking.

IV. Calculation of $\Delta\alpha$

The change of the step size $\Delta\alpha(n)$ is computed by using a fuzzy logic based algorithm. The input and output of the fuzzy system are quantitative measures of misadaption $u(n)$ and the change of the step size $\Delta\alpha(n)$, respectively. Inside the fuzzy logic (FL) block, they are converted into, and treated as, fuzzy variables, although they have crisp values. Here in this paper the norm of cross-correlation between the estimation error and the input data as a measure of misadoption is Uk, where $Uk/2$ is the magnitude of the gradient vector $\nabla_k e^2(k)$.

$$U_k = \|e(k)\, X(k)\|$$

The linguistic control rules are as given below.

R_1: IF $U(k)$ is ZE THEN $\Delta\alpha(n)$ is ZE
R_2: IF $U(k)$ is VS THEN $\Delta\alpha(n)$ is VS
R_3: IF $U(k)$ is S THEN $\Delta\alpha(n)$ is S
R_4: IF $U(k)$ is M THEN $\Delta\alpha(n)$ is M
R_5: IF $U(k)$ is L THEN $\Delta\alpha(n)$ is L
R_6: IF $U(k)$ is VL THEN $\Delta\alpha(n)$ is VL

The linguistic variables ZE, VS, S, M, L and VL represent the fuzzy subsets zero, very small, small, medium, large and very large, respectively. The membership functions of the fuzzy variables $U(k)$ and $\Delta\alpha(n)$ with respect to the linguistic variables are shown in Figure 3. The above fuzzy rules are aggregated by an OR operation given by

$$R = R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5 \cup R_6$$

The inference mechanism of the fuzzy logic based algorithm produces a fuzzy output $\Delta\alpha(n)$ from which the crisp value of $\Delta\alpha(n)$ is obtained through a defuzzification procedure. The output of the FL block is computed by

$$\Delta\alpha(n) = \frac{\sum_i \tau_i \Delta\alpha_i}{\sum_i \tau_i} \tag{16}$$

where τ_i is the firing strength of the ith rule and $\Delta\alpha(n)$ is the centre of gravity of the output fuzzy subset of the ith rule. The computing process being very simple does not affect the simplicity of the linear combiner.

The power system signal model presented in equation (4) does not show any noise term. Thus, if a random noise is added to the signal model, the accuracy of the linear combiner will be affected in the presence of the random noise. Therefore to provide a better noise rejection term, the error term is fed back recurrently and the input to the linear combiner will, for the model containing a d.c. component and harmonics, become

$$X(k) = [1 \frac{\theta}{\omega} \sin\theta \cos\theta \ldots \sin N\theta \cos N\theta \; e(k)e(k-1) \; e(k-2)]^T \tag{17}$$

V. Simulation results

In order to check the validity and performance of the proposed algorithm, numerical experimentation on the simulated waveforms has been carried out using the MATLAB software package. The simulations fully confirmed the correctness of the presented approach. The linear combiner algorithm is initialized by starting from a null weight vector. A sample rate of 64 based on the 50 Hz frequency is chosen for the estimation of signal amplitude and phase for all the studies. Owing to limited space, we are presenting some illustrative results in order to show

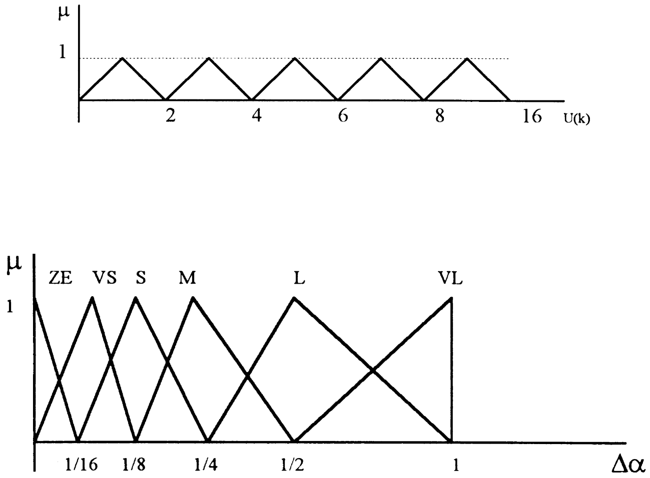

Figure 3. (i) Membership function of $U(k)$. (ii) Membership function of $\Delta\alpha$

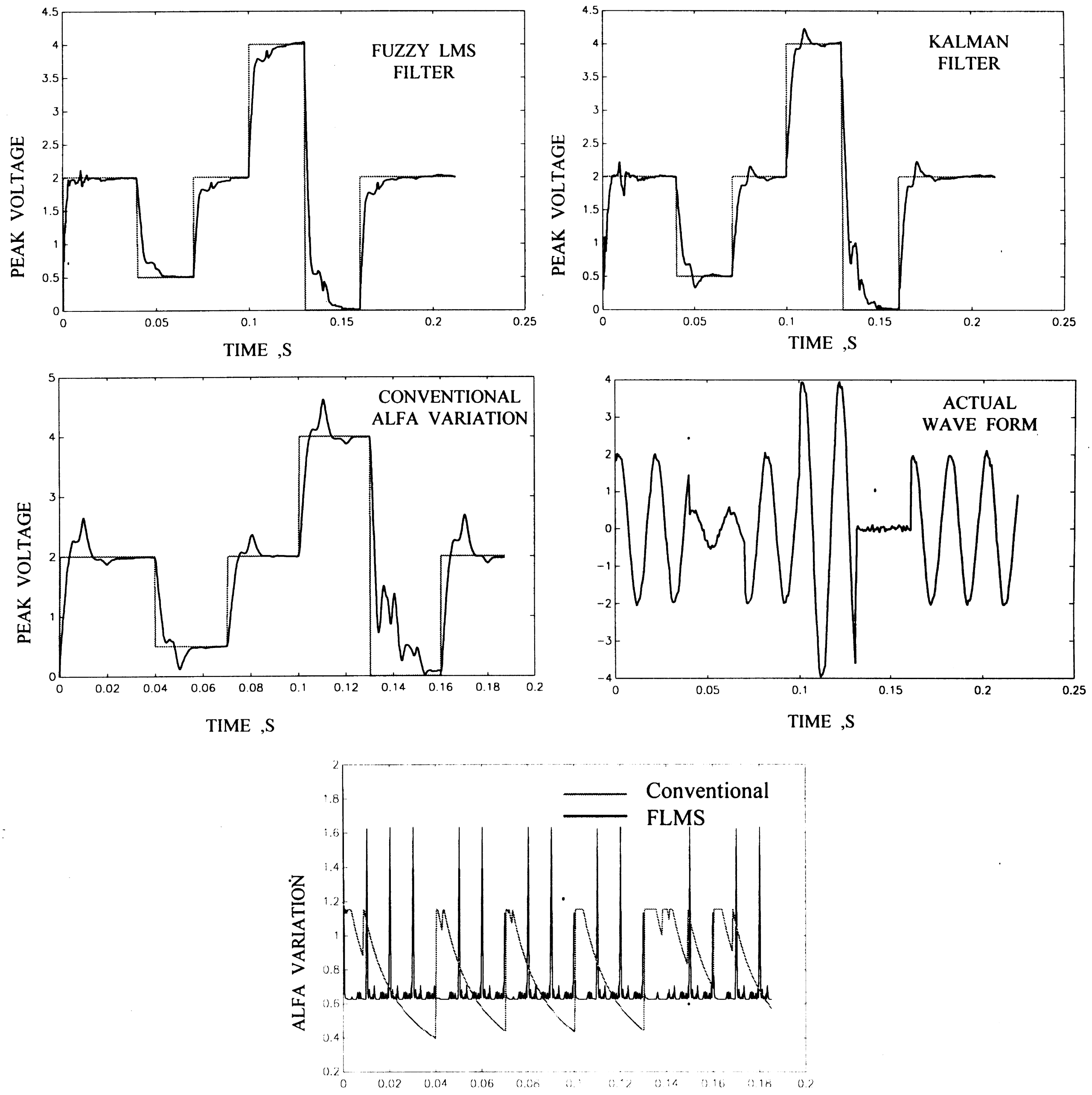

Figure 4. Tracking a distorted signal under conditions of sag, swell and outage

the accurate tracking capability of the FLMS (fuzzy LMS) estimator.

V.1 *Case 1*

Figure 4 shows the typical voltage waveform (short-duration RMS variations in the voltage waveform) encountered for a distribution feeder during a single-line to ground fault. The depression of voltage is usually known as sag, which has a magnitude around 80% of the fundamental and a duration of 2–10 cycles. The swell is characterized by the voltage rise of the unfaulted phase (to a value nearly 120% of the fundamental component) and outage is characterized by zero voltage on the faulted phase. From the figure it is observed that the linear combiner provides fast tracking of the peak voltage magnitudes and the fundamental voltage waveform very accurately in less than 1 cycle. The variation of α is also shown in this figure. This example clearly demonstrates the feasibility of using a linear combiner and fuzzy LMS

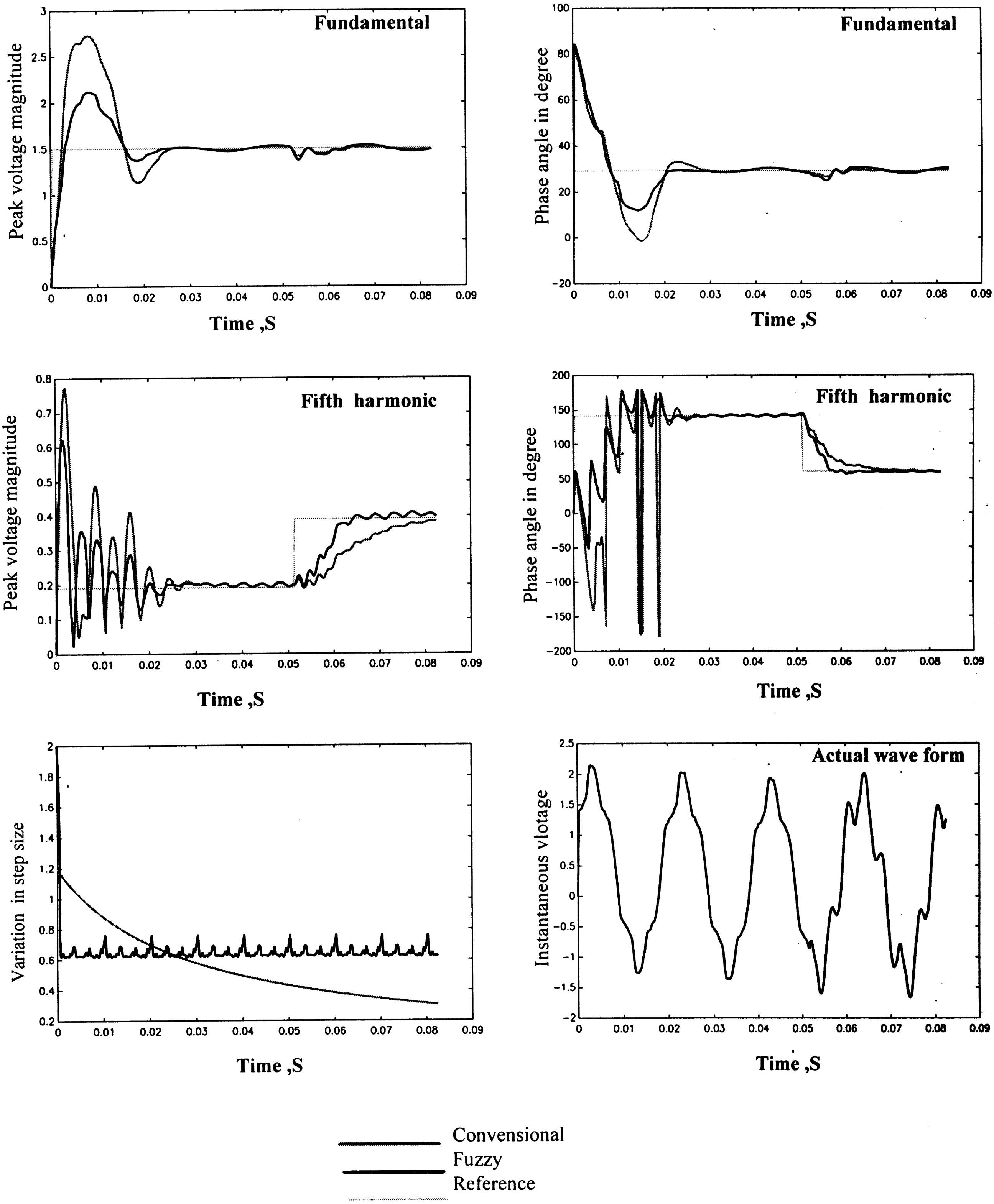

Figure 5. Tracking of fundamental and harmonic components

algorithm for power quality monitoring. An optimized Kalman filter is used to provide a meaningful comparison with the new approach presented in this paper. The Kalman filter shows overshoots and a larger settling time in comparison to the linear combiner–fuzzy LMS approach. The figure also shows the membership grades of the change in $\alpha[\Delta\alpha(k)]$ and the actual value of α. The fuzzy output for producing the change in α is either zero or 1 and the actual value of α is calculated according to equation (12).

V.2 *Case 2*

The signal that is of considerable importance in power networks is the fault current, which changes from its nominal value to a large value during the fault period. Such a signal is represented as

$$\begin{aligned} y(t) = {} & 1.5\sin(\omega t + 29.3°) + 0.5\exp(-15t) \\ & + 0.2\sin(5\omega t + 141.6°) + 0.017\sin(7\omega t + 86.2°) \\ & + 0.022\sin(11\omega t - 99.4°) \\ & + 0.024\sin(13\omega t - 179.2°) \\ & + 0.012\sin(17\omega t - 1.3°) + 0.016\sin(19\omega t - 89.6°) \\ & + K\,rand(t) \end{aligned}$$

where $K = 0.05$.

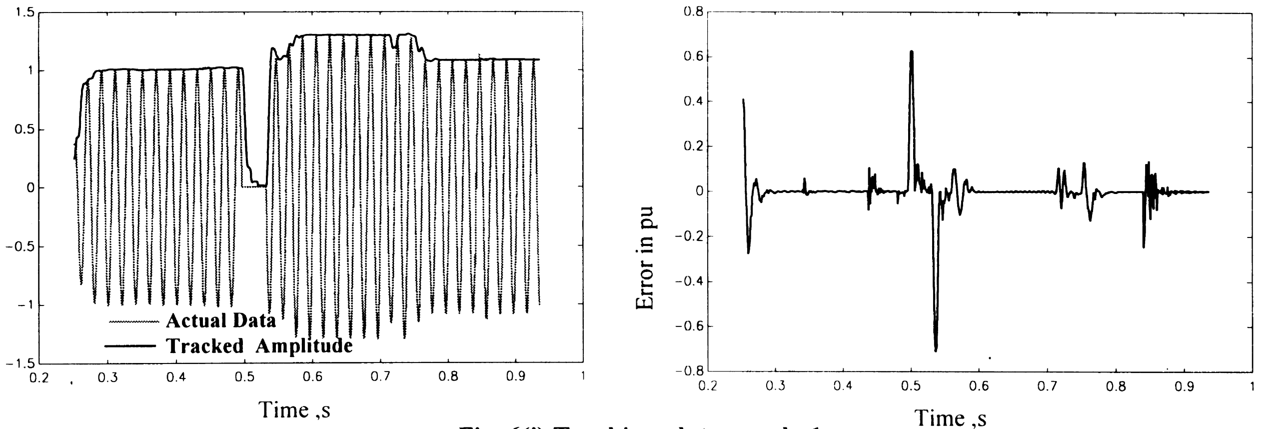

Fig .6(i) Tracking datasample-1

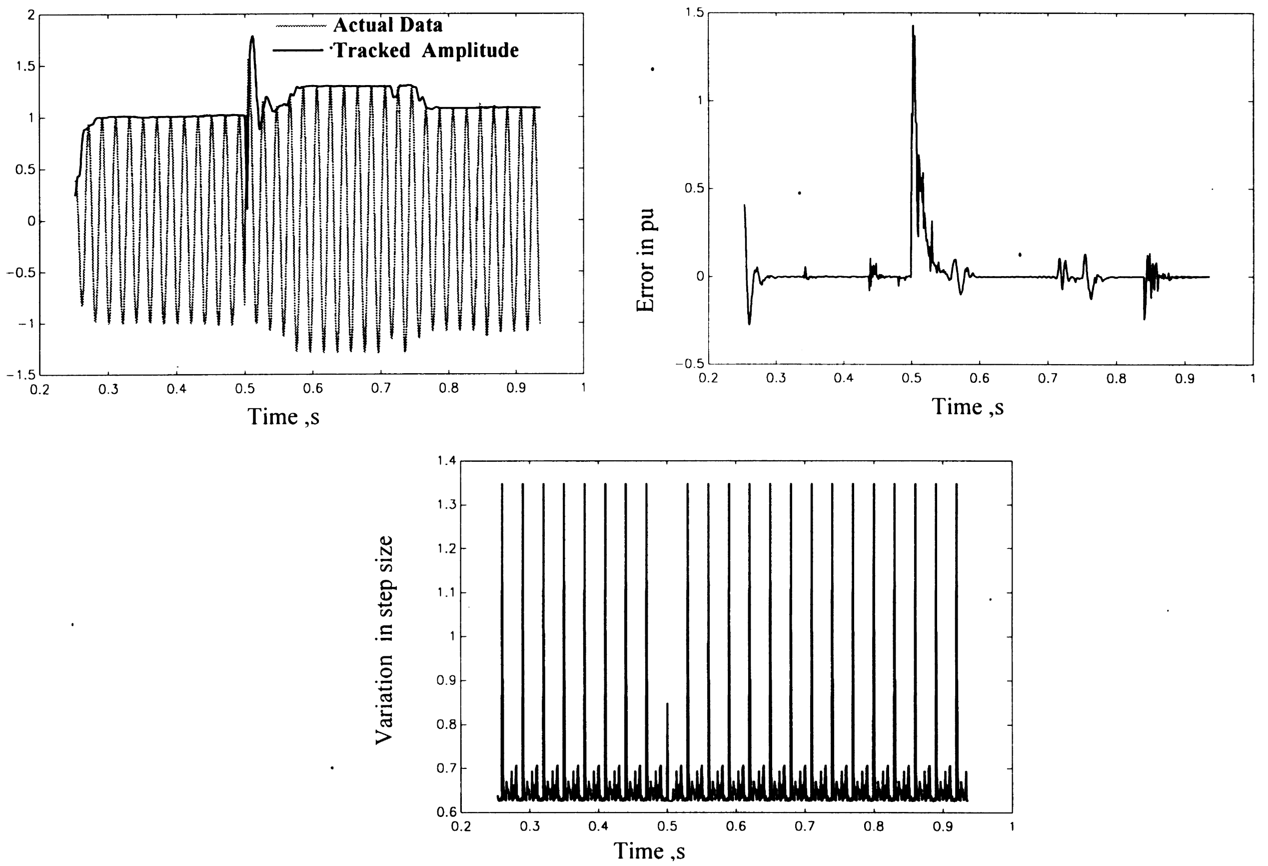

Figure 6. Tracking fundamental amplitude of simulated data samples by fuzzy LMS. (i) Tracking datasample-1; (ii) tracking datasample-2

The value of K is set at 0.05 for this study, which is equivalent to superimposing a 0.2 p.u. peak noise level on the faulted current signal. The function $rand(t)$ has a zero mean, normal distribution and variance unity.

Figure 5 shows the tracking of the fundamental and fifth harmonic components of the distorted signal presented above using both the fuzzy LMS algorithm and the Kalman filter. From the figure it can be seen that in the presence of a decaying d.c. and harmonics, the Kalman filter shows a larger error and converges to the true value slowly in more than four cycles. However, using the fuzzy LMS algorithm (Figure 6), the convergence is obtained in almost one cycle in the case of the fundamental and in less than two cycles in the case of the fifth harmonic. This result is found to be significant in comparison to the computation of harmonics in the presence of a decaying d.c. and large random noise.

VI. Conclusions

The adaptive linear combiner represents a very realistic and promising approach for the fast estimation of power network signal parameters corrupted by noise, decaying d.c. components, and harmonics. The non-linear learning parameter weight vector adjustment for the linear combiner is done using a difference error equation and provides an accurate estimation of the amplitude and phase of a voltage phasor corrupted by harmonics and noise. By taking a suitable fixed learning parameter, one can also obtain the same accuracy of results but with sudden changes in the input vector (which are quite obvious in power system waveforms), the magnitude of the learning parameter must be suitably changed. Again, it is difficult to optimize the initial value of the learning parameter, which is generally carried out heuristically. In this paper, an automatic adjustment of the learning parameter is achieved depending upon the changes in the input vector. Computer simulation experiments have shown that the non-linear weight adaptation along with an adaptive fuzzy LMS algorithm yield a more accurate and faster estimation of voltage phasor in comparison with the well-known Kalman-filter-based algorithm.

VII. Acknowledgements

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