Sensorless Sliding Mode Control of Induction Motor Drives

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Abstract- **This paper demonstrates that invariant regulator and superior servo performance can be obtained, compared to fixed gain controllers, through sliding mode speed control of a decoupled induction machine. The speed control scheme consists of a standard indirect vector control implementation for torque control, with a sliding mode speed controller. Assuming the bounds on the parameter variations are known, the control law is derived. The chattering of single component sliding mode controller, is eliminated by introducing a boundary layer. The performance of single component sliding mode controller with boundary layer is further improved by dual component sliding mode controller. Robustness to variation of parameters, like rotor resistance and moment of inertia are demonstrated through simulation. Results are compared with fixed gain controllers. A speed estimation algorithm is implemented and used in the sliding mode control scheme. Sensorless sliding mode control is found to be superior to fixed gain control. Further, it is established through tests that, dual component sliding mode control performs better than single component sliding mode control.**

I. INTRODUCTION

With increasing power and reducing costs of DSPs, high performance induction motor drives with advanced control techniques combined with sensorless control [1, 2] are widely commercialized. Induction motors are also getting more acceptance in robotic applications, where the actuator has to reproduce complex trajectories specified for manipulator movement. A control scheme is therefore needed that performs well in the general servo situation, and is not limited to providing satisfactory responses just to step demands. Field oriented induction motor drives [1, 2] with fixed gain controllers (such as PID controllers) fail to provide satisfactory response for trajectory tracking in servo applications. Problem is more sever under varying system parameters, load disturbances and measurement noise. Induction motor drive with sliding mode control [3-5] is suitable for both servo and regulator applications. It is computationally simple (compared to adaptive controllers with parameter estimation) and has robustness to parameter variations within known bounds and load disturbances. Disadvantage of sliding mode control method is chattering of the system states due to high control activity. In [5] sliding mode control is applied to position control loop of an indirect vector controlled induction motor drive, without rotor resistance identification scheme. A sliding mode based adaptive input-output linearizing control is presented in [6]. The motor flux and speed are separately controlled by sliding mode controllers with variable switching gains. Sliding mode controller and fuzzy sliding mode controller are designed in [7] for speed control of induction motor, and their performance compared.

In indirect field oriented control of induction motors, knowledge of rotor speed is required. These speed sensors have many disadvantages. Research has been carried out on design of speed sensorless control schemes [1, 2]. Two such speed estimation schemes are reported in [8]. In these schemes the speed is estimated based on measurement of stator voltages and currents. But the estimation is heavily dependent on machine parameters. The parameter uncertainties impose a challenge in the control performance. This paper presents a sensorless decoupling control scheme. A speed estimation algorithm is reported in section-III, which overcomes the necessity of the speed sensor. A sliding mode control is discussed in section-IV, to compensate the uncertainties that are present in the system. Sliding mode control with speed estimation algorithm works well together. Test results are discussed in section-V, and some concluding remarks are stated in section-VI.

II. FIELD ORIENTED INDUCTION MOTOR

Field orientated control of induction machine is already implemented with several variations. Synchronously rotating d-q reference frame current control is used in this report, for field orientation. To achieve field orientation, the flux (d-axis) component of stator current i_{ds} is aligned in the direction of the rotor flux, ψ_r, and the torque component of current, i_{qs} is aligned in the direction perpendicular to it. At this condition:

$$\psi_{qr} = 0 \text{ and } \psi_{dr} = \psi_r \tag{1}$$

The electromagnetic torque of field oriented induction motor is given by:

$$T_e = \frac{3p}{2}\frac{L_m}{L_r}\psi_{dr}\, i_{qs} = K_T \psi_{dr} i_{qs} \tag{2}$$

where K_T is the torque constant, and p is number of pole pairs. L_m and L_r are magnetizing and rotor self inductances.

With field orientation the dynamic equations of stator current components and rotor flux are given by [8]:

$$\dot{i}_{ds} = -a_1\, i_{ds} + a_2\psi_{dr} + \omega_e\, i_{qs} + v_{ds}/(\sigma L_s) \tag{3}$$

$$\dot{i}_{qs} = -\omega_e i_{ds} - a_1 i_{qs} - a_3 p\omega_r\psi_{dr} + v_{qs}/(\sigma L_s) \tag{4}$$

$$\dot{\psi}_{dr} = -(\psi_{dr}/\tau_r) + (L_m\, i_{ds}/\tau_r) \tag{5}$$

where, ω_e is the synchronous speed (electrical) in rad/s,
ω_r is rotor speed (mechanical) in rad/s,
v_{ds} and v_{qs} are d- and q-axis components of stator voltage,
$\sigma = 1 - L_m^2 / L_s L_r$ is the leakage coefficient,
$\tau_r = L_r / R_r$ is the rotor time constant,

$$a_1 = (R_s L_r + L_m^2 / \tau_r)/(\sigma L_s L_r),$$

$$a_2 = L_m / (\sigma L_s L_r \tau_r),\ a_3 = L_m / (\sigma L_s L_r),$$

R_s and R_r are stator and rotor resistances.
Slip frequency for obtaining indirect field orientation is [2]:

$$\omega_{sl} = \omega_e - p\,\omega_r = L_m\, i_{qs} / (\tau_r\, \psi_{dr}) \qquad (6)$$

The decoupling of torque and flux is guaranteed with field orientation technique. Induction motor can be controlled linearly as a separately excited dc motor. However, due to the presence of the rotor time constant τ_r in eqn. (6), the indirect field oriented control is highly parameter sensitive. Unpredictable parameter variations, external load disturbances, unmodelled and nonlinear dynamics adversely affect the control performance of the drive system. On-line adaptation to achieve ideal field orientation is an important but very difficult issue. Sliding mode control [5] is proved to be a good robust control technique under these conditions. Two types of sliding mode controller are discussed and compared in section IV. The speed information required in sliding mode control is estimated by the algorithm presented in the following section.

III. Speed Estimation Using Rotor Flux Observer

Many schemes have been devised to estimate motor speed from measured terminal quantities. Most of these estimation techniques are based on simplified motor models. In order to obtain a better estimation of the motor speed, it is necessary to have dynamic representation based on the stationary $(\alpha - \beta)$ reference frame. Since motor voltages and currents are measured in a stationary frame, it is also convenient to express these equations in stationary $(\alpha - \beta)$ reference frame.

From the stator voltage equations in stationary reference frame [2]:

$$\dot{\psi}_{\alpha r} = \frac{L_r}{L_m} v_{\alpha s} - \frac{L_r}{L_m}(R_s i_{\alpha s} + \sigma L_s \dot{i}_{\alpha s}) \qquad (7)$$

$$\dot{\psi}_{\beta r} = \frac{L_r}{L_m} v_{\beta s} - \frac{L_r}{L_m}(R_s i_{\beta s} + \sigma L_s \dot{i}_{\beta s}) \qquad (8)$$

where, ψ_r is the rotor flux linkage. Stator voltage is denoted by v_s. Stator current is i_s. α and β in the subscripts, denote the corresponding components of rotor flux linkage, or stator voltage or current.

Eliminating the current derivatives from (7) and (8), the rotor flux dynamic equations in the stationary frame are obtained as follows.

$$\dot{\psi}_{\alpha r} = (L_m / \tau_r)\, i_{\alpha s} - p\,\omega_r \psi_{\beta r} - \psi_{\alpha r} / \tau_r \qquad (9)$$

$$\dot{\psi}_{\beta r} = (L_m / \tau_r)\, i_{\beta s} + p\,\omega_r \psi_{\alpha r} - \psi_{\beta r} / \tau_r \qquad (10)$$

The electrical angle θ_e of the rotor flux vector ($\overline{\psi}_r$) with the α-axis of the stationary reference frame is given by:

$$\theta_e = \tan^{-1}(\psi_{\beta r} / \psi_{\alpha r}) \qquad (11)$$

The time derivative of this flux angle is the instantaneous electrical synchronous speed.

$$\omega_e = \dot{\theta}_e = (\psi_{\alpha r}\dot{\psi}_{\beta r} - \psi_{\beta r}\dot{\psi}_{\alpha r}) / \psi_r^2 \qquad (12)$$

Substituting (9) and (10) in (12), we get

$$\omega_e = p\,\omega_r + (\psi_{\alpha r} i_{\beta s} - \psi_{\beta r} i_{\alpha s}) L_m / (\tau_r \psi_r^2) \quad (13)$$

Estimated speed is obtained from (12) and (13) as [2]:

$$\hat{\omega}_r = [\psi_{\alpha r}\dot{\psi}_{\beta r} - \psi_{\beta r}\dot{\psi}_{\alpha r} - (\psi_{\alpha r} i_{\beta s} - \psi_{\beta r} i_{\alpha s})(L_m / \tau_r)]/(p\,\psi_r^2) \qquad (14)$$

Using measured stator voltages and currents, and the rotor flux observer based on (7)-(8), rotor speed is estimated by (14). This method of speed estimation is highly machine parameter sensitive and tends to give poor accuracy of estimation.

IV. Sliding Mode Control

The control problem is to get the motor speed ω_r to track a specific time varying command ω_r^* in the presence of model imprecision, load torque disturbances and measurement noise. In sliding mode control, the system is controlled in such a way that the tracking error, $e = \omega_r - \omega_r^*$ and its rate of change $\dot{e}$ always move towards a sliding surface. The sliding surface is defined in the state space by the scalar equation

$$s\,(e,\ \dot{e},\ t) = 0$$

where, the sliding variable, s is:

$$s = \dot{e} + \lambda\, e \qquad (15)$$

where, λ is a positive constant that depends on the bandwidth of the system.

The problem of tracking is equivalent to remaining on the sliding surface for all time, and the sliding variable, s is kept at zero. For a second order system the switching surface is a line. Control input is applied to drive the system state onto the switching line, and once on it, the system is constrained to remain on the line. The distance of the error trajectory from the sliding surface and its rate of convergence are used to decide the control input. The sign of the control input must change at the intersection of tracking error trajectory with the sliding surface. In this way, the error trajectory is forced to move always towards the sliding surface. Once it reaches the sliding surface, the system is constrained to slide along this surface to the equilibrium point. The condition of sliding mode [5] is:

$$\frac{1}{2}\frac{d}{dt}s^2 = s\dot{s} \le -\eta|s| \quad (16)$$

where, η is a positive constant.

This eqn. (16) is stricter than the general sliding condition: $s\dot{s} \le 0$. Equation (16) is equivalent to

$$\dot{s} \cdot \mathrm{sgn}(s) \le -\eta \quad (17)$$

To design a sliding mode speed controller for the field oriented induction motor drive system described in section-II, the steps are as follows. Substituting (15) involving speed error e in (17):

$$(\ddot{\omega}_r + \lambda\dot{e} - \ddot{\omega}_r^*)\,\mathrm{sgn}(s) \le -\eta \quad (18)$$

The speed dynamics is given by:

$$J\dot{\omega}_r + B\omega_r + T_L = T_e = K_T\psi_{dr}i_{qs} \quad (19)$$

Equivalently, (19) can be expressed as:

$$\dot{\omega}_r = g_1 - (T_L/J) \quad (20)$$

where,

$$g_1 = (-B\,\omega_r + K_T\,\psi_{dr}\,i_{qs})/J$$

is a function.

Differentiating (20) with respect to time and simplifying:

$$\ddot{\omega}_r = G + d + b\,v_{qs} \quad (21)$$

where,

$$G = (-B\,g_1 + K_T\,\psi_{dr}^*\,g_2)/J \quad (22)$$

$$g_2 = -(a_1 + \tau_r^{-1})\,i_{qs} - p\,\omega_r(1 + a_3L_m)\,i_{ds}$$

$$b = K_T\psi_{dr}^*/(\sigma L_s J)$$

G is a function, which can be estimated from measured values of currents and speed and d is the disturbance due to the load torque, and error in estimation of G, which may occur due to measurement inaccuracies. Third term is the control effort, as the q-axis stator voltage command is responsible for changing torque. In the most basic sliding mode controller, no measurement or estimation is done. It does not take G and d into consideration. It is defined as:

$$v_{qs}^* = -K \cdot \mathrm{sgn}(s) \quad (23)$$

where, K is a positive constant, which is the gain of the sliding mode controller. But this controller gives unacceptable performance due to high control activity, resulting in chattering of control variable and system states. To reduce chattering a boundary layer of width ϕ is introduced on both sides of the switching line. This amounts to reduction of the control gain inside the boundary layer and results in a smooth control signal. Then the control law of (23) modifies to [5]:

$$v_{qs}^* = -K \cdot \mathrm{sat}(s/\phi) \quad (24)$$

Where, $$\mathrm{sat}(s/\phi) = \begin{cases} s/\phi & \text{if } |s| \le \phi \\ \mathrm{sgn}(s) & \text{if } |s| > \phi \end{cases}$$

The controller defined by (24) is referred to as the single component sliding mode controller. A suitable value of K ensures sliding to occur. It is usually the maximum value of the control effort possible. But this may be unnecessarily wasteful in terms of control strategy. To improve the performance, Slotine proposed the dual component sliding mode controller, which is obtained as follows.

Substituting (21) in (18) and simplifying:

$$(G + d + \lambda\dot{e} - \ddot{\omega}_r^*)\,\mathrm{sgn}(s) + b\,v_{qs} \cdot \mathrm{sgn}(s) \le -\eta \quad (25)$$

From (23) and (25) dual component sliding mode controller without boundary layer is obtained as:

$$v_{qs}^* = \{(-\hat{G} - \lambda\dot{e} + \ddot{\omega}_r^*)/\hat{b}\} - K \cdot \mathrm{sgn}(s) \quad (26)$$

The first term in (26), $(-\hat{G} - \lambda\dot{e} + \ddot{\omega}_r^*)$ is a compensation term and the second term is the controller. The compensation term is continuous and reflects knowledge of the system dynamics. The controller term is discontinuous and ensures the sliding to occur. Though dual component sliding mode controller given by (26) has better performance than single component sliding mode controller given by (24), some amount of chattering is still present. To reduce this chattering, dual component sliding mode controller with boundary layer as given by (27) is used.

$$v_{qs}^* = \{(-\hat{G} - \lambda\dot{e} + \ddot{\omega}_r^*)/\hat{b}\} - K \cdot \mathrm{sat}(s/\phi) \quad (27)$$

The estimation error on G is assumed to be bounded by some known function ΔG_{max}, where

$$|G - \hat{G}| \le \Delta G_{max} \quad (28)$$

The voltage gain parameter b is unknown but has known bounds.

$$0 < b_{min} \le b \le b_{max} \quad (29)$$

The estimate $\hat{b}$ of gain b is the geometric mean of the above bounds.

$$\hat{b} = (b_{min}b_{max})^{1/2} \text{ and } \beta = (b_{max}/b_{min})^{1/2} \quad (30)$$

From (18) and (26), the controller gain K is derived as [5]:

$$K \ge [\beta(\Delta G_{max} + \eta) + (\beta - 1)(-\hat{G} - \lambda\dot{e} + \ddot{\omega}_r^*)]/\hat{b} \quad (31)$$

The controller gain, K is determined using (31) and considering various conditions such as: increase in stator and rotor resistance due to temperature rise, and change in load torque. The induction motor whose specifications and parameters are given in Table-I, is used for simulation experimentation. The controller gain is obtained as: K = 169.

V. Results And Discussions

The 3-phase induction motor drive system, whose rating and parameters are given in Table-I, is subjected to various tests with different controllers. The addition of a fly-wheel arrangement meant that the inertia could be doubled. So the sliding mode controller is tested for robustness with two different moment of inertia values:

Standard inertia, J_{min} = 0.0088 Kg $\cdot m^2$, and

High inertia (with fly-wheel), J_{max} = 0.0176 Kg $\cdot m^2$.

Both types of sliding mode controllers: single component and dual component are subjected to trajectory tracking test and regulator performance test. Results are compared among both types of controllers, for standard inertia and high inertia load to demonstrate the robustness. These results are also compared with those obtained using fixed gain P-I controller. The fixed gain speed control comprised of nested speed and torque control loops with Proportional-Integral (P-I) controllers. The design procedure of P-I controllers is described in [9]. The constants of the fixed gain controller were chosen to give the best transient response. The gains were: speed loop KPw=0.261, KIw=1.98, torque loop KPT=100, KIT=29877.

TABLE – I
RATING AND PARAMETERS OF THE INDUCTION MOTOR

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: $R_s = 6.37\ \Omega$, $R_r = 4.3\ \Omega$
Stator and rotor self inductances: $L_s = L_r = 0.26$ H
Mutual inductance between stator and rotor: $L_m = 0.24$ H
Moment of Inertia of motor and load: $J = 0.0088$ Kg $\cdot$m^2
Viscous friction coefficient: $B = 0.003$ N $\cdot$m $\cdot$s/rad

A. *Trajectory Tracking Performance*

In the case of trajectory tracking using sliding mode control, if the error trajectory moves away from the origin of the phase plane, it returns by being driven onto the switching line and then sliding along it. Within the boundary layer the error trajectory is not driven onto the switching line. Error trajectories having entered the boundary layer are constrained to move within it. The particular form of the periodic reference speed trajectory used for experiments is shown in Fig. 1. The command speed is increased linearly from 0 at t=0.2s to 147rad/s at t=0.5s. It is kept constant at 147rad/s till t=1.5s, and decreased linearly to −147rad/s at t=2.1s. Then command speed is kept constant at −147rad/s till 3.1s, and increased linearly to 0 at t=3.4s. Load torque of 5N·m (rated torque) is applied from t=0.75s to t=1.25s and −5N·m is applied from t=2.35s to t=2.85s. The same trajectory is used for (*i*) single component sliding mode controller, (*ii*) dual component sliding mode controller, and (*iii*) fixed gain P-I controller, and results are compared. The trajectory following results of the two types of sliding mode controller are presented for the two different moment of inertia, i.e., with standard inertia (J_{min}), and with high inertia load (J_{max}). With the single component sliding mode controller, the speed error responses are shown in Fig. 2 for both values of inertia. Fig. 3 shows the control output, which is the reference q-axis stator voltage with standard inertia load. With the dual component sliding mode controller, the speed error responses are shown in Fig. 4 for both values of inertia. This result demonstrates the robustness of the drive system with dual component sliding mode controller. However single component sliding mode controller is not as robust as dual component sliding mode controller. With high inertia load, single component sliding mode controller gives more error while accelerating compared to that with standard inertia. But the error is less during load torque changes with high inertia compared to that with standard inertia. More over the tracking error is less with dual component sliding mode controller compared to those with single component sliding mode controller. Thus dual component sliding mode controller is better than single component sliding mode controller. Fig. 5 shows the variation of switching variable, s for both single and dual component sliding mode implementation. Speed error and control output for fixed gain P-I controllers with standard inertia load are shown in Fig. 6. Though fixed gain controller gives satisfactory performance for step changes (usually small) in speed command, it is not suitable for tracking fast changing trajectory as taken in this report. When the machine inductances vary due to variation in core flux, torque constant K_T will vary. If the torque constant gets doubled from the nominal value, the speed error response for trajectory tracking is shown in Fig. 7. Thus, sliding mode controller gives superior performance compared to fixed gain controller for tracking fast changing trajectory. It also gives robust control against parameter variations.

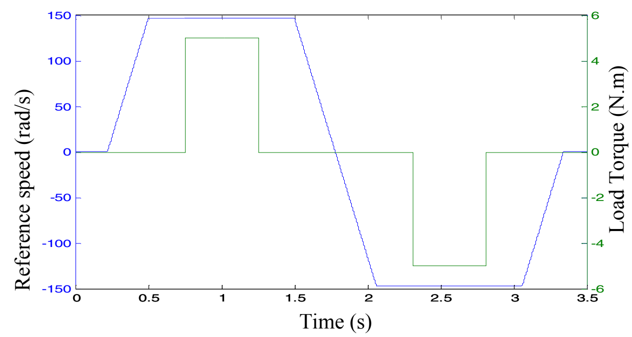


Fig. 1 Reference speed and load torque trajectories

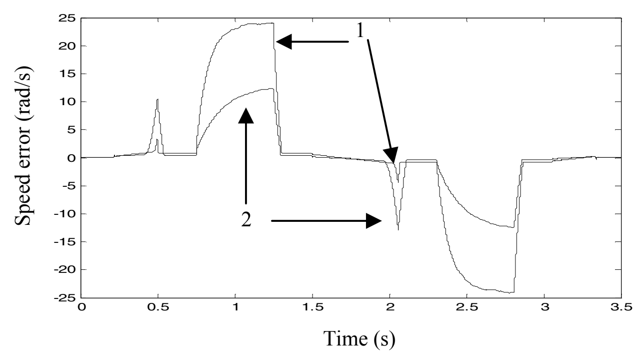


Fig. 2 Speed error for single component sliding mode controller with 1.standard inertia, 2. high inertia load

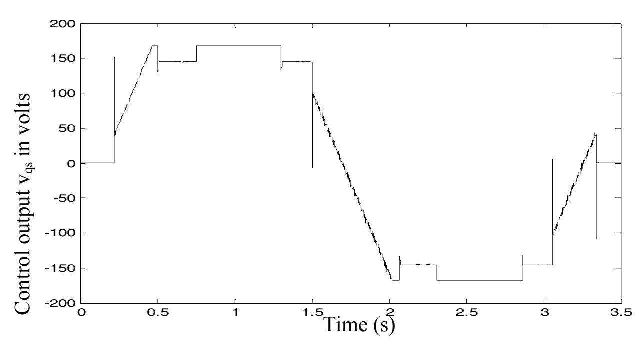


Fig. 3 Control output, i.e., reference q-axis stator voltage for single component sliding mode controller with standard inertia load

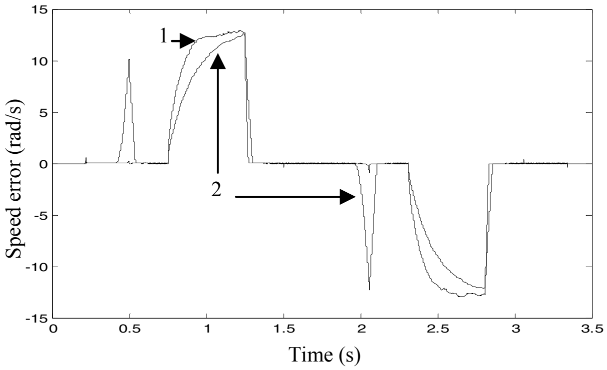


Fig. 4 Speed error for dual component sliding mode controller with 1. standard inertia, 2. high inertia load

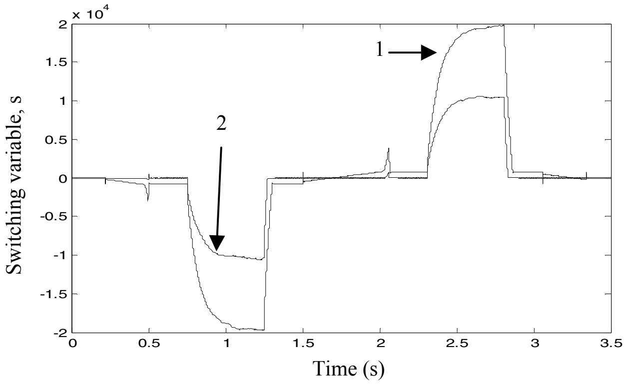
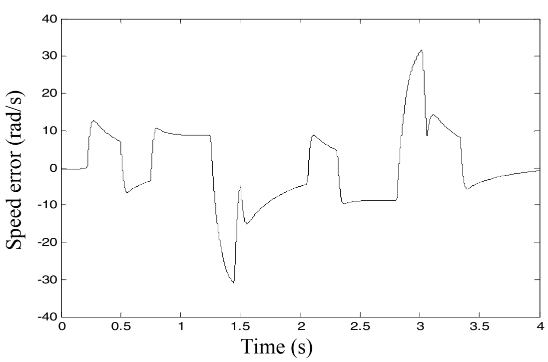
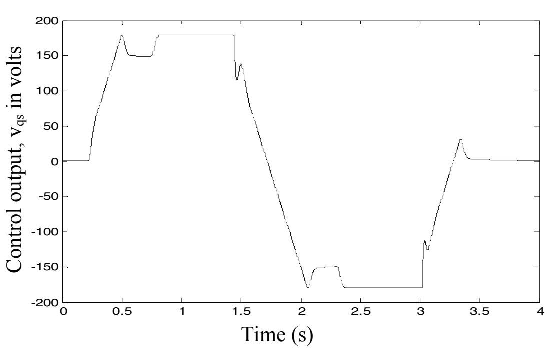


Fig. 5 Variation of switching variable, s for 1.single and 2. dual component sliding mode implementation



(a)



(b)

Fig. 6 (a) Speed error, and (b) Control output, i.e., reference q-axis stator voltage, for fixed gain P-I controllers with standard inertia load

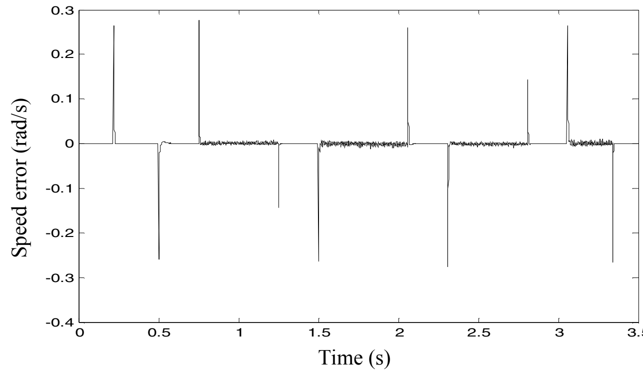
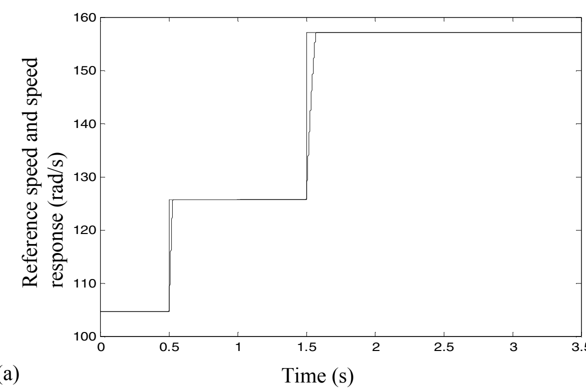


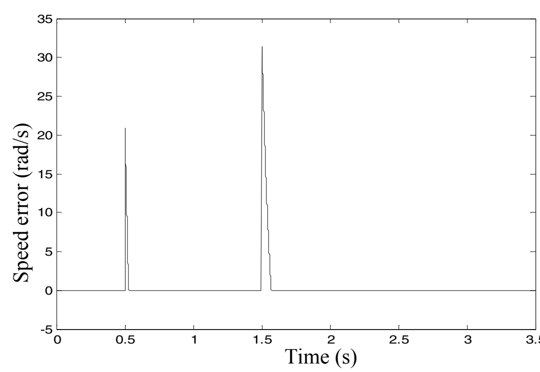
Fig. 7 Speed error for dual component sliding mode controller with high inertia load with K_T is twice the nominal value

B. Regulator Performance

The drive system with dual component sliding mode controller is subjected to step increases in speed command from 105 rad/s to 126 rad/s and then to 157 rad/s. Speed error response is shown in Fig. 8. The speed error response spike is below 30 rad/s. This shows sliding mode controller is also suitable for regulator application.



(a)



(b)

Fig. 8 (a) Step changes in reference speed and speed response, (b) speed error with dual component SMC

C. Sensorless Control

Sensorless control of the drive is implemented with speed estimation algorithm presented in section-III. The estimated speed is used in the dual component sliding mode controller. For step changes in speed command from 105rad/s to 126 rad/s, and then to 157rad/s the response of speed, estimated speed and error in speed estimation are shown in Fig. 9. In the speed estimation scheme, the same sampling time is used for both current and speed. Current contains ripples due to very fast sampling. So, the estimated speed is corrupted by the current ripples.

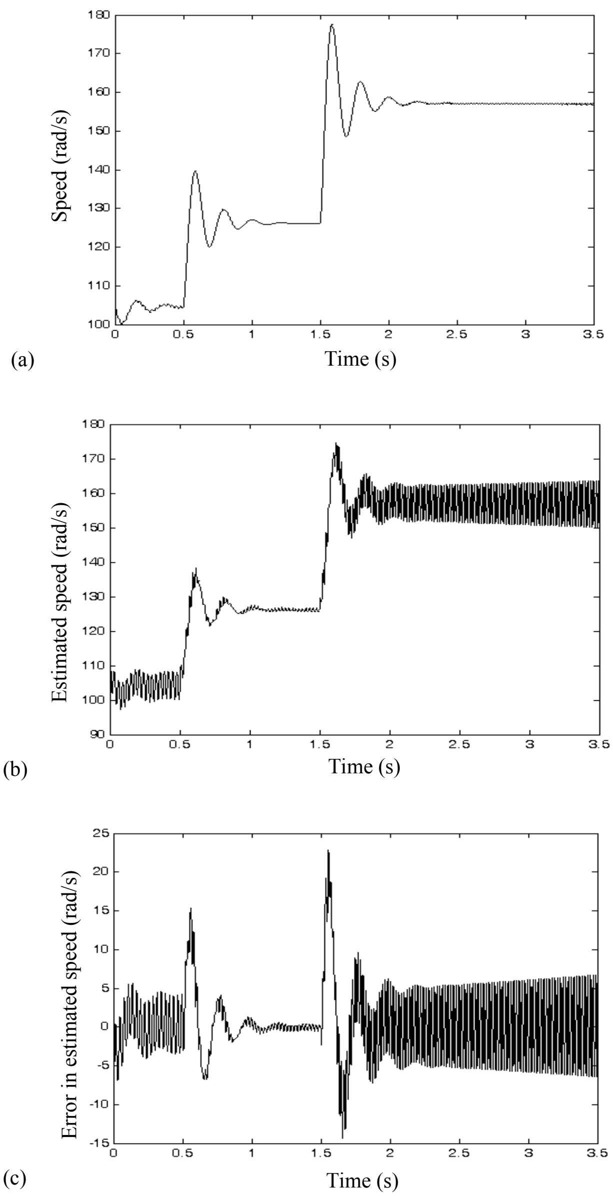


Fig. 9 (a) Speed response, (b) Estimated speed (c) Error in speed estimation, for step changes in reference speed with sensorless control

VI. CONCLUSIONS

Simulation results illustrate that the error produced by the fixed gain controller is larger than either of the sliding mode controller implementations. The dual component sliding mode controller with compensation term is superior to the single component sliding mode controller in terms of tracking performance. During accelerating period, tracking error is large with high inertia load, compared to that with standard inertia, for both types of sliding mode controllers. During change of load torque, with dual component sliding mode controller, speed error is nearly the same for both standard inertia and high inertia, where as with single component sliding mode controller, speed error for standard inertia is more than that with high inertia. Robustness with sliding mode controller is demonstrated through simulation. The regulator performance with sliding mode controller is also good. Sensorless sliding mode control scheme works very well with the present speed estimation algorithm. Only the estimated speed contains ripples, due to mismatch of current and speed sampling times. But the speed response of drive system, with estimated speed being used in the control scheme, is satisfactory.

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