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SIMILARITY SOLUTIONS FOR LAMINAR FREE CONVECTION FLOW OF A THERMOMICROPOLAR FLUID PAST A NON-ISOTHERMAL VERTICAL FLAT PLATE

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Abstract—Laminar free convection boundary layer flow of a thermomicropolar fluid past a non-isothermal vertical flat plate has been studied in detail. It has been established that the flow problem has similarity solutions when the variation *in the temperature of the plate is a linear function of the distance from the leading edge measured along the plate.* The resulting system of the nonlinear ordinary differential equations has been solved numerically by "Shooting Method" for various values of the material parameters. The effects of these parameters has been studied on the velocity and microrotation fields graphically. Also "Tables" have been given for the values of temperature, skin-friction parameter, microrotation gradient on the wall and Nusselt number. Two types of boundary conditions are prescribed for the microrotation on the wall.

1. INTRODUCTION

FREE CONVECTION has been of considerable interest to engineers and scientists because of its various applications in heat transfer. Representative field of interest in which combined heat and mass transfer—under conditions of free convection—are important include: design of chemical processing equipment, formation and dispersion of fog, distributions of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing and pollution of the environment.

The free convection problem of a non-isothermal vertical plate under boundary layer approximation for Newtonian fluids has been extensively studied by several authors[1]. Mathur[2] studied the free convection flow of an elastico-viscous fluid past a nonuniformly heated vertical plate. A detailed account of the study of this problem for Newtonian and non-Newtonian fluids has also been given in[2].

In the present paper, we have studied the similarity solutions for the laminar free convection flow of a thermomicropolar fluid past a non-isothermal vertical flat plate. The theory of thermomicropolar fluids has been developed by Eringen[3] by extending the theory of micropolar fluids[4]. This theory deals with viscous fluids in which the microconstituents are rigid and spherical or randomly oriented. Polymeric fluids, liquid crystals, fluid suspensions, animal blood, etc. can be characterized by this fluid model. On the basis of this theory, the experimentally observed phenomenon of drag reduction[5, 6] in the flow past a rigid body of fluids containing minute amount of polymeric additives can be explained satisfactorily. Very recently, Riha[7] has applied this theory for the adequate representation of fluid suspensions of rigid particles in a Newtonian fluid.

Balram and Sastry[8] have studied free convection flow of micropolar fluids in a parallel plate vertical channel. Sastry and Maiti[9] have obtained numerical solutions of combined convective heat transfer of micropolar fluids in an annulus of two vertical pipes. In[8, 9], it has been found that the boundaries are cooled and buoyancy force influences the flow of micropolar fluids to a considerable extent. In Section 2 of this paper, we have given the formulation of the problem of the laminar boundary layer free convection flow of a thermomicropolar fluid past a non-isothermal vertical flat plate. Section 3 deals with the possibility of the existence of similarity solutions for this flow problem. It has been established that similarity solutions are possible only when the *variation in the temperature of the plate is a linear function of the distance from the leading edge measured along the plate.* Under this thermal boundary condition, the governing system of partial differential equations is transformed into a system of non-linear ordinary differential equations. Using Shooting Method, this system of nonlinear ordinary differential equations has been solved numerically for some prescribed values of the

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various material parameters involved in the problem. The numerical solutions have been obtained by the use of DEC-10 Computer.

Finally in Section 4, we have presented the results of the present investigation.

2. FORMULATION OF THE PROBLEM

We choose a 2-dimensional Cartesian co-ordinate system (x, y) in which "x" is measured along the vertical flat plate and "y" is normal to the plate. The equations governing the steady laminar boundary layer flow of an incompressible thermomicropolar fluid[3] in this co-ordinate system are

Continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0. \tag{2.1}$$

Momentum

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=(\mu_v+k_v)\frac{\partial^2 u}{\partial y^2}+k_v\frac{\partial \nu}{\partial y}+\rho g\beta(T-T_\infty). \tag{2.2}$$

Moment of momentum

$$\rho j\left(u\frac{\partial \nu}{\partial x}+v\frac{\partial \nu}{\partial y}\right)=\gamma_v\frac{\partial^2 \nu}{\partial y^2}-k_v\left(\frac{\partial u}{\partial y}+2\nu\right). \tag{2.3}$$

Energy

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K_c\frac{\partial^2 T}{\partial y^2}+\alpha^*\left(\frac{\partial T}{\partial x}\frac{\partial \nu}{\partial y}-\frac{\partial T}{\partial y}\frac{\partial \nu}{\partial x}\right). \tag{2.4}$$

Where

- u, v components of velocity along and normal to the vertical flat plate
- ν component of microrotation whose direction of rotation is in the xy-plane
- ρ, T density and temperature of the fluid
- μ_v, k_v, γ_v viscosity, vortex viscosity and spin-gradient viscosity
- j, K_c, α^* micro-inertia density, thermal conductivity and micropolar heat conduction coefficient
- g, β, C_p acceleration due to gravity, coefficient of expansion and specific heat of the fluid at constant pressure.

There are two more material parameters β_v (the gradient viscosity) and β^* (micropolar heat conduction) which will appear in the expressions for the couple stress components and the rate of heat transfer. Eringen[3] has given the inequalities to be satisfied by the various material parameters. These inequalities, which arise from the thermodynamic restrictions, are

$$\begin{gathered} k_v\geq 0,\ 2\mu_v+k_v\geq 0,\ \beta_v+\gamma_v\geq 0,\ K_c\geq 0, \\ (\alpha^*-\beta^*T^{-1})^2\leq 2K_cT^{-1}(\gamma_v-\beta_v). \end{gathered} \tag{2.5a}$$

In addition, we must have

$$j\geq 0. \tag{2.5b}$$

Wall boundary conditions

Velocity field

$$u(x,0)=v(x,0)=0. \tag{2.6}$$

Microrotation field

We assume the following two types of boundary conditions for microrotation

$$\nu(x,0) = 0 \text{ (no spin boundary condition)}, \tag{2.7a}$$

$$\nu(x,0) = -\frac{1}{2}\left(\frac{\partial u}{\partial y}\right)_{y=0} \tag{2.7b}$$

(Antisymmetric part of stress vanishes at the wall).

Temperature field

$$T(x,0) = T_w(x) \tag{2.8}$$

(variable temperature of the wall).

At the boundary layer, we must have

$$y \to \infty: u \to 0, \nu \to 0, T \to T_\infty \tag{2.9}$$

where T_∞ is the constant temperature of the fluid outside the boundary layer.

The details of the derivation of the boundary layer eqns (2.1)–(2.4) are available in [10, 11].

In the energy eqn (2.4), the viscous dissipation terms have been neglected. This is indeed a permissible simplification in this flow problem since the velocities usually encountered in natural convection are rather small. It has also been recently shown by Mathur *et al.* [11] that viscous dissipation has very little effect on the temperature field and the rate of heat transfer for the flow of an incompressible thermomicropolar fluid past a circular cylinder placed in such a way that its axis is normal to the oncoming free stream.

The boundary conditions (2.7a) and (2.7b) correspond, respectively, to the strong and weak concentration of microelements near the boundary.

3. METHODS OF SOLUTION

The continuity eqn (2.1) is identically satisfied by introducing the stream function $\Psi(x, y)$ such that

$$u = \frac{\partial \Psi}{\partial y} \text{ and } v = -\frac{\partial \psi}{\partial x}. \tag{3.1}$$

Now to explore the possibility for the existence of similarity, we assume

$$\psi = Ax^aF(\eta), \ \eta = Byx^b,$$

$$\nu = Cx^cG(\eta), \ T_w - T_\infty = Nx^n, \tag{3.2}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}$$

where A, B, C and N, a, b, c and n are constants.

Substituting from (3.1) and (3.2) in the eqns (2.1)–(2.4), we obtain

$$A^2B^2x^{2a+2b-1}[(a+b)F'^2 - aFF''] = \frac{1}{\rho}(\mu_v + k_v)AB^3x^{a+3b}F''' + \left(\frac{k_v}{\rho}\right)BCx^{b+c}G' + g\beta Nx^n\theta, \tag{3.3}$$

$$ABCx^{a+b+c-1}(cF'G - aFG') = \left(\frac{\gamma_v}{\rho j}\right)B^2Cx^{2b+c}G'' - \left(\frac{k_v}{\rho j}\right)(2Cx^cG + AB^2x^{a+2b}F''), \tag{3.4}$$

$$AB x^{a+b+n-1}(n\theta F' - aF\theta') = \left(\frac{K_c}{\rho C_p}\right) B^2 x^{2b+n}\theta'' + \left(\frac{\alpha^*}{\rho C_p}\right) BC x^{b+c+n-1}(n\theta G' - c\theta' G), \tag{3.5}$$

where a prime denotes differentiation with respect to η. For similarity to exist, the eqns (3.3)–(3.5) must hold for all values of x. This is only possible when

$$\begin{aligned} 2a+2b-1 &= a+3b = b+c = n \\ a+b+c-1 &= 2b+c = c = a+2b, \\ a+b+n-1 &= 2b+n = b+c+n-1. \end{aligned} \tag{3.6}$$

The solution of the system of algebraic eqns (3.6) is

$$b = 0, a = c = n = 1. \tag{3.7}$$

Thus, similarity exists for this flow problem.

Making use of (3.7), we get

$$\begin{aligned} &\psi = AxF(\eta), \nu = CxG(\eta), \eta = By, \\ &T_w - T_\infty = Nx, T = T_\infty + Nx\theta(\eta), \\ &u = \frac{\partial\psi}{\partial y} = ABxF'(\eta), v = -\frac{\partial\psi}{\partial x} = -AF(\eta). \end{aligned} \tag{3.8}$$

From eqns (3.8), it is evident that the constants A, B, C and N have, respectively, the dimensions of velocity, the reciprocal of length, the reciprocal of the product of length and time, and of the ratio (temperature/length).

Making use of dimensional analysis, we obtain

$$\begin{aligned} &A = [\bar{\alpha}^2 Ng\beta]^{1/4}, B = [(Ng\beta)/\bar{\alpha}^2]^{1/4} \\ &C = [(Ng\beta)^3/\bar{\alpha}^2]^{1/4}, \bar{\alpha} = \frac{K_c}{\rho C_p}, Pr = \frac{\mu_v C_p}{K_c} \\ &N_1 = \frac{k_v}{\mu_v}, N_2 = \frac{\rho j (Ng\beta)^{1/2}}{\mu_v}, N_3 = \frac{\gamma_v \rho (Ng\beta)^{1/2}}{\mu_v^2}, \\ &N_4 = \frac{\rho\beta_v (Ng\beta)^{1/2}}{\mu_v^2}, N_5 = \frac{\alpha^*(Ng\beta)^{1/2}}{K_c}, N_6 = \frac{\beta^*(Ng\beta)^{1/2}}{\mu_v C_p T_\infty} \end{aligned} \tag{3.9}$$

where Pr is the Prandtl number and $\bar{\alpha}$ is the thermal diffusivity.

In view of (3.7)–(3.9), the eqns (3.3)–(3.5) and the boundary conditions (2.6)–(2.9) reduce to the following equations

$$F'^2 - FF'' = Pr(1+N_1)F''' + PrN_1 G' + \theta, \tag{3.10}$$

$$N_2(F'G - FG') = PrN_3 G'' - N_1(2G + F''), \tag{3.11}$$

$$F'\theta - F\theta' = \theta'' + N_5(\theta G' - \theta' G), \tag{3.12}$$

$$\left.\begin{array}{l}\eta=0;\ F=F'=0\ \text{(a)}\ G=0\\ \qquad\qquad\qquad\text{or}\\ \qquad\qquad\text{(b)}\ G=-\dfrac{1}{2}F''\end{array}\right\},\ \theta=1,$$

$$\eta\to\infty;\ F'\to 0,\ G\to 0,\ \theta\to 0. \tag{3.13}$$

In the eqns (3.10)–(3.13), the dimensionless parameters N_1, N_2, N_3 and N_5, respectively, characterize the vortex viscosity, microinertia density, spin-gradient viscosity and the micropolar heat conduction. The parameters N_4 and N_6 will appear in the expressions for couple stress components and the rate of heat transfer. In terms of these parameters, the inequalities (2.5) become

$$\begin{gathered}N_1\geq 0,\ N_1+2\geq 0,\ N_3+N_4\geq 0,\ Pr\geq 0,\\ (N_5-PrN_6\Phi^{-1})^2\leq 2\Phi^{-1}(N_3-N_4)\frac{PrE}{R}\end{gathered} \tag{3.14a}$$

where $\Phi=(T/T_\infty)$ (dimensionless temperature), $E=A^2/C_pT_\infty$ (like Eckert number) and $R=\rho A/B\mu_v$ (like Reynolds number). Further, we must have

$$N_2\geq 0. \tag{3.14b}$$

N_1, N_2, N_3, N_4, N_5 and N_6 must be chosen in such a way that the inequalities (3.14) are satisfied.

Numerical solutions of the eqns (3.10)–(3.12) together with the boundary conditions (3.13) have been obtained by Shooting Method employing Taylor series at an interval $\Delta\eta=0.05$ for the following values of the parameters

$$N_1=0.1,\ 0.25;\ N_2=0.002,\ N_3=0.017,\ 0.02,\ Pr=9.0,\ N_5=1.$$

These values satisfy the restrictions given by the inequalities in eqn (3.14).

Ahmadi[12] and Tözeren and Skalak[13] have stated that the parameter N_1 depends on the shape and concentration of the microelements. For a given shape of the microelements, N_1 directly gives a measure of concentration of the microelements. The parameters N_2 and N_3 can be thought of fluid properties depending on the relative size of microstructure in relation to a geometrical length.

Skin friction and wall couple stress

The skin-friction coefficient C_f is defined by

$$C_f=\frac{(t_{yx})_{y=0}}{\rho A^2}$$

(A = characteristic velocity).

In terms of the non-dimensional quantities, we have

$$C_f=Pr[(1+N_1)F''(0)+N_1G(0)]\bar{x}$$

where $\bar{x}=Bx$.

The dimensionless couple stress on the wall is given by

$$M_w=\frac{B}{\rho A^2}(m_{yz})_{y=0}=Pr[PrN_3\bar{x}G'(0)+(R/E)N_5\theta(0)].$$

Heat transfer coefficient

The non-dimensional heat transfer coefficient called Nusselt number $N(\bar{x})$ is defined as

$$N(\bar{x}) = \left(\frac{q_L}{K_c T_\infty}\right) = \left(\frac{q}{BK_c T_\infty}\right)$$

where

$$q = \left(K_c \frac{\partial T}{\partial y} + \beta^* \frac{\partial \nu}{\partial x}\right)_{y=0}.$$

In terms of non-dimensional variables, we have

$$N^*(\bar{x}) = \frac{N(\bar{x})}{\dfrac{N}{BT_\infty}} = \frac{N(\bar{x})BT_\infty}{N} = \bar{x}\theta'(0) + \Lambda N_6 Pr G(0),$$

where

$$\Lambda = \frac{BT_\infty}{N}$$

(Dimensionless ratio of free stream temperature to characteristic wall temperature).

4. RESULTS AND DISCUSSION

Velocity field

In Fig. 1, we have plotted the velocity profiles $F'(\eta)$. It is seen that increase in N_1 results in the decrease of $F'(\eta)$ for both types of boundary conditions on microrotation. This means that when the concentration of microelements near the boundary increases, the fluid velocity decreases. We also note that the velocity decreases with increasing N_3 irrespective of the boundary condition on microrotation. Further, we see that the fluid velocity is more in the case of antisymmetric part of the stress vanishing on the plate as compared to the no relative spin on the plate. This happens because the vanishing of antisymmetric part of the stress on the boundary corresponds to weak concentration of microelements while no relative spin on the boundary indicates strong concentration of microelements near the plate.

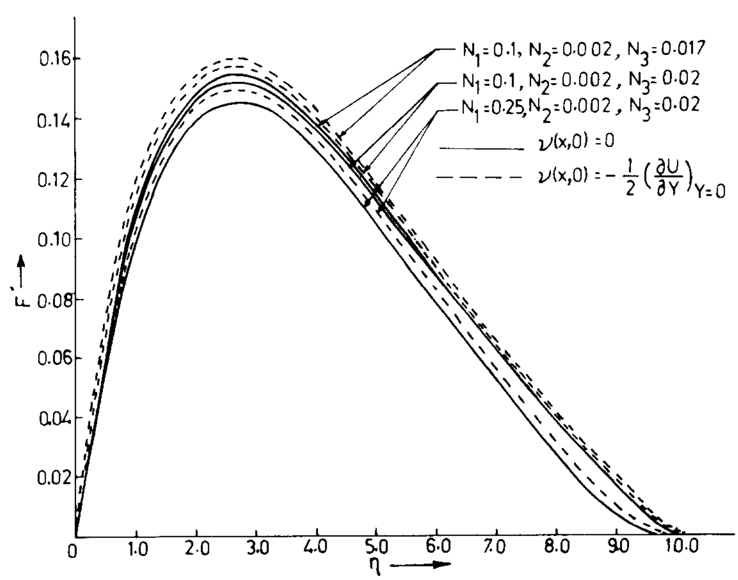

Fig. 1. Velocity distribution showing the effect of the variation of micropolar fluid parameters with different types of boundary conditions on microrotation.

Microrotation field

Figure 2 shows the effect of variation of N_1 and N_3 on the microrotation profiles for two types of boundary conditions on the microrotation. The nature of the microrotation profiles for the no spin boundary condition is the same as obtained in[10] for the stagnation point flow of a micropolar fluid. The condition of vanishing of the antisymmetric part of the stress on the boundary results in a drastic change of the microrotation profiles.

Temperature field

We have recorded in Table 1, the values of the dimensionless temperature for different values of the similarity variable "η" and showing the effect of variation of the micropolar fluid parameters on the temperature field for different types of boundary conditions on microrotation.

It is observed that for no relative spin condition, the temperature increases with increasing N_1 and N_3. The variation with N_3 is insignificant. For the boundary condition of vanishing of antisymmetric part of the stress, the temperature increases with increasing N_1 while it decreases slightly with increasing N_3.

Table 2 shows the effect of variation of N_1 on the skin-friction parameter $F''(0)$, microrotation gradient and temperature gradient on the plate. We note that the skin-friction decreases with increasing N_1 while the temperature gradient increases. This is true irrespective of the boundary condition on microrotation. $F''(0)$ and $\theta'(0)$ have greater values for the boundary condition of vanishing of anti-symmetric part of the stress as compared to no spin boundary condition, which means that the skin-friction and the wall temperature gradient are more for weak concentration of microelements in comparison to strong concentration of microelements near the boundary.

With the known values of $F''(0)$, $G'(0)$ and $\theta'(0)$, C_f, M_w and $N(\bar{x})$ can be calculated for the prescribed values of N_1, N_3, N_5, N_6, Pr, R, E and Λ. In Table 3, we have given the values—$N^*(\bar{x})$, the dimensionless rate of heat transfer, showing the effect of variation of N_1 and N_6 on it. From Table 3, we observe that $-N^*(\bar{x})$ decreases with increasing N_1 while it increases with increasing N_6. Similar results have been obtained in[10, 11] and[14].

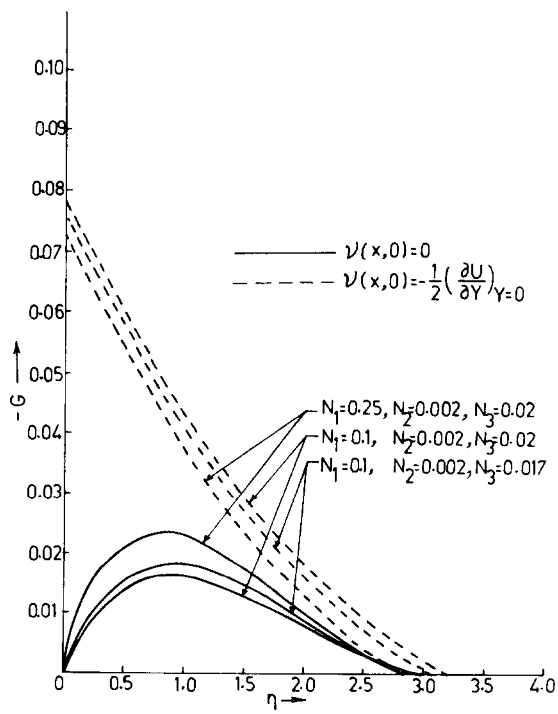

Fig. 2. Microrotation profiles showing the effect of the variation of micropolar fluid parameters with different types of boundary conditions on microrotation.

Table 1. Variation of temperature with N_1 and N_3 for different types of boundary conditions for $Pr = 9.0$ and $N_2 = 0.002$

η	θ: Boundary condition $\nu = 0$			Boundary condition $\nu = -\frac{1}{2}(\frac{\partial u}{\partial y})_{y=0}$		
	$N_1 = 0.1$, $N_3 = 0.02$	$N_1 = 0.25$, $N_3 = 0.02$	$N_1 = 0.1$, $N_3 = 0.017$	$N_1 = 0.1$, $N_3 = 0.02$	$N_1 = 0.25$, $N_3 = 0.02$	$N_1 = 0.1$, $N_3 = 0.017$
0.0	1.0	1.0	1.0	1.0	1.0	1.0
1.0	0.6435	0.6545	0.6433	0.6488	0.6572	0.6532
2.0	0.3747	0.3915	0.3747	0.3761	0.3872	0.3824
3.0	0.1992	0.2153	0.1992	0.1982	0.2882	0.2043
4.0	0.0978	0.1102	0.0978	0.0962	0.1037	0.1008
5.0	0.0448	0.0532	0.0447	0.0435	0.0484	0.0466
6.0	0.0193	0.0244	0.0193	0.0185	0.0214	0.0203
7.0	0.0078	0.0106	0.0078	0.0074	0.0089	0.0083
8.0	0.0029	0.0041	0.0029	0.0027	0.0034	0.0031
9.0	0.0008	0.0012	0.0008	0.0007	0.0010	0.0009
10.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Table 2. The effect of variation of N_1 on the skin-friction parameter, gradients of microrotation and temperature on the surface for $Pr = 9.0$, $N_2 = 0.002$ and $N_3 = 0.02$, $N_5 = 1$

N_1	$F''(0)$	$-G'(0)$	$-\theta'(0)$
	For the boundary condition: $\nu(x,0) = 0$.		
0.1	0.1493	0.0442	0.3881
0.25	0.1350	0.0781	0.3825
	For the boundary condition $\nu(x,0) = -\frac{1}{2}(\frac{\partial u}{\partial y})_{y=0}$.		
0.1	0.1558	-0.0365	0.3675
0.25	0.1480	-0.0389	0.3561

Table 3. The effect of variation of N_1 and N_6 on the rate of heat transfer, $-N^*(\bar{x})$, for $Pr = 9.0$, $N_2 = 0.002$, $N_3 = 0.02$, $N_5 = 1$, $\Lambda = 1$

$\bar{x}$	For the boundary condition $\nu(x,0) = 0$.		For the boundary condition $\nu(x,0) = -\frac{1}{2}(\frac{\partial u}{\partial y})_{y=0}$			
	$N_1 = 0.1$	$N_1 = 0.25$	$N_1 = 0.1$, $N_6 = 0.01$	$N_1 = 0.1$, $N_6 = 0.05$	$N_1 = 0.25$, $N_6 = 0.01$	$N_1 = 0.25$, $N_6 = 0.05$
0	0	0	0.0067	0.0335	0.0061	0.0305
1	0.3881	0.3825	0.3948	0.4216	0.3886	0.4130
2	0.7762	0.7650	0.7829	0.8097	0.7711	0.7955
3	1.1643	1.1475	1.1710	1.1978	1.1536	1.1780

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II. Preparation of Text and Figures

(1) Manuscripts should be typed on good quality white paper, measuring 27.5 × 40 cm (11 × 16 inches).

(2) It is imperative that a black typewriter ribbon be used; blue does not reproduce. Electrical type script is preferred; small and italic typefaces are unsuitable. Care should be taken to ensure a clean, clear impression of the letters. Avoid erasure marks, smudges, pencil or ink corrections and creases.

(3) The typing area of page 1 must be 19.5 × 28 cm ($7\frac{1}{4}$ × 11 inches); the typing area of all other pages must be 19.5 × 31 cm ($7\frac{1}{4}$ × 12 inches). Each page should be completely filled with typing and/or diagrams.

(4) The title should be all in CAPITAL LETTERS, except formulae, centered on the width of page 1, and beginning 5 cm (2 inches) below the top edge of the paper.

(5) Allow a 1.5 cm ($\frac{1}{2}$inch) space between the title and the name(s) of the Authors(s). Follow immediately below, and on a separate line, with the affiliation(s) of the Author(s).

(6) Allow a 2.5 cm (1 inch) space between the Author's affiliation and the Abstract. Type the word ABSTRACT in capitals, beginning at the left hand margin. Start the margin for the entire Abstract at the end of the word ABSTRACT. Then type the Abstract itself in lower case lettering and single spacing.

(7) Allow a 1.5 cm ($\frac{1}{2}$ inch) space between the Abstract and the first major heading. Major headings, e.g. INTRODUCTION, METHODS, RESULTS, DISCUSSION, REFERENCES, etc., should be typed in capitals and lower case letters, centered on the width of the page, and underlined. Subsidiary headings, if used, should begin at the left hand margin.

(8) Spacing between text lines: $1\frac{1}{2}$. (Use double spacing if $1\frac{1}{2}$ is not available).

(9) Tables should be typed as part of the text, but in such a way as to avoid confusion with the text. The word TABLE should be capitalized and centered with the Table number above the Table. The heading should have the first letter of all main words in capitals. Authors should use discretion to ensure that a single Table does not overlap onto the next page. All Tables should have headings.

(10) Any material that cannot be typed, such as symbols and formulae, should be inked carefully in black.

(11) Line diagrams should be supplied, preferably in the form of glossy prints, at least of the size in which they are intended to appear in the *Letters.* They should NOT be pasted in, but appropriate space for each Figure should be left above the descriptive caption. The Figure number and Author's name should be clearly indicated on the reverse side of each illustration. Care should be taken to ensure that the caption does not become confused with the text. The abbreviation FIG. should be capitalized and, with the Figure number, centered above the caption. The caption itself should be in single space typing. Allow 3 spaces between end of caption and text which follows. If the diagrams are larger than they are intended to appear in the *Letters,* they may be separately supplied, but sufficient space for their final versions must be allowed in the text, and captions must be provided in these locations.

(12) Half-tone pictures should be supplied in triplicate as glossy prints in the actual size (or slightly larger) in which they are to appear in the *Letters.* Handle captions as under (11).

(13) Do not type the page numbers, but number each sheet lightly near the bottom preferably with a blue pencil.

(14) Footnotes should be typed single spaced, 3 spaces below the text at the bottom of the appropriate page, and separated from the text by a short line. They should be wholly within the allowed typing space.

(15) References should be indicated in the text by consecutive numbers in brackets, thus, [1, 2], as part of the text and not raised above it. The full reference should be cited in a numbered list at the end of the text in single spacing. There should be double spacing between successive references. References should contain the names of all authors of any one paper together with their initials, the title of the journal (with generally accepted abbreviation, if possible), volume number, first page number and year, as illustrated below. References to books should contain the publisher's name and location.

1. ERINGEN, A. C., Nonlocal Polar Elastic Continua, *Int. J. Engng. Sci.*: **10**, 1, 1972.
2. LANDAU, L. D., and LIFSHITZ, E. M., *Electrodynamics of Continuous Media*, Oxford, Pergamon Press, p. 57, 1960.
3. NABARRO, F. R. N., *Theory of Crystal Dislocations*, London, Oxford University Press, p. 384, 1967.

(16) Use only standard symbols and abbreviations in the text and illustrations.

(17) Manuscripts, Figures, and Diagrams should NOT BE FOLDED.