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A comparative study on estimation techniques with applications to power signal frequency

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An extended least square (ELS) technique has been proposed in this paper for power system frequency estimation. The validation of the above technique has been made by comparing its performance with the existing techniques such as Kalman filter (KF) and least mean square (LMS) technique etc. It has been observed through a series of simulation studies on frequency estimation that the ELS technique exhibits better performance in comparison to both the LMS and KF methods of power system frequency estimation. In Kalman filter, the determination of covariance matrix is very crucial leading to delay in convergence. LMS algorithm becomes complicated with the incorporation of correlation matrix, which may affect the convergence. On the contrary extended least square algorithm seems to be very simple and attractive without the implementation of covariance and correlation matrix. The feasibility of the ELS algorithm for frequency estimation has been tested with a signal buried with noise. The above estimation technique can be applied in real-time implementation, which will be immensely helpful for the power system protection. A comparative study on performance of the KF, LMS and ELS techniques for power system estimation has been made and included in the paper.

Key words: extended least square (ELS) technique, Kalman filter, least mean square (LMS) technique, power system parameters

1. Introduction

In a complex power system the fast and accurate estimation of supply frequency, voltage and its variation in real-time is essential. Variations in system frequency from its nominal value indicate the occurrence of a corrective action for its restoration to its original value. In this context a large number of numerical methods are available for frequency estimation from the digitized samples of the system voltage. Conventional methods assume that the power system voltage waveform is purely sinusoidal and therefore the time between two zero crossings is an indication of system frequency. Discrete Fourier transforms, least error squares technique, recursive Newton-type algorithm, adaptive notch filters [7, 8, 9] etc. are some of the popular signal processing techniques

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used for frequency measurements of power system signals. A large number of numerical techniques and their practical implementations are found in the literature but these set of approaches suffer from inaccuracies due to the presence of noise and harmonics and other system changing conditions such as change in fault inception angle, change in fault resistance etc. with amplitude, phase and frequency of a signal buried with noise and harmonics. It may be noted that the extended least square technique has attracted widespread attention due to its fast and accurate estimations. In view of addressing the difficulties of the existing methods and to achieve fast and accurate estimation of nominal and off-nominal power system frequency, the extended least square technique [4, 5] has been suggested for power system frequency estimation problems.

2. Different algorithms for estimation of frequency

2.1. Kalman filter

Kalman filter [1, 2] is a stochastic state estimator for parameter estimation. From the discrete values of the three phase voltage signal of a power system, a complex voltage vector is formed using the well known $\alpha-\beta$ transformation. A non-linear state space formulation is obtained by Kalman filter approach and frequency is modeled as a state here. The estimation of the state vector yields the unknown power system frequency. The mathematical formulation and implementation of the Kalman filter is given by the set of equations.

$$\left.\begin{aligned} V_{an} &= V_m\cos(\omega n\Delta T+\varphi)+\varepsilon_{an} \\ V_{bn} &= V_m\cos(\omega n\Delta T+\varphi-\frac{2\pi}{3})+\varepsilon_{bn} \\ V_{cn} &= V_m\cos(\omega n\Delta T+\varphi+\frac{2\pi}{3})+\varepsilon_{cn} \end{aligned}\right\} \tag{1}$$

where V is the test signal, V_m is the amplitude of the signal, ω is the angular frequency, ε is the noise term, ΔT is the sampling interval, n is the sampling instant, φ is the phase of fundamental component. The complex form of signal derived from the three-phase voltages is obtained by transform as shown below

$$\begin{aligned} V_\alpha &= \sqrt{2/3}(V_a-0.5V_b-0.5V_c), \\ V_\beta &= \sqrt{2/3}(0.866V_b-0.866V_c). \end{aligned} \tag{2}$$

A complex voltage can be obtained from above as

$$V_n = V_{\alpha n}+jV_{\beta n}, \tag{3}$$

$$h = \begin{bmatrix} 0 & 1 \end{bmatrix}, \tag{4}$$

$$k = \hat{p}^{*T}(h\hat{p}h^{*T}+r)^{-1}, \tag{5}$$

where k is the Kalman gain, h is the observation vector, p is the covariance matrix, r is the variance of the signal. Thus the covariance matrix is related with Kalman gain with the following equation

$$\hat{p} = \hat{p} - kh\hat{p}. \tag{6}$$

The updated estimated state is related with previous state with the following relation

$$\hat{x} = \hat{x} + k(V - h\hat{x}), \tag{7}$$

$$f = \frac{1}{2\pi\Delta T}\left(\sin^{-1}\mathrm{Im}(\hat{x})\right), \tag{8}$$

where f is the estimated frequency of the signal.

2.2. Least mean square algorithm

To enhance the convergence characteristics of a power system signal, least mean square (LMS) algorithm is adopted where the formulated structure looks very simple and this algorithm is found accurate under various systems changing condition to estimate correct measure of frequency. The complex voltage signal as expressed in Kalman filter is given by

$$V_n = V_{\alpha n} + jV_{\beta n}. \tag{9}$$

The voltage can be modeled as

$$\begin{aligned} V_n &= Ae^{j(\omega n\Delta T+\varphi)} + \varepsilon_k, \\ \hat{V}_n &= V_{n-1}\, e^{j\omega n\Delta T}\,. \end{aligned} \tag{10}$$

This model is utilized in the proposed frequency estimation algorithm and the scheme that describes the estimation process. The error signal in this case is

$$e_n = V_n - \hat{V}_n \tag{11}$$

where V_n is the estimated value of voltage at the nth instant, and

$$\hat{V}_n = W_n\hat{V}_{n-1}$$

where $W_n = e^{j\hat{\omega}_n\Delta T}$ denotes the weight, $\hat{\omega}$ is the estimated angular frequency. The significance of the above model is that the input vector contains one element only and so also the weight vector. The complex LMS algorithm is applied to estimate the state. The algorithm minimizes the squared error recursively by altering the complex weight vector W_n at each sampling instant as

$$W_n = W_{n-1} + \mu_n e_n \hat{V}_n^* \tag{12}$$

where $*$ represents the complex conjugate of that value and μ is the convergence factor controlling the stability and rate of convergence of the algorithm. The step size μ_n is varied for better convergence of the LMS algorithm in the presence of noise. For complex states, the equations are modified as

$$\mu_{n+1} = \lambda\mu_n + \gamma p_n p_n^* \tag{13}$$

where p_n represents the autocorrelation of e_n and e_{n-1} and is computed as

$$p_n = \rho p_{n-1} + (1-\rho)e_n e_{n-1} \tag{14}$$

where ρ is an exponential weighting parameter and $0 < \rho < 1$. Parameters λ $(0 < \lambda < 1)$ and $\gamma > 0$ control the convergence time. Parameter μ_{n-1} is set to $\mu_{\max}$ or $\mu_{\min}$ when it falls below or above the lower and upper boundaries, respectively. These values are chosen on the base of signal statistics. At each sampling interval, the frequency is calculated as

$$f_n = \frac{1}{2\pi\Delta T}\sin^{-1}\left[\mathrm{Im}\left(W_n\right)\right] \tag{15}$$

where $\mathrm{Im}(\cdot)$ stands for the imaginary part of a quantity.

2.3. Extended least square algorithm

Let the signal buried with noise is represented by the following structure

$$z(n) = A_1\sin(\omega_0 n + \phi_1) + \mu(n). \tag{16}$$

Normally, signal to noise ratio in a power system can be taken from the range 30dB to 40dB. Thus for the purpose of estimation

$$z(n) = [\sin\omega_0 n \;\; \cos\omega_0 n]\,[\alpha \;\; \beta]^T \mu(n) \tag{17}$$

or in the standard form

$$z(n) = \phi(n)\theta + \mu(n). \tag{18}$$

Here $z(n)$ is the noisy measurement, $\phi(n)$ is the system structure matrix, θ is the vector of unknown parameter. The estimate for the required parameter is expressed as follows

$$\hat{\theta} = [\phi(n)\phi^T(n)]^{-1}\phi(n)z(n). \tag{19}$$

α, β are the parameters to be estimated and are given by

$$\alpha = A_1\cos\phi_1, \tag{20}$$

$$\beta = A_1\sin\phi_1. \tag{21}$$

After the estimation of θ (unknown parameter), amplitude A_1 and phase ϕ_1 can be calculated as

$$A_1 = \sqrt{\alpha^2 + \beta^2} \tag{22}$$

$$\phi_1 = \tan^{-1}\frac{\beta}{\alpha} \tag{23}$$

where $\alpha = \theta_{11}$ and $\beta = \theta_{21}$. Once the estimate of amplitude and phase is done, it is required to estimate the frequency. This can be evaluated from equation(16) as

$$f_0 = \frac{1}{2\pi n}\left[\sin^{-1}\left(\frac{z(n)-\mu(n)}{A_1}\right) - \phi\right]. \tag{24}$$

3. Simulation results

A synthetic signal of 1 p.u amplitude, 50 Hz frequency and 0.5 p.u phase angle is generated in MATLAB platform. Then the algorithms such as Kalman filter, LMS and ELS are implemented with the sampling interval of 1 millisecond. The three phase signal with 1 p.u amplitude in each phase is also generated in MATLAB platform. From the complex signal a two phase signal is generated by $\alpha - \beta$ transformation. The initial covariance matrix is taken as ρI where I is the identity matrix and $\rho > 1$. Here the observation vector h is taken as $[0 \;\; 1]$. The signal to noise ratio (SNR) is taken as 30dB. The frequency estimation can be done with the steps illustrated in the Kalman filter algorithm.

Similarly for LMS algorithm the complex signal is also generated as that of the method adopted in Kalman filter. The complex weight matrix is updated with the right choice of step size $(\mu = 0.18)$ and correlation matrix $(P = 0)$. From the complex weight matrix the estimation of frequency is made.

For the extended least square technique the signal is expressed in parametric form. Evaluating the pseudo inverse of the system structure matrix does the estimation of the parameter ϕ. The sampling time is taken as 1 millisecond. The estimation of frequency is done in three steps. The first two steps as described in mathematical formulation are made to estimate amplitude and phase, followed by estimation of frequency.

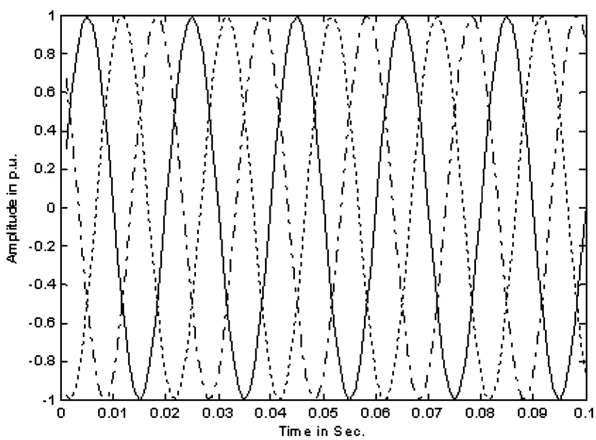

Figure 1. Three phase signals.

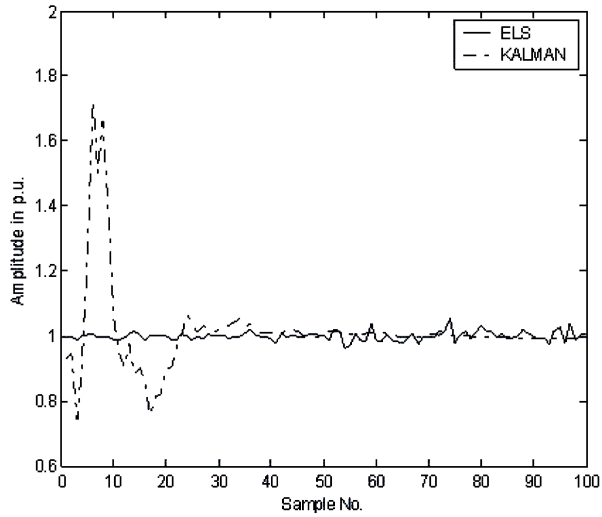

Figure 2. Comparison of estimation of amplitude.

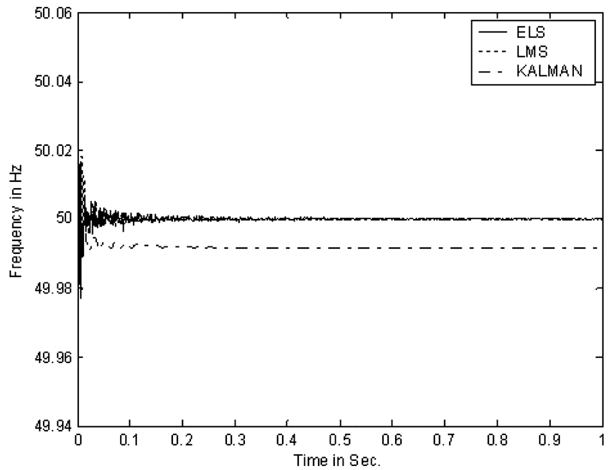

Figure 3. Comparison of estimation of frequency.

4. Discussion

Three phase signal of 1 p.u amplitude is generated in MATLAB and is shown in Fig. 1. Estimation of amplitude for real signal is done by Kalman filter and ELS method. From the Kalman filter algorithm it is found that the estimated amplitude is 1.068 p.u

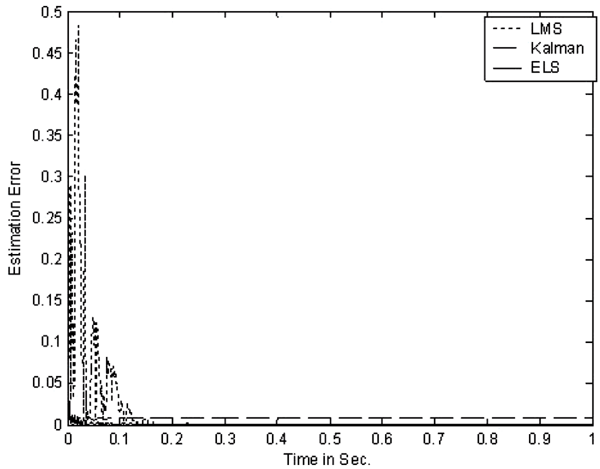

Figure 4. Comparison of estimation error.

for 0.003 sec. and 1 p.u for the rest period but in case of ELS algorithm the estimated amplitude is 1 p.u for the entire simulation period which is shown in Fig. 2.

Estimation of frequency by different algorithms is superimposed in Fig. 3. From Fig. 3 it is reflected that Kalman filter approach exhibits more oscillation and does not settle around 50 Hz. At the same time LMS approach exhibits oscillations in the few initial samples and finally settles around 50 Hz. But ELS algorithm exhibits fewer oscillations and also settles around 50 Hz. So the estimation of frequency by ELS algorithm is preferred.

Estimation error is the difference between actual frequency and estimated frequency. Estimation of error by different algorithms is done in Fig. 4. In the case of Kalman filter the estimation error is almost constant and comes near about 0.0152 Hz. In LMS algorithm the estimation error is more for few initial samples and it is found to be 0.0086 Hz for the rest of samples. However, for ELS algorithm the estimation error is reduced for the total simulation time and found as 0.0001 Hz which is definitely superior as compared to previous methods.

It can be seen from Table 1 that the computational time for estimation of frequency by ELS algorithm is 0.2340 sec. which is less in comparison to other two methods.

Table 1. Comparative assessment of methods

Method	Estimation error (RMS)	Computational time in sec.
KF	0.0152	1.1870
LMS	0.0086	0.3440
ELS	0.0001	0.2340

5. Conclusions

This paper presents the estimation of frequency of a synthetic signal by various signal processing techniques. However, choice of the covariance matrix is very crucial at the initial instant for Kalman filter algorithm. Improper choice of covariance matrix leads to longer computational time with larger estimation error. LMS algorithm seems very complex due to the implementation of correlation matrix and necessity of proper choice of step size. But at the same time ELS algorithm is very simple by representation of the parametric form of the signal. With the one step computation the amplitude and phase of the signal is determined followed by the estimation of frequency. The computational time is less due to the simplicity of the algorithm and estimation error is also lower. Validation of the extended least square algorithm can be done in MATLAB platform with various system changing conditions and all possible types of faults. Real time implementation of the algorithm can be realized with a DSP processor.

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