FREQUENCY ESTIMATION OF DISTORTED POWER SYSTEM SIGNALS USING EXTENDED COMPLEX KALMAN FILTER

P. K. Dash, A. K. Pradhan, G.Panda
Regional Engineering College, Rourkela
India

Abstract - The paper proposes an extended complex Kalman filter and employs it for the estimation of power system frequency in the presence of random noise and distortions. From the discrete values of the 3-phase voltage signals of a power system, a complex voltage vector is formed using the well known $\alpha\beta$-transform. A nonlinear state space formulation is then obtained for this complex signal and an extended Kalman filtering approach is used to compute the true state of the model iteratively with significant noise and harmonic distortions. As the frequency is modeled as a state, the estimation of the state vector yields the unknown power system frequency. Several computer simulations test results are presented in the paper to highlight the usefulness of this approach in estimating near nominal and off-nominal power system frequencies.

Keywords- Power system frequency, Frequency estimation, Extended Kalman filter, Nonlinear filter

1. Introduction

Digital control and protection of power systems require the estimation of supply frequency and its variation in real-time. Variations in system frequency from its normal value indicate the occurrence of a corrective action for its restoration. A large number of numerical methods is available for frequency estimation from the digitized samples of the system voltage. Conventional methods assume that the power system voltage waveform is purely sinusoidal and therefore the time between two zero crossings is an indication of system frequency. Discrete Fourier transforms [1], Least error squares technique [2,3], Kalman filtering [4], Recursive Newton-type algorithm [5], Adaptive notch filters [6] etc. are known signal processing techniques used for frequency measurements of power system signals. A new numeric technique and its practical implementation are presented in reference [7]. This approach suffers from inaccuracies due to the presence of noise and harmonics. An iterative technique for fast and accurate estimation of nominal and off-nominal power system frequency has been presented in [8]. This technique requires a correct guess of the system frequency for fast estimation and suffers from inaccuracies in the presence of noise (with an SNR value of 20 dB or less) and harmonics. Amongst the several numerical techniques described above both linear and extended Kalman filtering approaches have attracted widespread attention, as they accurately estimate the amplitude, phase and frequency of a signal buried with noise and harmonics.

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In this paper, a variation of nonlinear Kalman filter in the complex form is presented which simplifies the modeling requirement for frequency and amplitude estimation of a signal. It has been recently shown in reference [9] that the extended complex Kalman filter (ECKF) is more attractive than the real one from the point of view of modeling and stability considerations.

The discrete values of the three phase voltage signals of a power system are transformed into a complex vector using the well known $\alpha\beta$-transform used in power system analysis. This complex voltage vector is then modeled along with frequency in a nonlinear state-space form and the theory of extended Kalman filter is used to obtain the state vectors iteratively. The computation of Kalman gain and choice of initial covariance matrix is crucial in determining the speed of convergence of the new algorithm and its noise rejection property. A variety of simulated power system conditions is used for the application of this new technique and frequency estimation error is close to .01 Hz to .02 Hz in most cases. The application of this algorithm for frequency relaying in power system is expected to be simple with very little computation for 2-state complex Kalman filter.

2. ECKF for frequency estimation

The discrete representation of three phase voltages of a power system is obtained as

$$
\begin{aligned}
V_a(k) &= V_m \sin(\omega k\Delta T + \phi) + \in_a(k) \\
V_b(k) &= V_m \sin(\omega k\Delta T + \phi - 2\pi/3) + \in_b(k) \\
V_c(k) &= V_m \sin(\omega k\Delta T + \phi + 2\pi/3) + \in_c(k)
\end{aligned} \tag{1}
$$

where $\in_a(k), \in_b(k), \in_c(k)$ are noise terms that can be any combination of white noise and harmonics, ΔT is the sampling interval and k is the sampling instant (iteration count). The α-β components are obtained from the above discrete phase voltages as

$$
\begin{bmatrix} V_\alpha(k) \\ V_\beta(k) \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} V_a(k) \\ V_b(k) \\ V_c(k) \end{bmatrix} \tag{2}
$$

A complex voltage $V(k)$ is obtained from (2) as

$$
\begin{aligned}
V(k) &= V_\alpha(k) + jV_\beta(k) \\
&= Ae^{j(\omega k\Delta T + \phi)} + \eta(k)
\end{aligned} \tag{3}
$$

where A is the amplitude of the signal and $\eta(k)$ is the noise component.

The discrete observation signal $V(k)$ can now be modeled in a state-space form as

$$
\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & x_1(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \tag{4}
$$

$$
Y(k) = V(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \eta(k) \tag{5}
$$

where the states x_1 and x_2 are

$$
x_1(k) = e^{j\omega\Delta T} = \cos k\omega\Delta T + j\sin k\omega\Delta T
$$

$$
x_2(k) = Ae^{j(\omega k\Delta T + \phi)} \tag{6}
$$

and ΔT = sampling interval

The above linear stochastic filter is also equivalent to the following nonlinear one

$$
\begin{aligned}
x(k+1) &= F(x(k)) \\
y(k) &= Hx(k) + \eta(k)
\end{aligned} \tag{7}
$$

where

$x(k) = [x_1(k) \quad x_2(k)]$

$F(x(k)) = [x_1(k) \quad x_1(k)\, x_2(k)]$

$H = [0 \quad 1]$ (8)

F = a nonlinear function

Applying extended complex Kalman filter (ECKF) to the nonlinear system described in eqn.7 we obtain

$$
\begin{aligned}
\hat{x}(k/k) &= \hat{x}(k/k-1) + K(k)\,(y(k) - H\hat{x}(k/k-1)) \\
\hat{x}(k+1/k) &= F(\hat{x}(k/k))
\end{aligned} \tag{9}
$$

$$
K(k) = \hat{P}(k/k-1)\, H^{*T}\left[H\hat{P}(k/k-1)\, H^{*T} + R\right]^{-1} \tag{10}
$$

$$
\hat{P}(k/k) = \hat{P}(k/k-1) - K(k)\, H\, \hat{P}(k/k-1) \tag{11}
$$

$$
\hat{P}(k+1/k) = F_1(k)\hat{P}(k/k)\, F_1^{*T}(k) \tag{12}
$$

where

$$
\begin{aligned}
F_1(k) &= \left.\frac{\partial F(x(k))}{\partial x(k)}\right|_{x(k)=\hat{x}(k/k)} \\
&= \begin{bmatrix} 1 & 0 \\ \hat{x}_2(k/k) & \hat{x}_1(k/k) \end{bmatrix}
\end{aligned} \tag{13}
$$

$K(k)$ = Kalman gain matrix

$\hat{P}(k/k)$ or $\hat{P}(k+1/k)$ = Covariance matrix

H = Observation vector

*, T represent conjugate and transpose of a complex quantity respectively.

In the above formulation, the state-space representation given in equation (4) can be expanded to include decaying dc and harmonic components if necessary. For example, if there is a fifth harmonic(x_3) in the signal, the state-space model becomes

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & x_1(k) & 0 \\ 0 & 0 & x_1^{5}(k) \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} \quad (14)$$

This nonlinear filter is quite stable regardless of the initial conditions of the states X_1 and X_2, provided the observation signal is bounded, which is usually true in a practical system like the power system. The choice of initial covariance matrix P_0 and noise covariance R is crucial. R is taken as 1 in the numerical examples presented in this paper. After the convergence of the state vector is attained, the frequency is calculated as

$$\hat{f}(k) = \frac{1}{2\pi \Delta T} \sin^{-1}\left[\mathrm{Im}(\hat{x}_1(k))\right] \quad (15)$$

3. Computer Simulation Tests

The proposed technique is applied to several power system steady and dynamic operating conditions using MATLAB programming environment for the estimation of the fundamental frequency. A sampling rate of 3.2 KHz (64 samples per cycle) is used for the estimation process. The initial covariance matrix is chosen as $P_0=\rho*I$, where $\rho>1$, and the initial estimate of the fundamental frequency is assumed to lie between 40 to 60 Hz for the estimation of the nominal power system frequency of 50 Hz. The effect of initial estimates of 40, 50 and 60 Hz on the estimation of 50 Hz frequency for a sinusoidal signal with no noise is shown in Fig.1. Conversely from an initial estimate of 50 Hz, the convergence characteristics to obtain final frequencies of 40, 55 and 60 Hz are depicted in Fig.2.

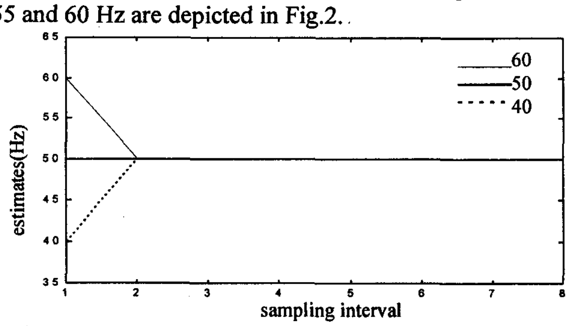

Fig.1. Frequency estimates of 50 Hz input signal with initial estimates of 40,50 and 60Hz

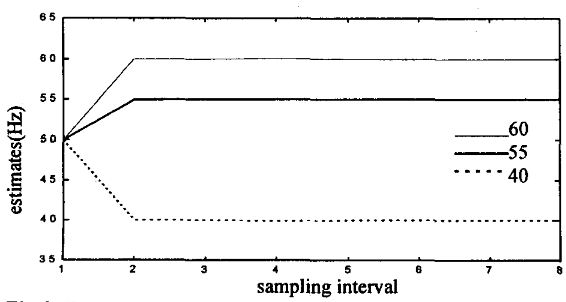

Fig.2. Frequency estimates of 40, 55 and 60 Hz input signals with initial estimate of 50 Hz

After studying the effect of initial estimate of the frequency, it is desirable to study the response of the ECKF in the presence of harmonics, sudden change in system frequency and noise. These aspects are addressed in the following section.

Case 1 : Presence of harmonics in the signal

The input voltage signal is assumed to contain 1 p.u. of fundamental, 0.12 p.u. of second harmonic, 0.3 p.u. of third harmonic and .05 p.u. of fifth harmonic. The proposed approach is used to estimate the frequency of the fundamental component which is taken as 51 Hz. Fig.3 presents the estimated frequency and its convergence to the true value in nearly 2 cycles and the frequency error compared to the true value is 0.01 Hz. It is also seen from the simulations that due to the α-β transformation, the third harmonic component in the observed signal is eliminated and hence does not influence the estimation process. However, the presence of second and fifth harmonic component would increase the time to converge to the true value.

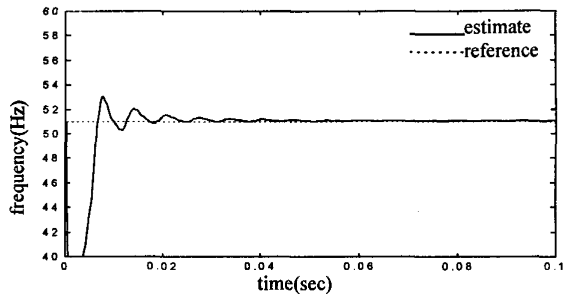

Fig.3. Response to signal of 51 Hz plus harmonics

Case 2 : Change in amplitude and phase

Fig.4 shows the response to a constant 50 Hz signal when the amplitude is suddenly increased from 1.0 p.u. to 1.5 p.u. The ECKF is found to be insensitive to the amplitude changes unlike the earlier approaches and the estimated value converges to the true value in less than quarter of a cycle (5 ms).

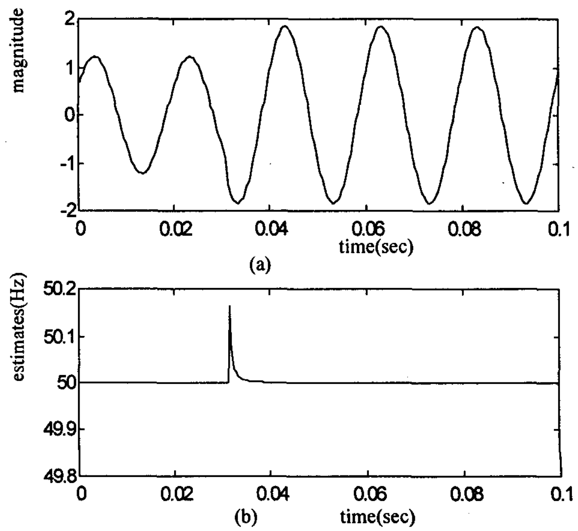

Fig.4.(a)50 Hz voltage signal with instantaneous amplitude increment from 1.0p.u. to 1.5p.u.(b)its frequency estimates

Fig.4(c) shows the response of the algorithm to a sudden change in the phase angle of the system voltage by -10^0. From response it can be seen that the new method produces a very quick estimate of the system frequency in a time frame less than one cycle. This is definitely a significant improvement than the earlier approaches used for frequency estimation.

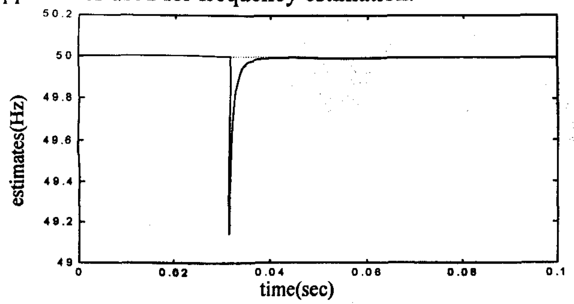

Fig.4(c).Frequency estimates of 50 Hz voltage signal with sudden decrement of phase angle by 10^0

Case 3 : Change in frequency

The test signals in this case are the noise free three phase voltage and step change in frequency from 50 Hz to 45 Hz and from 45 Hz to 52 Hz are affected at .0313s and .0625s respectively. Fig.5(a) shows the performance of the proposed algorithm for step changes in system frequency and it is observed from the figure that the true estimates of the frequency are obtained mostly within 10 ms (half a cycle of the voltage waveform).

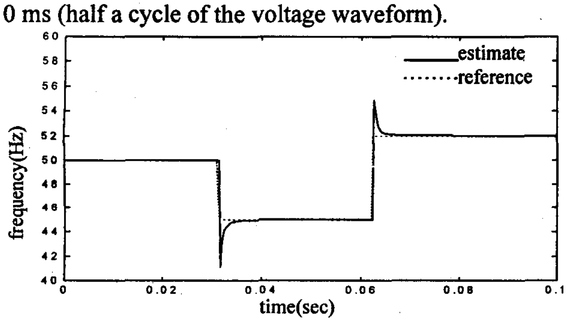

Fig.5.(a) Response to signal with step changes in frequency

Fig. 5(b) shows the response to a dynamic change (decay) in system frequency at 32 Hz/sec and the signal is assumed to contain no noise. It is observed from the figure that the proposed approach follows the actual system frequency very closely with a delay of 2 to 3 ms. The tests are repeated with frequency rise or decay ranging from 5 Hz/sec to 60 Hz/sec giving similar delays as mentioned for the 32 Hz/sec case.

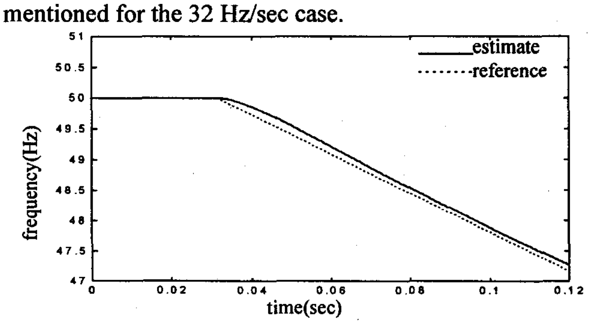

Fig.5.(b) Response to signal with 32 Hz/sec decay in frequency

Case 4 : Presence of noise

The performance of the proposed algorithm is evaluated in the presence of random noise with zero mean and Gaussian distribution and SNR (signal to noise ratio) varying from 20 dB to 80 dB . For low signal to noise ratio that is 20 dB, the convergence to the true frequency of the signal 50 Hz is obtained in almost 2 cycles (40 ms) as shown in Fig.6. However, if the SNR is reduced to 40 dB, the time required for convergence is reduced to less than 20 ms. This observation is quite significant as the earlier algorithms [3,8] reported estimate with high errors of nearly 0.2 Hz to 0.8 Hz for SNR = 40 dB.

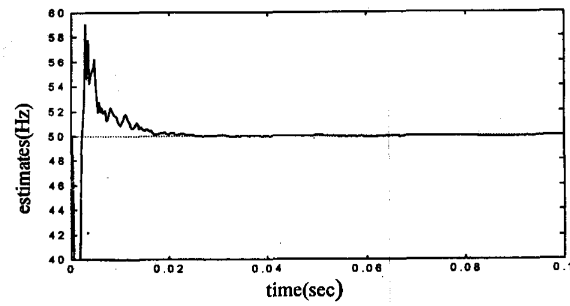

Fig.6. Response to signal of 50 Hz plus noise (20 dB SNR)

The frequency errors computed in the presence of noise when SNR varied from 20 dB to 80 dB are shown in Fig.7. It is observed that with SNR 40 dB the corresponding error is .0034 Hz which is a very significant improvement in comparison to the earlier approaches.

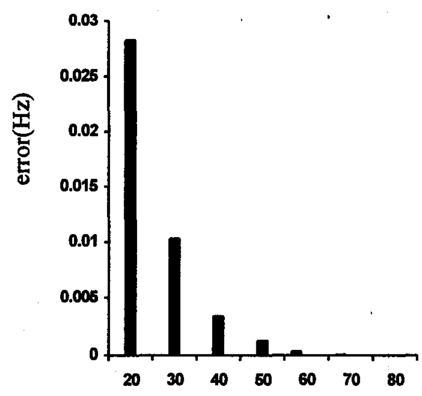

Fig.7. Estimation error for noisy input signal

Fig.8(b) shows the performance of the algorithm in the presence of 20 dB noise and 30% of third harmonic and 10% of fifth harmonic in addition to the 100% 50Hz component. The convergence to the true value is obtained in nearly 3 cycles (60 ms) even for this highly distorted signal, shown in Fig.8(a) .

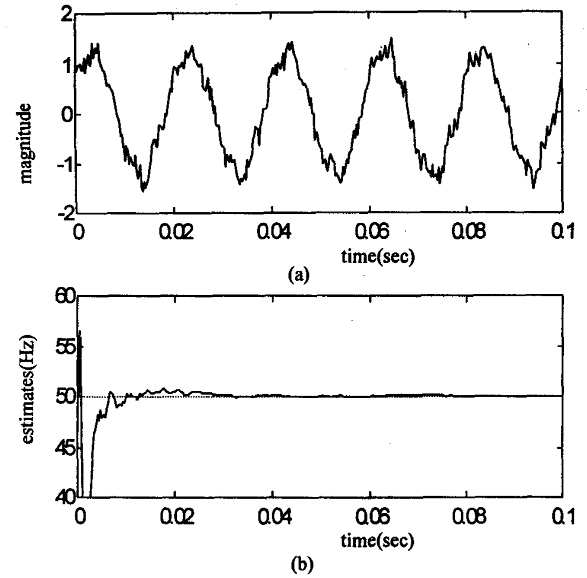

Fig.8(a)50 Hz voltage signal with third harmonic (30%), fifth harmonic(10%) and noise(20dB SNR)(b)its frequency estimates

In the cases of sudden jump and rise/decay of the system frequency, the new approach produces accurate estimation of the frequency in the presence of noise. Figs.9(a) and 9(b) present the response with SNR value of 40dB to sudden jumps in system frequency and rise in frequency from 50 Hz with a rate of 32 Hz/sec. From the results it is observed that the new filter produces significant convergence speed in the estimation of frequency.

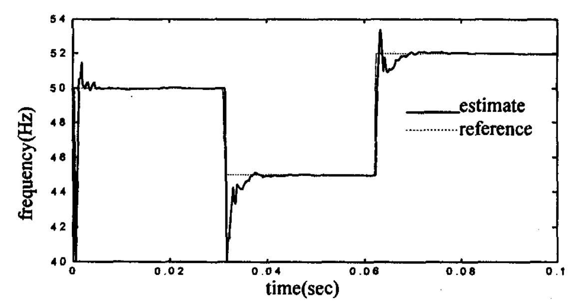

Fig.9(a) Response to signal with noise for step changes in signal frequency case

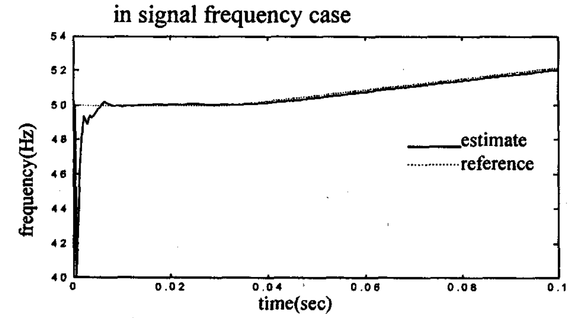

Fig.9(b) Response to signal with noise for32 Hz/sec rise in signal frequency case

Case 5 : Unbalances in three phase voltages

The proposed method works excellent in the presence of unbalances in the magnitudes of the phase voltages. For example, for $V_a = 1.0$ p.u., $V_b = 1.1$ p.u. and $V_c = 0.9$ p.u. (all peak values), the true frequency of 50 Hz is obtained in nearly 3 cycles and frequency estimation error is nearly .01 Hz, which is acceptable. Fig.10 demonstrates for such a case.

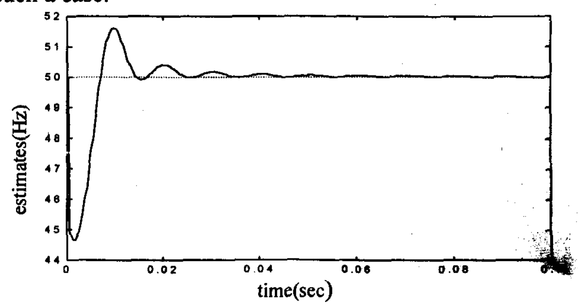

Fig.10. Response to 50 Hz signal at unbalanced condition

4. Conclusion

The paper presents an extended complex Kalman filter for the estimation of power system frequency in the presence of harmonics and random noise. The method uses the sampled values of three phase voltages as inputs and $\alpha\beta$- transform to convert these inputs to a complex observation vector. In the presence of significant noise and harmonics, the speed of convergence is reduced to 3 cycles and this can be improved significantly if harmonics are also considered in the state space formulation. This new approach is found to be very stable and yields significant frequency estimation accuracy of the order of .01 Hz -.02 Hz in the presence of noise, less than 40 dB or so. The present approach is found to work very well for step changes and decay or rise in system frequency.

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Biographies

P.K.Dash was educated at the Utkal University and I.I.Sc., Bangalore. He was a post-doctoral fellow at the University of Calgary, Canada and held several visiting appointments with North American Universities, BBC Brown Boveri, Switzerland, and Bristol Aerospace, Canada. His recent collaborations are with Virginia Polytechnic Institute and State University, U.S.A. Dr. Dash is a Professor of Electrical Engineering and Chairman of the Centre of Applied Artificial Intelligence, Regional Engineering College, Rourkela, India. During 1993-94 and 1996-97 he was a visiting staff at National University of Singapore.

A.K.Pradhan received his B.Sc.(Engg.) degree in Electrical Engineering in 1990 and M.Sc.(Engg.) in Power System Engineering in 1992 from Sambalpur University. He has joined as a lecturer in Electrical Engineering department at University College of Engineering in 1992. Currently he is pursuing his Ph.D. degree at Regional Engineering College, Rourkela. His doctoral research focuses on the areas of power system estimation and adaptive protection. His research interests also include power system analysis and applications of Fuzzy Logic and Neural Network to power system protection.

Ganapati Panda obtained first class degree with Honours in Electrical Engineering in 1971 and M.Sc. Engg. in Communication Systems in 1977 from Sambalpur University. He received Ph.D. degree in "Digital Signal Processing" in 1982 from I.I.T., Kharagpur. He was awarded Common Wealth Academic Fellowship and pursued Post Doctoral work in "Adaptive Signal Processing" at University of Edinburg , U.K., during 1984-86. He was a Lecturer during 1979-88, Reader during 1979-88 at University College of Engineering, Burla. In 1988 he joined Regional College of Engineering, Rourkela as Professor and acted as founder Head in Department of Applied Electronics and Instrumentation Engineering. He is continuing in this post till date.

He has published more than 100 research papers in Conferences and reputed Journals. He has also edited a book on "Digital Signal Processing". He has been honoured with prestigious Samanta Chandra Sekhar award for outstanding contribution to Engineering and Technology by Govt. of Orissa in 1993. He acts as a reviewer of IEE Proceeding (UK) and IEEE Transactions(USA). He is a Life Fellow of Institute of Engineers, India, Life Member of C.S.I., Life Member of I.S.T.E. and senior member of IEEE (USA). His name has been referred in Biography International 1993, Asia's Who's Who-Men and Women of Achievements, Indo-Europeean Who's Who, Indo-Arab's Who's Who, The Twentieth Century Award for Achievement by International Biographical Center, Cambridge. He has visited USA, UK, France, Japan and Singapore on various academic assignments.