Dynamics and Control of a Computer Controlled Bubble Cap Distillation Column

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**Dynamics and Control of a Computer Controlled Bubble Cap Distillation Column:**

**ABSTRACT**

In this work a seven plate, pilot plant type continuous bubble cap distillation column is used. Process dynamic equations are derived using lumped parameter approach, state variables and state equations. The plate efficiency used is 57%, which is calculated using total reflux condition. Using state equations, theoretical step responses for feed composition change, feed flow rate change, and reflux rate changes are calculated. These are compared with experimental data.

Closed loop response studies are done using P and P-I controller to control top product composition by varying reflux. Since D-action software is not working, so P-D and P-I-D are not tested.

Conclusions are drawn about the settings to be used on computer - controlled distillation column.

**Key words:** digital control, analog control, distillation column, computer control, state equations, state variables.
NOTATIONS

B  Bottom product flow rate, moles/time
D  Distillate flow rate, moles/time
F  Feed flow rate, moles/time
f  Feed liquid flow rate change, moles/time
H  Molar liquid hold up on trays, moles
h  Molar liquid hold up change on trays or deviation variable of H, moles
K_C  Proportional gains of the controller
K_u  Ultimate Proportional gain
L  Liquid flow rate at rectifying section, moles/time
L_R  Re reflux flow rate, moles/time
L_S  Liquid flow rate at stripping section, moles/time
l  Liquid flow rate change, moles/time
m  Equilibrium constant
R  Reflux ratio
R_1  Constant
T  Temperature, °C
T_b  Bubble point of feed, °C
T_f  Feed temperature, °C
T_i  Integral time, min
T_set  Set point temperature, °C
T_u  Ultimate time period, min
t  Time, min
V  Vapor flow rate in rectifying section, moles/time
V_S  Vapor flow rate in stripping section, moles/time
X  Mole fraction of more volatile component in liquid phase
x  Perturbation variable or deviation variable of X
Y  Mole fraction of more volatile component in vapor phase
y  Perturbation variable or deviation variable of Y
β  Hydraulic time constant
λ  Latent heat of vaporization of feed, kcal/mole
τ  Time constant, min
γ  Hydraulic time constant

Superscripts & Subscripts

i  Component
j  State number
n  Stage number
d  Hold up of reflux drum, moles
b  Hold up of re-boiler and column base, moles
*  Steady state condition
′  Transpose of matrix
INTRODUCTION

A large number of articles have been published on process dynamics and simulation studies of continuous distillation columns\textsuperscript{1-4}. Some of the earlier articles on control of distillation columns are\textsuperscript{5-12}. The above articles are mostly using analog type of electrical, electronic controllers, pneumatic, electro-pneumatic controllers, and digital controllers.

The authors observed step response data on process dynamics and feed back control of a computer-controlled distillation column. The experimental step response data are compared with model equations. Inferences are drawn on this computer controlled continuous distillation column.

THEORY

The model equations used for the plate column are given below. Neglecting vapor hold up on each plate, responses are fast for flow rate changes, the component balance equations are:

\[
\frac{H_n^* dx_n}{dt} = L_{n-1}^* x_{n-1} - L_n^* x_n + V_{n+1}^* y_{n+1} - V_n^* y_n \quad (1a) \quad \text{(for } n \neq \text{ feed plate)}
\]

\[
\frac{H_n^* dx_n}{dt} = L_{n-1}^* x_{n-1} - L_n^* x_n + V_{n+1}^* y_{n+1} - V_n^* y_n + F^* x_f \quad (1b) \quad \text{(for } n = \text{ feed plate)}
\]

The overall material balance equations are:

\[
\frac{dh_n}{dt} = L_{n-1} - L_n + V_{n+1} - V_n \quad (2a) \quad \text{(for } n \neq \text{ feed plate)}
\]

\[
\frac{dh_n}{dt} = L_{n-1} - L_n + V_{n+1} - V_n + f \quad (2b) \quad \text{(for } n = \text{ feed plate)}
\]

1) For liquid rate perturbation only, in the absence of any vapor rate perturbation.

\[V_{n+1}=v_n=0 \text{ and } h_n=\beta_n h_0, \quad \beta_n = f (H_n^*, L_n^*) \quad (3)\]

Equations 1a, 1b, 2a, and 2b reduces to,

\[
\frac{dh_n}{dt} = \beta_n \frac{dl_n}{dt} = l_{n-1} - l_n \quad (4a) \quad \text{(for } n \neq \text{ feed plate)}
\]
\[
\frac{dh_n}{dt} = \beta_n \frac{dl_n}{dt} = l_{n-1} - l_n + f \quad \text{for } n = \text{feed plate} \tag{4b}
\]

\[
\frac{H_n^* dx_n}{dt} = L_{n-1}^* x_{n-1} - L_n^* x_n + V_{n+1}^* y_{n+1} - V_n^* y_n + l_{n-1}(X_{n+1}^* - X_n^*) \quad \text{for } n \neq \text{feed plate} \tag{5a}
\]

\[
\frac{H_n^* dx_n}{dt} = L_{n-1}^* x_{n-1} - L_n^* x_n + V_{n+1}^* y_{n+1} - V_n^* y_n + l_{n-1}(X_{n+1}^* - X_n^*) + X_n^* f \quad \text{for } n = \text{feed plate} \tag{5b}
\]

2) For vapor rate perturbations only, in the absence of any liquid perturbations:

\[
l_{n-1} = l_n = 0, \text{ and } h_n = \gamma_n v_n, \quad \gamma_n = f(H_n^*, V_n^*) \tag{6}
\]

Equations 1a, 1b, 2a, and 2b reduce to,

\[
\frac{dh_n}{dt} = \gamma_n \frac{dV_n}{dt} = V_{n+1} - V_n \quad \text{for } n = \text{feed plate} \tag{7a}
\]

\[
\frac{H_n^* dx_n}{dt} = L_{n-1}^* x_{n-1} - L_n^* x_n + V_{n+1}^* y_{n+1} + Y_n^* v_{n+1} - V_n^* y_n - Y_n^* v_n + X_n^* (v_{n+1} - v_n) \quad \text{for } n = \text{feed plate} \tag{7b}
\]

3) For small perturbation in any input variable

\[
y_n = m_n x_n\tag{8}
\]

where \(m_n\) is a constant for plate \(n\).

**Case - 1:** Feed composition change

The material balance equations for each plate of the two component 9-plate continuous distillation column (Fig.-1) including condenser and re-boiler using equations 1a, 1b, and 8 are:

\[
H_0^* \frac{dx_0}{dt} = L_0^* x_0 - (L_0^* x_0 + D)x_0 + V_1^* y_1 - V_0^* (m_0 x_0), \text{ for } n = 0
\]

\[
\frac{dx_0}{dt} = - \frac{(L_0^* + D)x_0}{H_0^*} + \frac{m_1 V_1^* x_1}{H_0^*}; V_0^* m_0 = 0
\]

Similarly for each plate, after simplification, the equations are:
\[ n = 0; \quad \frac{dx_0}{dt} = -\frac{(L_0 + D)x_0}{H_0^*} + \frac{m_1V^*_1}{H_0^*} \]  
(Condenser)

For plates, \( n = 1 \) to \( n = 7 \), the equations using Kronecker delta function

\[ n = 1, 2 \ldots 7; \quad \frac{dx_n}{dt} = \frac{L_{n-1}^*}{H_n^*}x_{n-1} - \frac{L_n^* + m_nV^*_n}{H_n^*}x_n + \frac{m_{n+1}V^*_{n+1}}{H_n^*}x_{n+1} + \delta_{nf}F^*x_f; \]

\( \delta_{nf} = 0 \) for \( n \neq \text{feed plate} \), \( \delta_{nf} = 1 \) for \( n = \text{feed plate} \).

\[ n = 8; \quad \frac{dx_8}{dt} = \frac{L_7^*}{H_8^*}x_7 - \frac{L_8^* + m_8V^*_8}{H_8^*}x_8 \]  
(Re-boiler)

The above equations are arranged in state form \( \dot{X} = AX + Bu \) (where \( \dot{X} = \frac{dx}{dt} \)), neglecting the condenser not being an equilibrium stage and are solved for step response of feed composition, using MATLAB.
Case - 2: Feed flow rate change

For feed rate perturbations $f$, using equation 4b,

$$\frac{dh_n}{dt} = \beta_n \frac{dl_n}{dt} = l_{n-1} - l_n + f$$

$$\beta_4 \frac{dl_4}{dt} = l_3 - l_4 + f \quad (\because l_4 = l_3 = l_2 = l_1 = 0)$$

\[ l_4 = \frac{f}{(\beta_4 s + 1)} \]

\[ l_5 = \frac{f}{\prod_{j=4}^{5} (\beta_j s + 1)} \]

\[ l_6 = \frac{f}{\prod_{j=4}^{6} (\beta_j s + 1)} \]

\[ l_7 = \frac{f}{\prod_{j=4}^{7} (\beta_j s + 1)} \]

\[ l_8 = \frac{f}{\prod_{j=4}^{8} (\beta_j s + 1)} \]

Substituting the values of $l_0, l_1, l_2, \ldots, l_8$ in equations 5a and 5b,

For, $n = 4$ (feed plate)

$$\frac{dx_4}{dt} = \frac{L_4^* x_3}{H_4^*} - \frac{(L_4^* + m_4 V_4^*) x_4}{H_4^*} + \frac{m_5 V_5^*}{H_4^*} x_3 + \frac{X_4^* f}{H_4^*}$$

For, $n = 5$ (stripping section)

$$\frac{dx_5}{dt} = \frac{L_5^* x_4}{H_5^*} - \frac{(L_5^* + m_5 V_5^*) x_5}{H_5^*} + \frac{m_6 V_6^*}{H_5^*} x_4 + \frac{(X_4^* - X_5^*) f}{(\beta_5 s + 1)}$$

For, $n = 3$ (rectifying section)

$$\frac{dx_3}{dt} = \frac{L_3^* x_2}{H_3^*} - \frac{(L_3^* + m_3 V_3^*) x_3}{H_3^*} + \frac{m_4 V_4^*}{H_3^*} x_2$$

Equations are obtained for other plates and are put in the state equation form, $\dot{X} = A x + B_2 U$; where $B_2$ is
\[ B_2 = \begin{bmatrix}
0 \\
0 \\
0 \\
X_f^* \\
\frac{H_4^*}{(X^*_4 - X^*_5)} \\
\frac{1}{\beta_5 s + 1} \\
\frac{1}{(X^*_5 - X^*_6)} \\
\frac{1}{\prod_{j=4}^{5} (\beta_j s + 1)} \\
\frac{1}{(X^*_6 - X^*_7)} \\
\frac{1}{\prod_{j=4}^{6} (\beta_j s + 1)} \\
\frac{1}{(X^*_7 - X^*_8)} \\
\frac{1}{\prod_{j=4}^{7} (\beta_j s + 1)} \\
\end{bmatrix} \]

\( \beta \) is tray hydraulic time constant. It is 3 to 4 seconds and it is taken 3 seconds here because of small column. The above equations are solved for step response of feed flow rate using MATLAB.

**Case - 3: Reflux flow rate change**

For reflux perturbation, \( l_R \) from equation 4a, \( l_0 = l_R \)

\[
\beta_1 \frac{dl_1}{dt} = l_R - l_1; \quad l_1 = \frac{l_R}{(\beta_1 s + 1)}
\]

\[
\beta_2 \frac{dl_2}{dt} = l_1 - l_2; \quad l_2 = \frac{l_R}{(\beta_1 s + 1)(\beta_2 s + 1)}
\]

\[
l_3 = \frac{1}{\prod_{j=1}^{3} (\beta_j s + 1)}
\]

so on

\[
l_8 = \frac{1}{\prod_{j=1}^{8} (\beta_j s + 1)}
\]

For \( n = 4 \) (feed plate);

\[
H_4^* \frac{dx_4}{dt} = L_3 x_3 - L_4 x_4 + V_5^* y_5 - V_4^* y_4 + I_3 (X^*_3 - X^*_4) + X_f^* f
\]
Substituting for \(l_3\),

\[
\frac{dx_4}{dt} = \frac{L_3^* x_3}{H_4^*} - \frac{(L_4^* + m_4 V_4^*) x_4}{H_4^*} + \frac{m_5 V_5^*}{H_4^*} x_5 + \frac{(X_3^* - X_4^*)}{\Pi(\beta_j s + 1)}
\]

For \(n=3\);

\[
\frac{dx_3}{dt} = \frac{L_2^* x_2}{H_3^*} - \frac{(L_3^* + m_3 V_3^*) x_3}{H_3^*} + \frac{m_4 V_4^*}{H_3^*} x_4 + \frac{(X_2^* - X_3^*)}{\Pi(\beta_j s + 1)}
\]

Similarly equations for other plates are written.

They are arranged in state variable form of \(\dot{X} = A_x + B_x U\); where \(B_3\) is

\[
B_3 = \begin{bmatrix}
\frac{(X_0^* - X_1^*)}{\Pi(\beta_j s + 1)} \\
\frac{X_1^* - X_2^*}{\Pi(\beta_j s + 1)} \\
\frac{X_2^* - X_3^*}{\Pi(\beta_j s + 1)} \\
\frac{X_3^* - X_4^*}{\Pi(\beta_j s + 1)} \\
\vdots \\
\frac{X_7^* - X_8^*}{\Pi(\beta_j s + 1)} \\
\end{bmatrix}
\]

The above state equation is solved for step response of reflux flow rate using MATLAB.

**CLOSED LOOP RESPONSE**

The assumptions are:

1) Top tray temperature is independent of the dynamics of the section below and depends only on reflux rate and its temperature

2) The loop for reflux drum and top tray is of first order.

3) The time lags are neglected, and

4) The liquid on the tray is assumed to be perfectly mixed.
\[
\frac{T(s)}{R(s)} = \frac{R_I}{\tau_s + 1}
\]

where, \(T\) is temperature of top tray in deviation form and \(R\) is change of reflux flow rate. The block diagram for the closed loop is shown below.

**Proportional control:** The closed loop response relation is:

\[
\frac{T(s)}{T_{set}(s)} = \frac{R_2}{\tau_s + 1}; \quad T(t) = R_2 * A(1 - e^{-t/\tau_1})
\]

where, \(A\) is set point change, \(R_2 = \frac{K_C R_I}{1 + K_C R_I}\) and \(\tau_1 = \tau / (K_C R_I + 1)\)

**Proportional-integral control:** The closed loop response relation is:

\[
\frac{T(s)}{T_{set}(s)} = \frac{\tau_s + 1}{\tau_s^2 s^2 + 2\xi \tau_s + 1}
\]

where, \(\tau_1^2 = \frac{\tau_1 \tau}{K_C R_I} \) and \(2\xi \tau_2 = \frac{(K_C R_I + 1)}{K_C R_2 \tau_1}\)

The solution is a combination of impulse and step response of 2nd order.

**EXPERIMENTAL SET-UP**

The column consists of seven plates, having three bubble caps on each plate. A steam heated shell and tube type re-boiler, water-cooled condenser, reflux drum, feed tank, top product tank, and bottom product tank are other units with column. The arrangement is shown in Figure-1. The details of the column are:

- **Material of construction:** Stainless steel
- **Column:** diameter-110 mm; plates-7, caps per plate-3
- **Feed tank capacity:** 8.5 liters. Plus 2 liters auxiliary.
- **Mini compressor:** For feed circulation and for pneumatic control.
- **Rotameters:** For feed, condenser water, reflux, distillate and bottom product.
- **Reboiler:** area 0.30 sq.m, shell and tube-vertical, steam-cell side.
Condenser: area 0.30 sq.m shell and tube-horizontal.

Temperature sensor: Type RTD, 11 numbers with indicator, sensitivity-0.01 °C.

Mini steam generator: 6 KW electrical heated.

Control panel: temperature indicator, output indicator, transmitters, power supplies for transmitters, converters, heater control switches.

Temperature indicator: digital, 11 channels.

Thyristor: for temperature control.

Output indicator: for reflux control.

E/P converter: for control valve actuation.

Power supply: for E/P converter.

Control valve: pneumatic type for reflux control.

ADC/DAC card: input-16, output-2.

Software: PID control, data logging, display, printing, analysis.

The liquid feed enters either to plate 3 or 4 or 5 by air pressure from the feed supply tank.

The system used is: Benzene-Toluene.

The control to the column is a single point control to control the purity of top product. For the set point changes and tuning of the controller, the screen-VI on the PC of the ON-LINE mode is used.

PROCEDURE

Steam is generated in the mini-steam generator. When steam is ready the liquid feed is sent into the column from feed tank by air pressure. The feed enters the column through Rotameter F4 and flows through the stripping section to the re-boiler. The level in re-boiler is maintained at 75%. Steam from steam generator is used to heat liquid in re-boiler. The RTDs and other measuring devices are made ‘ON’. Vapour from re-boiler passes up the entire column. The vapours rising through rectifying section are completely condensed in the overhead condenser and the condensate is collected in the accumulator / reflux drum. The overflow of reflux drum is collected as product. The column is operated until steady state is reached without using
automatic control and then step changes in feed composition, feed flow rate, and reflux rate are given. Then plate temperatures are noted through RTDs in PC and flow rates in Rota meters for open loop response.

In the open loop response, the theoretical and experimental results for step response in reflux rate are not agreeing well. Due to this a set of experiments are done again to determine the open loop response of top tray temperature to step change in reflux rate, keeping others constant. This experimental step response is used for the study of closed loop response.

RESULTS AND DISCUSSION

The reflux drum hold up is 6.87 moles ($H_d$). This is obtained by collecting the volume of liquid hold up to level controller (625 ml) and top product composition (86 wt% benzene). The hold up in re-boiler is 11.93 moles. The hold up in each plate is 68.33ml (assuming equal in each plate) and the efficiency of the column is obtained by running the column at total reflux condition, which is 0.57. The input data for step change in feed composition, feed flow rate, reflux flow rate, and $m$ values are shown in Table-1.

The comparison of theoretical and experimental step response values of feed composition, feed flow rate, and reflux flow rate are shown in Figures-2, 3, and 4 respectively. The error between theoretical and experimental values is around 20% in the beginning and reduces to 5% at steady state.

The errors are mainly due to: varying reflux flow rate (gravity flow), assuming linear equilibrium relationship ($y = mx$) and neglecting energy balance.

A few experimental and theoretical closed loop responses using P-controller with different $K_c$ values are shown in Figure-5.

A few experimental and theoretical closed loop responses using P-I controller with different $K_c$ and $T_i$ values are shown in Figure-6, 7, and 8.

For evaluating optimum settings of P and PI-controllers, the responses are shown in Figure-9 and 10.

The theoretical and experimental closed loop response values agrees nearly and the assumption that the system is nearly first order is not far from the limit for manipulated variable (reflux flow rate).
For proportional control, high \( K_c \) (low proportional band) is working well for this column. For PI-controller, high \( K_c \) and low \( T_i \) are working well. The Ziegler-Nichols settings are working well using P-controller for this column. For PI-controller, using the suggested \( K_c \) of Ziegler-Nichols and operating with \( T_i \) of \( T_u/1.2 \) (suggested by Ziegler-Nichols) is not working well for this column. It is observed that \( T_i \) of \( 2^*T_u \) is working well in this column.

**Table - 1: Input data for step responses**

a) **Feed composition change**

\[
A = \begin{bmatrix}
-2.0808 & 2.3886 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1.3554 & -3.7962 & 2.8872 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.3842 & -4.3332 & 3.1986 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1.1436 & -6.438 & 4.2192 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 3.2382 & -7.5702 & 4.6572 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 3.3036 & -8.055 & 5.109 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 3.3690 & -8.5800 & 5.5896 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.1957 & -0.3632 & 0
\end{bmatrix}
\]

Step change in feed composition = 0.60 - 0.4558 = 0.144 (input control vector \( U \)).

\[B_1' = [0 \ 0 \ 0 \ 1.3566 \ 0 \ 0 \ 0 \ 0 \ 0]\]

b) **Feed flow rate change**

\[B_2' = [0 \ 0 \ 0 \ 0.002464 \ 0.00912 \ 0.0103 \ 0.0106 \ 0.01201]\]

\[U_2 = 1.209 \ (input \ control \ vector)\]

Feed flow rate change: 95 ml to 190 ml / min.

c) **Reflux flow rate change**

\[B_3' = [0.00857 \ 0.008756 \ 0.00877 \ 0.00865 \ 0.0082 \ 0.0077 \ 0.00637 \ 0.00545]\]

\[U_3 = 1.154 \ (input \ control \ vector)\]

Reflux flow rate change: 47.5 to 95 ml / min.
d) $m$ values

$m_0 = 1.0, m_1 = 1.1375, m_2 = 1.2, m_3 = 1.42, m_4 = 1.54, m_5 = 1.653, m_6 = 1.79, m_7 = 1.925, m_8 = 2.065.$

REFERENCES

Figure 1 Experimental Set-up
Figure 2  Step Response of feed composition change

Figure 2: Step Response of feed composition change
Figure 3  Step Response of feed flow rate change
Figure 4  Step Response of reflux rate change
Figure 5  Proportional control action
Figure 6 Proportional integral control action
Figure 7  Proportional integral control action
Figure 8 Proportional integral control action
Figure 9  Optimum settings for proportional control action (Ziegler-Nichols)
Figure 10A  Optimum settings for proportional integral control action
Figure 10B  Optimum settings for proportional integral control action
List of Figures

1. Figure 1 Experimental Set-up
2. Figure 2 Step Response of feed composition change
3. Figure 3 Step Response of feed flow rate change
4. Figure 4 Step Response of reflux rate change
5. Figure 5 Proportional control action
6. Figure 6 Proportional integral control action
7. Figure 7 Proportional integral control action
8. Figure 8 Proportional integral control action
9. Figure 9 Optimum settings for proportional control action (Ziegler-Nichols)
10. Figure 10A Optimum settings for proportional integral control action
11. Figure 10B Optimum settings for proportional integral control action