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DIGITAL IMPEDENCE PROTECTION OF POWER TRANSMISSION LINES USING A SPECTRAL OBSERVER

P. K. DASH AND D. K. PANDA
Regional Enegineering College, Rourkela-8, INDIA

ABSTRACT

The paper presents a new recursive discrete-time filter for calculating the impedences from digitized voltage and current samples at relay location. From an interpolating signal model comprising a decaying d.c. component and components of fundamental and harmonic frequencies, an interpolating spectral observer is constructed to give the desired Fourier co-efficients recursively. The recursive interpolators are also especially useful for sampling rates other than Nyquist rate and unevenly spaced samples. The proposed algorithm is tested using the fault data recorded at the Saskatchewan Power corporation producing fast and reliable tripping conditions. The performance of the spectral observer is further enhanced by suitable placement of poles of the observer producing a fast operation of the digital relay. The performance of the protection scheme with unevenly spaced data samples is also presented in the paper.

INTRODUCTION

Computer hardware technology has considerably advanced since early 1960's . Newer generations of mini-and micro-computers tend to make digital computer relaying a viable alternative to the traditional relaying systems. Most of the work reported in the literature concentrates on the development of digital computer algorithms for relaying of EHV transmission lines.

Computerized distance relaying systems use digitized samples of voltage and current wave forms to calculate the apparent impedence between the relay point and the fault. A number of techniques for digital transmission line protection have been proposed over the past decade. A very suitable number of these are given in references (1) through (10). The various schemes proposed advocate a fairly wide spectrum of techniques to determine the impedence of the transmission line under fault conditions. Most of these algorithms generally meet the basic constraints of speed and accuracy although compromises have to be made depending upon the relaying point within the system.

This paper presents a new approach to the interpolation of voltage and current signal samples for a transmission line using a recursive discrete-time filter which generates interpolating function co-efficients from input signal samples. The algorithm explicitly filters the decaying d.c. components and some harmonics (if present after analog filtering), determines the real and imaginary components of the fundamental frequency current and voltage phasors and calculates the impedence as seen from the relay location. The decay rate of the d.c. component is not assumed in advance because it is affected by both the resistance of the arc at the fault and effective resistance of the system. The interpolating filter has the following advantages:

1) It is recursive in nature involving repetitive serial computation.
2) It performs one iteration for sample updating, incorporating a new sample and discarding all effects of the oldest sample affecting the previous iteration.
3) It lacks restriction as to sampling rate. In case of discrete Fourier transform, Nyquist rate sampling is not necessary.
4) The algorithm is computationally efficient, and is particularly attractive for non-uniform signal samples resulting in simple S/H and A/D converters.
5) The fast computational capability of the algorithm makes it suitable for microprocessor applications.
6) The algorithm produces fast tripping times by suitable pole placement for the gain vector matrix.

The interpolting filter for harmonic sinusoidal signals is known as a 'spectral observer'. The spectral observer technique is used in this paper to compute the impedence seen from the relay location for three sets of data recorded during actual faults on a 200-mile , 230 kV transmission line of the Saskatchewan Power Corporation.

DIGITAL FILTER

The voltage and current waveforms presented to the relay during a fault consist of components of decaying d.c., fundamental frequency and high frequency components. The harmonic components, whose frequencies mainly are determined by the distance of the fault, are due to travelling wave phenomena. Additionally, harmonics of the 60 Hz components will be produced because of nonlinearity. By appropriate choice of low pass filter, these components can be eliminated from the waveforms. Eliminating the high frequency components, the current and voltage can be described by the following two expressions

$$v(t) = V_{dc}\, e^{-t/\tau} + \alpha_v \cos \omega t + \beta_v \sin \omega t \quad (1)$$

$$i(t) = I_{dc}\, e^{-t/\tau} + \alpha_i \cos \omega t + \beta_i \sin \omega t \quad (2)$$

and τ = transmission line time constant.

The above expressions after expansion using Taylor series take the general form

$$y(t) = a_2 t^2 + a_1 t + a_o + \alpha_1 \cos \omega t + \beta_1 \sin \omega t \quad (3)$$

where $\omega = 2\pi/T$, T is the period of the function, α and β are functions of the Fourier coefficients.

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A linear time invariant homogeneous differential equation with characteristic polynomial

$$\left\{ \frac{d^3}{dt^3} \right\}\left\{ \frac{d^2}{dt^2} + \omega^2 \right\} y(t) = 0 \qquad (4)$$

has a general solution of the form (3), where the Fourier coefficients σ_1 and β_1 and polynomial coefficients and a_1, a_2 and a_0 are arbitrary constants dependent upon the system initial conditions.

A convenient state variable representation for this homogeneous system is the following

$$\dot{x} = Ax\ ,\ y = C^T x \qquad (5)$$

where $x^T = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5]$,

$$C^T = [1\ 0\ 1\ 0\ 0],$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -\omega^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad (6)$$

Although the band limited periodic signal $y(t)$ may not be actually produced by (6), it may be considered to be the output of the system. The harmonic and polynomial components of $y(t)$ are the signals

$$\begin{aligned} x_1(t) &= \sigma_1 \cos \omega t + \beta_1 \sin \omega t \\ x_2(t) &= \dot{x}_1(t) = -\omega\sigma_1 \sin \omega t + \omega\beta_1 \cos \omega t \\ x_3(t) &= a_2 t^2 + a_1 t + a_0 \\ x_4(t) &= \dot{x}_3(t) = 2 a_2 t + a_1 \\ x_5(t) &= \dot{x}_4(t) = 2 a_2 \end{aligned} \qquad (7)$$

Discrete Fourier transformation is thus equivalent to the system (6)'s initial conditions

$$x_1(0) = \sigma_1\ ,\ x_2(0) = \omega\beta_1$$

$$x_3(0) = a_0\ ,\ x_4(0) = a_1\ ,\ x_5(0) = 2a_2$$

from the samples of $y(t)$.

RECURSIVE DISCRETE SIGNAL RESOLUTION

A discrete-time model of (5) and (6) with sampling interval Δt is

$$\begin{aligned} x(k+1) &= \exp(A\,\Delta t)x(k) = \phi\, x(k) \\ y(k) &= C^T x(k) \end{aligned} \qquad (8)$$

For observing the discrete-time model, the related system,

$$\begin{aligned} \xi(k+1) &= \phi\xi(k) + g\,[y(k\,\Delta t) - C^T \xi(k)] \\ &= (\phi - gC^T)\,\xi(k) + g\, y(k\,\Delta t) \end{aligned} \qquad (9)$$

$$\gamma(k) = C^T \xi(k)\ ,\ \Psi = \phi - gC^T$$

is termed as an identity observer of the system (8).

The matrix ϕ in equation (8) is given by

$$\phi = \exp(A\Delta t) = \begin{bmatrix} \cos \omega\Delta t & \frac{\sin \omega\Delta t}{\omega} & 0 & 0 & 0 \\ -\omega \sin \omega\Delta t & \cos \omega\Delta t & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta t & \frac{1}{2}\Delta t^2 \\ 0 & 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (10)$$

For finding the gain vector g in equation (9) the eigenvalues of $(\phi - gC^T)$ may be placed either at zero or at arbitrary location, so that the observed system pair (ϕ, C) is completely observable.

If the eigenvalues of Ψ are zero, the observer is termed as deadbeat. A deadbeat observer driven by samples of the signal $y(t)$, will thus converge to the state of the signal model in 5 steps or less.

For a dead beat observer, the gain matrix g is obtained by solving

$$C^T \phi\, g = 0 \qquad (11)$$

and a probable value of g to satisfy (11) is

$$g = \frac{1}{2N+1}\left[\cos 2\omega\Delta t, \omega \sin 2\omega\,\Delta t, 3\Delta t^2, -\frac{5}{2}\Delta t, 1\right] \qquad (12)$$

N=harmonic order

However, for any arbitrary sampling rate other than the Nyquist rate (dead beat observation), a general method of gain matrix calculation is given in Appendix - I.

For a given sampling rate and data window, g is calculated in advance of signal processing and spectral observer computations assume the following simplified form

$$\xi_1(k+1) = a_1\,\xi_1(k) + b_1\,\xi_2(k) + g_1\,[y(k\Delta t) - \gamma(k)] \qquad (13)$$

$$\xi_2(k+1) = d_1\xi_1(k) + d_1\xi_2(k) + g_2\,[y(k\Delta t) - \gamma(k)] \qquad (14)$$

$$\xi_3(k+1) = \xi_3(k) + \xi_4(k)\Delta t + \xi_5(k)\,\frac{\Delta t^2}{2} + g_3\,[y(k\Delta t) - \gamma(k)] \qquad (15)$$

$$\xi_4(k+1) = \xi_4(k) + \xi_5(k)\,\Delta t + g_4\,[y(k\Delta t) - \gamma(k)] \qquad (16)$$

$$\xi_5(k+1) = \xi_5(k) + g_5\,[y(k\Delta t) - \gamma(k)] \qquad (17)$$

$$\gamma(k) = \xi_1(k) + \xi_3(k) \qquad (18)$$

and

$$a_1 = \cos \omega\Delta t\ ,\ b_1 = \frac{\sin \omega\Delta t}{\omega}, \quad d_1 = -\,\omega \sin \omega\Delta t \qquad (19)$$

The coefficients a_1, b_1 and d_1 are computed in advance of signal processing and are stored in microcomputer memory for use for apparent impedence calculations. The gains $g_1, g_2, \ldots, g_5$ are cyclic and are computed only once and are stored in a similar way as for the a_1, b_1, d_1 constants. Although the above signal model does not contain harmonics, the extension is straight forward.[11] With a harmonic signal model, the interpolator is termed as a spectral observer. Spectral observers perform recursive discrete Fourier transform where progressive incorporation of new samples discarding the old as in running Fourier transformation, requires only a single iteration. These filters are also especially useful for sampling rate other than the Nyquist rate. For very noisy signal the interpolating observer is choosen to have eigenvalues placed at locations other than the origin of the complex plane to achieve a different noise performance. As shown in reference(11) a complete n-point DFT requires n^2 multiplications ($n = 2N+3$), N being the harmonic order. However, updating with a new sample, discarding the oldest sample as in running Fourier transform, involves only 12 multiplications for one cycle of the filter with dc offset. This filter is suitable for both Nyquist and non-Nyquist sampling rates. The computational overhead of this algorithm seems to be higher than the one cycle Fourier one without taking the dc offset into account.

Further simplification can be achieved by replacing $\sin \omega t$ in place of $\sin \omega\Delta t/\omega$ and $-\sin_2 \omega\Delta t$ for $-\omega \sin_2 \omega\Delta t$, and $\omega\Delta t$ for Δt and $1/2\omega^2 \Delta t^2$ for $1/2\ \Delta t^2$ in equation (10). Due to this scaling the values of a_1, b_1 and d_1 are obtained as $a_1 = \cos \omega\Delta t$, $b_1 = \sin \omega\Delta t$, $d_1 = -\sin \omega\Delta t$ and $\alpha_1 = \xi_1(k)$, $\beta_1 = \xi_2(k)$ and the magnitudes of the gains for the decaying dc terms in the signal are reduced.

TRANSIENT APPARENT IMPEDENCE CALCULATION

Analog low pass filtering of the relay input (V, I) signals is necessary in order to have reliable relay operation under faulted line conditions. Without such filtering high frequency transients due to line shunt capacitance result in erratic and unreliable digital relay operation. The filtered voltage and current signals are used as input to R - X calculation algorithm, which yields the apparent line resistance R, and reactance X, to the fault point. The signals used for voltage V and current I inputs to the relay depend on the type of relay being studied. For phase distance relays, the V and I inputs are delta voltages and currents, for ground distance relays the V and I inputs are 'phase-to-ground and compensated phase currents'. For example for B - C phase fault, the relaying signals are $V_B - V_C$ and $I_B - I_C$, respectively. For ground distance relays the V and I inputs are the 'phase-to-ground' voltages and compensated phase currents. For example the compensated phase current for 'A' phase is given by

$$I_A^C = I_A + KI_G \text{ where } K = \frac{Z_0 - Z_1}{3Z_1} \qquad (20)$$

and I_G is the neutral current.

For computing the apparent impedence for phase or ground relays, it is required to compute the fundamental component of the voltage and current phasors from their sampled values.

The apparent resistance and reactances of the transmission line are obtained as

$$R = \frac{\alpha_1^v \alpha_1^i + \beta_1^v \beta_1^i}{\alpha_1^i \alpha_1^i + \beta_1^i \beta_1^i}, \quad X = \frac{\alpha_1^i \beta_1^v - \alpha_1^v \beta_1^i}{\alpha_1^i \alpha_1^i + \beta_1^i \beta_1^i} \qquad (21)$$

The suffix v and i pertain to the voltage and current signals used for calculating the values of α_1 and β_1 of the spectral observer. Although a simple fault waveform is assumed for the present study, a general signal model involving decaying d.c. and high frequency harmonic components can be used for recursive signal interpolation. ξ_1, ξ_2 are updated discarding all effects of the oldest sample affecting the previous iteration. A linear transformation of the spectral observer state vector recovers samples of the expansion at any specific time or the weighting co-efficients of the expansion, as desired. Further it is possible to estimate the next signal sample from the output of the spectral observer ($y = C^T x$) and an error bound from the estimated and measured signal can be determined. The error gives an indication regarding the accuracy of the assumed fault voltage or current model waveform.

NON-UNIFORM SAMPLING RATE

For power transmission system if the signal model is of the form

$$Y(t) = a_0 + \alpha_1 \cos \omega t + \beta_1 \sin \omega t \qquad (22)$$

the discrete-time state equations are

$$\epsilon(k+1) = \phi(k)\,\epsilon(k), \quad y(t_k) = C^T \epsilon(k) \qquad (23)$$

$$\phi(k) = \begin{bmatrix} \cos \Delta t_k & \sin \Delta t_k & 0 \\ -\sin \Delta t_k & \cos \Delta t_k & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\epsilon(k) = \left[x_1(k)\ x_2(k)\ x_3(k)\right]^T \qquad (24)$$

where

$$\Delta t_k = t_{k+1} - t_k \qquad (25)$$

The state observer for the discrete-time model has the structure

$$\xi(k+1) = \Psi(k)\,\xi(k) + g(k)\,y(t_k)$$
$$\Psi(k) = \phi(k) - g(k)\,C^T(k) \qquad (26)$$

The observer state will converge to that of the discrete-time model (22) in three steps provided the gain sequences $g(0)$, $g(1)$, $g(2)$ are chosen as

$$g(0) = \frac{[\phi(0)]_{col\ 1}}{[C^T]_{col\ 1}},$$
$$\Psi(0) = \phi(0) - g(0)C^T \qquad (27)$$

$$g(1) = \frac{\phi(1)\,[\Psi(0)]_{col\ 2}}{C^T\,[\Psi(0)]_{col\ 2}},$$
$$\Psi(1) = \phi(1) - g(1)C^T \qquad (28)$$

$$g(2) = \frac{\phi(2)\,[\Psi(1)\Psi(0)]_{col\ 3}}{C^T\,[\Psi(1)\ \Psi(0)]_{col\ 3}},$$
$$\Psi(2) = \phi(2) - g(2)C^T \qquad (29)$$

The next gain in the sequence is

$$g(3) = \frac{\phi(3)\,[\Psi(2)\ \Psi(1)]_{col\ i}}{C^T\,[\Psi(2)\ \Psi(1)]_{col\ i}},$$
$$\Psi(3) = \phi(3) - g(3)C^T \qquad (30)$$

and so on, where i is any column number for which the scalar denominator is nonzero. The state obsverver with these time varying gains will, at each step, converge to the state of the signal model based upon the most recent three signal samples.

For a general nth order time varying model the gain vector is computed recursively as

$$g(k) = \frac{\phi(k)\left[\Psi(k-1)\Psi(k-2)..\Psi(k-n+1)\right]_{col\ i}}{C^T(k)\left[\Psi(k-1)\Psi(k-2).\Psi(k-n+1)\right]_{col\ i}} \quad (31)$$

$$\text{and} \quad \Psi(-1) = \Psi(2) = \ldots = \Psi(-n+1) = I \quad (32)$$

where I is an nxn identity matrix and cycling the column index i, a complete iterative algorithm is obtained for observer gain calculation for non-uniform sample rates. If the uneven samples are cyclic, the gain vector becomes cyclic and computation of equation (26) does not involve a large computational over head.With the availability of multiple and fast processors and floating point microchips, the computational overheads for unevenly spaced samples get reduced.

TESTS USING THE SYSTEM FAULT DATA

The algorithm described in this paper and a starting algorithm for determining the onset and type of fault are used to obtain the apparent impedance of a faulted transmission line digitally. The starting algorithm consists of comparing the magnitudes of the voltage samples recently observed with those observed one cycle ago (of 60 Hz). If any five consecutive voltage samples are found to have lower (or higher) value than the set limit, a significant disturbance is deemed to have occurred. The prefault current phasors are subtracted from the post fault currents and the resulting currents are compared to identify the type of fault experienced. The use of both voltage and current samples for detection and classification of faults is superior to that using either voltage or current samples alone and this provides discrimination against load switching. After the fault type is determined, the impedence of the faulted phase or phases can be calculated using algorithms presented in the preceding section of the paper.

The proposed scheme is tested using three sets of data recorded during actual fault on a 200-mile, 230 kV transmission line. These data that represent three single line-to-ground faults are recorded by Saskatchewan Power Corporation and given at a sample rate of 12 samples per cycle (5). All signals are pre-filtered using a lowpass analog Butterworth filter and are digitized using a 12-bit A/D converter. To obtain a sample rate of 4 per cycle based on a 60 Hz waveform, intermediate samples are skipped from the 12 samples percycle data. This improper analog filtering however, shall add some additional time to the digital relay operation. For the test case considered here the total length of the protected line represents a reactance of 0.8 p.u. (based on a 100 MVA, 230 kV voltage base). The value of K has been taken equal to 0.787. Three relay zones in the form of quadrilaterals are selected for inclusion in the relay design. The observer gain coefficients are calculated and tabulated(Table 1) to show the effects of sampling rate, inclusion of terms involving time in the Taylor series expansion and the pole placement of the observer gain matrix, on the spectral observer gain vector g .

TABLE 1

(a) With constant dc offset

Sampling rate N_s	g_1	g_2	g_3
4	.50	-.50	.50
8	.178	-.223	.143
12	.153	-.102	.091

(b) With one term of the Taylor-series included for dc offset.

Sampling rate N_s	g_1	g_2	g_3	g_4
4	-.192	.213	1.58	.32
8	-.139	.182	1.25	.318
12	-.018	.086	1.167	.309

(c) With two terms of the Taylor series included for dc offset.

Sampling rate N_s	g_1	g_2	g_3	g_4	g_5
4	-.25	.25	2.750	1.114	.203
8	-.18	.206	3.126	1.751	.405
12	-.029	.102	3.69	2.38	.608

(d) With two terms for dc offset(sample rate N_s = 12) and eigenvalues of the observer gain matrix are changed (Apendix-I).

Eigen-values	g_1	g_2	g_3	g_4	g_5
0.0	-.029	.102	3.69	2.389	.608
-0.5	-.036	-.198	4.72	3.264	.762
-1.0	-.048	-.293	5.13	3.88	.835

For testing the algorithm, the first set of data includes a ground wire fault from blue phase to ground at approximately the middle of the line. The second set of data represents a close in ground switch blue phase fault. The third set of data represents a blue phase lightning strike to ground at approximately the middle of the line.

Table 2 presents the impedence estimates for first set of data for a sampling rate of 12 samples per cycle. The effect of variation of the spectral observer gains (with eigenvalues at 0 , -.5 and -1.0) can be seen from this table. A careful look into the table reveals that by a suitable placement of the poles of the spectral observer, a faster operation of the digital relay can be achieved in comparison to one cycle Fourier transform technique.

Table 3 presents the impedence estimates for the same data as above for a sample rate of 4 samples per cycle. From the results presented in these tables it is found that a fault detection and tripping can be obtained within 1/2 to 3/4 of a cycle based on 60 Hz waveform.

Table 4 presents the apparent impedence calculations for 4 samples per cycle using the time reference calculation from the faulted sample. Using an observer with gains calculated by placing the poles at -.5, a high speed fault defection is achieved.

Tables 5 and 6 show the apparent impedence calculations, for 2nd and 3rd set of data respectively. From the results it can be found that a reliable trip decision can be made within a period ranging from 1/2 cycle to 3/4 of a cycle. The positive values of the apparent reactance for the 2nd set of data indicate a fault on the line side.

The proposed algorithm was also tested for a B-phase to C-phase fault using simulated data (8,9) for a 345 KV line faulted at a distance of 100 miles from the sending end. Analog low pass filtering of the voltage and current signals is necessary to remove high frequency transients due to line shunt capacitance. The relay operating characteristic is the quadrilateral polar characteristic, which is set at 85% of the impedence of the 100 mile line, and this results in a reactance reach of 0.539 p.u. (53.9 ohms) for the first protective zone (I). The reach of the second protective zone (II) of the relay is set at 0.746 p.u. (74.6 ohms).

Figs. 1(a) and 1(b) show the apparent impedence trajectory for B-C phase fault for the phase relays using delta type voltage and current signals. From the figure it is evident that the fault detection timing is improved to 1/2 cycle (based on 60 Hz wave form) using suitable pole placement for the spectral observer.

Fig. 2 shows the performance of the algorithm for unevenly spaced samples with a sample rate of 12 samples per cycle. The R-X trajectory hits the threshold value in slighty more than 1/2 a cycle based on 60 Hz waveform .

GENERAL REMARKS

The proposed interpolating observer is a powerful and versatile method for computing functional expansion of signals from the signal samples. Further there is freedom in choosing the signal model and some flexibility in selecting the size of the data window and time reference for fault computations. The freedom of choosing the system model allows prespecified harmonics and decaying d.c. components. The decay rate is not prespecified but is accounted for by the algorithm automatically taking into consideration the effect of system resistance and arc resistance at the fault. A harmonic signal model is useful for differential protection of power transformer. Spectral observers perform a recursive discrete Fourier transform and require only a single iteration. These filters are especially useful for sampling rate other than the Nyquist rate (deadbeat type). There is the possibility of placing the observer eigenvalues other than at the origin of the complex plane to obtain a different noise performance. In general, non-deadbeat observation will modify the updating behaviour of the interpolater to one involving a fading memory of all previous samples. Nonuniform sampling rate can be easily incorporated into the observer without much computational overhead.

TABLE 2 IMPEDENCE ESTIMATES FOR BLUE PHASE GROUND WIRE FAULT (SAMPLE RATE = 12)

Sample No.	Fourier		Spectral Observer Poles at 0		Poles at -.5	
	R	X	R	X	R	X
23	-5.83	-.468	-5.85	-.469	-5.88	-.481
24	-5.86	-.471	-5.86	-.473	-5.875	-.476
25	-6.49	-.713	-6.114	-.677	-6.133	-.535
26	.92	1.04	1.08	1.758	-5.58	-4.3
27	.62	1.30	.664	1.28	2.18	-3.5
28	.26	1.02	.33	.903	1.51	-.32
29	.116	.86	.004	.811	.8	.242
30	.0912	.874	.019	.687	.47	.349
31	.0386	.927	.055	.588	.19	.353
32	.088	.762	.076	.482	.08	.33
33	.084	.462	.089	.386	.072	.372
34	.082	.431	.087	.303	.081	.338
35	.078	.363	.0698	.341	.068	.341
36	.0764	.338	.072	.338	.071	.337
37	.0736	.342	.0715	.339	.069	.338
38	.0732	.340	.0742	.342	.072	.340
39	.0635	.342	.0724	.340	.071	.341
40	.0712	.338	.0726	.341	.072	.342

FAULT OCCURS ON SAMPLE NO.25
(Per Unit Impedence)

TABLE 3 IMPEDENCE ESTIMATE FOR BLUE PHASE GROUND FAULT (SAMPLE RATE = 4) USING SPECTRAL OBSERVER

Sample No.	Poles at 0		Poles at -.5		Poles at-1	
	R	X	R	X	R	X
16	-5.97	-.436	-5.78	-.438	-5.89	-.462
17	-7.63	-.395	-7.52	-.297	-7.70	-1.55
18	.095	.518	.091	.502	.39	.53
19	-.103	.432	.083	.27	-.091	.287
20	.062	.305	.078	.388	.046	.365
21	.072	.348	.075	.372	.062	.342
22	.078	.337	.073	.341	.076	.345
23	.074	.345	.074	.343	.069	.338
24	.071	.339	.072	.338	.073	.343

FAULT OCCURS ON SAMPLE NO.17
(Per Unit Impedence)

TABLE 4 IMPEDENCE ESTIMATE FOR BLUE PHASE GROUND WIRE FAULT (SAMPLE RATE = 4) USING SPECTRAL OBSERVER

Sample No.	Poles at 0		Poles at -.5		Poles at -1.0	
	R	X	R	X	R	X
13	-7.70	.005	-7.70	-.005	-7.703	-.005
14	.006	.48	.122	.475	.391	.434
15	-.124	.434	.091	.418	-.06	.247
16	.084	.374	.078	.348	.069	.352
17	.068	.371	.072	.351	.076	.348
18	.072	.356	.0742	.341	.074	.351
19	.0695	.344	.0708	.348	.071	.353

FAULT OCCURS ON SAMPLE NO.13
(Per Unit Impedence)

TABLE 5 IMPEDENCE ESTIMATE FOR CLOSE IN GROUND SWITCH FAULT

Sample No.	Fourier R	Fourier X	Spectral Observer (Poles at zero) R	Spectral Observer (Poles at zero) X
24	-4.07	-1.19	-4.14	-1.00
25	- .88	-4.86	1.6	.208
26	.58	.37	.36	.174
27	.17	.21	.113	.118
28	.039	.105	.009	.066
29	.038	.102	.007	.02
30	.031	.098	.005	.011
31	.024	.095	.004	.001
32	.015	.057	.001	-.003
33	- .035	.0358	- .015	-.008
34	- .023	- .0212	- .011	-.003
35	- .012	- .0156	- .003	-.021
36	- .008	- .0142	- .002	-.011
37	- .0109	- .01237	- .003	-.014

FAULT OCCURS ON SAMPLE NO.25
(Per Unit Impedence)

TABLE 6 IMPEDENCE ESTIMATE FOR BLUE PHASE LIGHTNING FAULT

Sample No.	Fourier R	Fourier X	Spectral Observer (Poles at zero) R	Spectral Observer (Poles at zero) X
24	-4.70	-1.93	-4.80	-1.79
25	-4.49	-2.66	-4.58	-1.72
26	-2.82	-1.71	.457	-1.24
27	1.42	.239	.421	- .04
28	.865	.386	.357	.078
29	.446	.598	.303	.102
30	.352	.613	.224	.143
31	.137	.663	.163	.289
32	.166	.531	.110	.311
33	.174	.44	.072	.348
34	.153	.410	.076	.351
35	.141	.326	.068	.368
36	.112	.346	.072	.361
37	.0928	.342	.078	.358

FAULT OCCURS ON SAMPLE NO.25
(Per Unit Impedence)

CONCLUSIONS

The paper presents a new spectral observer algorithm for digital impedance protection of transmission lines. Harmonic smoothing, filtering, and prediction are easily implemented with a flexible signal model to fit the incoming signal samples. The spectral observation technique produces fault detection and tripping timings within 1/2 cycle to 3/4 of a cycle based on 60 Hz waveform.

The algorithm is recursive in nature and lacks restrictions to sampling rate and a suitable plecement of observer eigenvalues can enhance the speed of operation. The restrictions on data window and time reference of computations are not there providing flexibility of computations. Signal samples placed arbitrarily in time can be used for fault detection and result in inexpensive A/D converters without S/H devices.

Further the spectral observer presented in this paper is general enough to be extended, applied in situations such as transformer differential faults and generator assymetrical faults. A future paper will present real time implementation of this algorithm on a 16-bit microcomputer (LSI-11/23 Microcomputer).

Acknowledgement

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REFERENCES

1. B.J.Mann, and I.F.Morrison, 'Digital calculation of impedance for transmission line protection', IEEE Trans. on PAS, Vol.PAS-90, No.1, Jan. 1971, pp.270-278.
2. J.Carr and R.V.Jackson, 'Frequency domain analysis applied to digital transmission line protection', IEEE Trans. on PAS, Vol.PAS-94, No.4, 1975, pp.1157-1166.
3. A.G.Phadke, T.Hibka, and M.Ibrahim, 'Digital computer system for EHV substations', IEEE Trans. on PAS, Vol.PAS-95, No.1, 1976, pp.291-301.
4. G.S.Hope, O.P.Malik, and P.K.Dash, 'A new algorithm for impedance protection of transmission lines', Paper No.A 76 413-6, Summer meeting of the IEEE PES, Vancouver, Canada, July 1979.
5. P.W.Davall, and G.Au Yeung, 'A software design for a computer based impedance relay for transmission line protection', IEEE Trans. On PAS, Vol.PAS-99, No.1, 1980, pp.235-245.
6. M.S.Sachdev, and M.A.Baribeau, 'A new algorithm for digital impedance relays', IEEE Trans. on PAS, Vol. PAS-98, No.6, 1979, pp. 2232-2240.
7. A.M.Ranjbar, and B.J.Cory, 'An improved method for digital protection of HV transmission lines', IEEE Trans. on PAS, Vol. PAS-91, No.2, 1975, pp.544-550.
8. B.Jeyasurya, T.H.Vu, and W.J.Smolinski, 'Determination of Transient apparent impedances of faulted transmission line', IEEE Trans. on PAS, Vol. PAS-102, No.10, pp.3370-3378.
9. B.Jeysurya, and W.J.Smolinski, 'Identification of a best algorithm for digital distance protectionof transmission lines', ibid, pp.3358-3369.
10. A.A.Girgis, 'A new Kalman filtering based on digital distance relay', IEEE Trans. on PAS, Vol. PAS-101, No.9, 1982, pp.3471-3480.
11. G.Hostetter, 'Recursive discrete Fourier transformation', IEEE Trans. ASSP, Vol.ASSP-28, April 1980, pp.184-190.
12. G.H.Hostetter, 'Fourier analysis using spectral observers', Proc. IEEE, Vol.68, pp.284-285, Feb. 1980.
13. D.G.Luenberger, 'An introduction to observers', IEEE Trans. Automat. Contr., Vol.AC-16, pp.596-602, 1971
14. I.H.Mufti, 'On the reduction of a system to canonical (phase-variable) form', IEEE Trans. Automat. Contr. Vol.AC-10, pp.206-207, Apr.1965
15. D.S.Rane, 'A simplified transformation to (phase-variable) canonical form', IEEE Trans. Automat.Contr. ,Vol.AC-11, pp.608- July, 1966.
16. P.K.Dash, D.K.Panda, 'A new algorithm for Microcomputer based diffeential protection of power transformes', a companion paper submitted to IEEE PAS, 1986 WPM.

APPENDIX - I

The spectral observer may operate with any sampling interval Δt. In general 2N+1 samples are required for convergence to determine the harmonic content of the band limited signal y(t). A general method of gain coefficient calculation is to temporarily transform the

system to the dual of the phase variable canonical form (13),(14),(15).

$$\xi' = Q\,\xi$$

$$\xi'(k+1) = (Q\phi Q^{-1})\,\xi'(k) + Qg\left[y - C^T Q^{-1}\,\xi'(k)\right]$$

$$\gamma(k) = (C^T Q^{-1})\,\xi'(k)$$

where

$$\phi' = Q\phi Q^{-1} = \begin{bmatrix} p_1 & 1 & 0 & \cdots & 0 & 0 \\ p_2 & 0 & 1 & \cdots & 0 & 0 \\ p_3 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ p_{2N} & 0 & 0 & \cdots & 0 & 1 \\ p_{2N+1} & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$C^T Q^{-1} = \left[\,1 \quad 0 \quad 0 \;\cdots\; 0 \quad 0\,\right]$$

and

$$g' = Qg = \left[\,g_1' \quad g_2' \;\cdots\cdots\; g_{2N+1}'\,\right]^T$$

The characteristic equation of ϕ' is easily shown to be

$$Z^{2N+1} - p_1 Z^{2N} - p_2 Z^{2N-1} \cdots - p_{2N} Z - p_{2N+1} = 0$$

The characteristic equation of the transformed feedback system is

$$Z^{2N+1} - (p_1 - g_1')Z^{2N} - (p_2 - g_2')Z^{2N-1} - \cdots$$
$$\cdots - (p_{2N} - g_{2N}')Z - (p_{2N+1} - g_{2N+1}') = 0,$$

each coefficient of which may be chosen at will by appropriate choice of the elements of g'. For a deadbeat error system

$g_i' = p_i$, $i = 1,2,\ldots,2N+1$.

Then $g = Q^{-1} g'$.

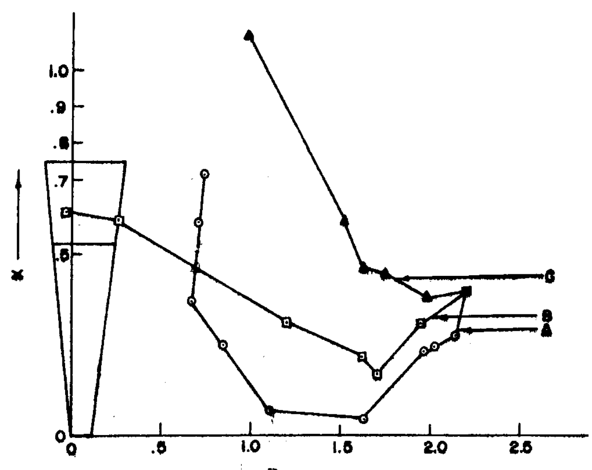

Fig.1(a) Apparent impedance trajectory for yellow to blue phase fault (poles at -.5) N_s =12

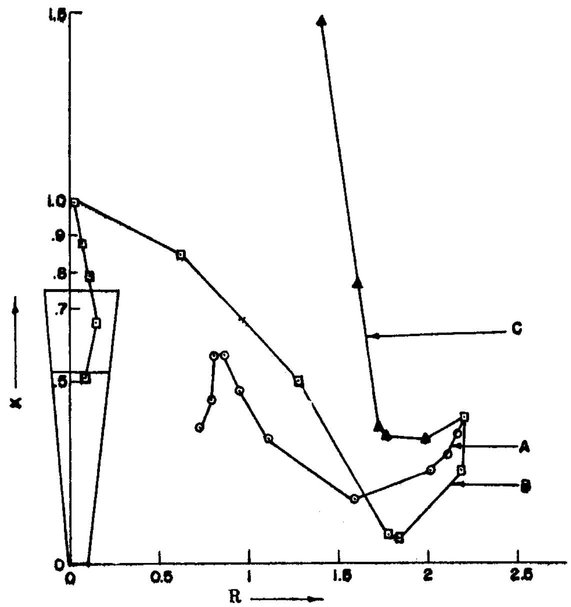

Fig.1(b) Apparent impedance trajectory for yellow to blue phase fault (poles at 0,0), N_s =12

A→ A-B phase relay
B→ B-C phase relay
C→ C-A phase relay

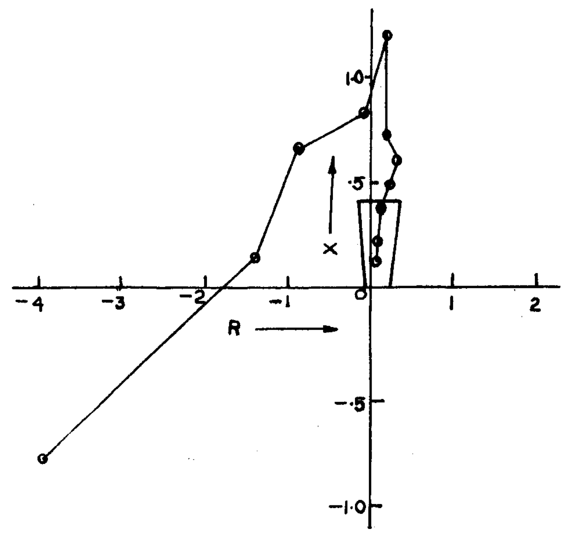

Fig.2 Apparent impedance trajectory for blue phase to ground wire fault(with Nonuniform - sampling rate)

For the example of transmission line protection in the paper the value of Q matrix is choosen in accordance with the formulas given in reference(14). The values of $p_1, p_2, \ldots, p_5$ are then obtained from $Q\phi Q^{-1}$ matrix and the g matrix is obtained as

$$g = Q^{-1}\left[p_1, p_2, p_3, p_4\;,\; p_5\right]^T$$

However, arbitrary pole placement can be achieved by the appropriate choice of the values of $p_1, p_2, p_3, \ldots p_5$.

Discussion

G. D. Rockefeller (Rockefeller Associates, Inc., St. Rose, LA): The authors have not convinced me that their method is superior based on the limited number and scope of test cases studied. Actual faults, two midpoint and one close-in fault cases, and a comparison with just the Fourier algorithm (of a multitude of possible competitive techniques), constitute the authors' substantiation. The spectrum of the inputs and the cutoff frequency of the anti-aliasing filter were not specified. Also, the correct reactance wasn't given for the midpoint cases.

Manuscript received August 5, 1986.

M. A. Rahman and **B. Jeyasurya** (Memorial University of Newfoundland, St. John's, NF, Canada): The authors should be commended for this paper. Using a recursive discrete-time filter, the paper outlines its adaptability for digital protection of transmission lines. Equation (21) provides the values for line impedances, although there is a mix-up of superfix v and i.

Availability of fast microprocessors with built-in large memories makes the spectral observer technique uniquely suitable for relaying in power systems. Field experiences indicate significant noise and irregularity of the sampled signals, particularly arising out of saturations of electromagnetic devices used. One such example is the power transformer. Would the authors expand on the suitability of the spectral observer technique for transformer protections, where nonlinearity and saturation are the norms? The test results of the algorithm are at best elementary. The results are only simulated values for an off-line case. It would be nice if the authors had confirmatory on-line test results.

Manuscript received August 14, 1986.

M. S. Sachdev, J. R. MacCormack, and **M. Nagpal** (University of Saskatchewan, Saskatoon, Sask., Canada): The paper presents the design of a spectral observer and discusses the feasibility of using it for distance protection of transmission lines. The spectral observer design is based on the premise that the signals to be processed consist of voltages and currents of the fundamental frequency and decaying dc components only.

The impedances calculated using the proposed design and the system data collected by the Saskatchewan Power Corporation are listed in Tables 2 through 6. The authors have compared the results with those obtained using the Fourier algorithm. This comparison is not fair because the Fourier algorithms are designed for use when the functions of filtering the components of high frequencies and computing the impedances are to be performed simultaneously. The spectral observer as designed in the paper does not have the filtering qualities that the Fourier algorithm has.

We have repeated the impedance calculations for the case of Table 2 reported in the paper. For this purpose we used the Kalman filtering and the three sample least error squares algorithms and the data used for the study reported in Table 2. The calculated impedances are listed in Table D1.

The impedances listed in the paper are in per-unit values; the base value used is 100 ohms. We fail to find from the paper how this value was arrived at. Leaving this argument aside, we have also used 100 ohms as the base value so that the readers can compare the results for themselves. A study of Tables 2 and D1 and a comparison of the results reveals the following.

1) The calculated values of the resistance and the reactance decrease from large numbers to the final expected values as time elapses after the inception of the fault.
2) One could use the criteria that an estimate is within acceptable limits of accuracy if the calculated value of the reactance is within 10 percent of the final value and the calculated value of the resistance is within 100 percent of the final value. If these criteria are applied to the results presented in Table 2 of the paper, it is found that the first acceptable solutions become available on processing sample Nos. 33 and 32 when the poles of the spectral observer are at 0 and -0.5, respectively. Compared to this, a study of Table D1 reveals that the first acceptable solution becomes available on processing sample 29 when either the Kalman filtering or the three point least error squares algorithm is used. The fault is stated to have occurred at the instant when sample 25 was taken.
3) The calculated values provided by the Kalman filtering algorithm are more stable than the values provided by the least error squares algorithm. This is because the Kalman filtering algorithm is a recursive least error squares algorithm that filters the noise as it processes the input data. The results of the least error squares algorithm can also be improved by using noise filtering techniques, such as running averaging, exponential smoothing, etc.

We have also repeated the impedance calculations for the cases the authors have reported in Tables 5 and 6 of the paper. In these studies we again used the Kalman filtering and the three sample least error squares algorithms and the data used by the authors for those cases. A comparison of the results obtained by using the Kalman filtering and the least error squares approaches with those presented in Tables 5 and 6 also leads to the conclusions that are listed above.

The computations required by the Kalman filtering and the three sample least error squares techniques are substantially fewer than the computations required by the spectral observer approach. It seems to us that the authors have not shown why the proposed approach is superior to the other algorithms that are designed using the same premise the authors have used in the paper.

Manuscript received August 18, 1986.

TABLE D1
Calculated Impedances for Blue-Phase Ground Wire Fault Case when Kalman Filtering and Three Sample Least Error Squares Algorithms are Used

Sample No.	Impedances Calculated Using the Algorithm			
	Kalman filtering		Least Error Squares	
	R	X	R	X
23	-5.910	-0.436	-6.077	-0.255
24	-5.867	-0.418	-5.888	-0.872
25	-6.303	-0.674	-2.989	1.514
26	1.472	-0.724	-0.223	-0.003
27	0.747	0.319	0.038	0.404
28	-0.021	0.498	0.075	0.393
29	0.081	0.334	0.090	0.311
30	0.108	0.358	0.082	0.321
31	0.104	0.341	0.086	0.329
32	0.081	0.327	0.122	0.319
33	0.084	0.331	0.077	0.344
34	0.090	0.331	0.090	0.334
35	0.089	0.332	0.088	0.334
36	0.090	0.339	0.081	0.323
37	0.088	0.333	0.117	0.344
38	0.091	0.338	0.091	0.344
39	0.092	0.339	0.094	0.343
40	0.096	0.337	0.093	0.346

P. K. Dash and **D. K. Panda:** The authors acknowledge with thanks the various discussions made by Mr. Rockefeller, Prof. Sachdev et al., and Prof. Rahman and Prof. Jeyasurya.

Regarding the points raised by Mr. Rockefeller, the authors agree that more numerical tests are required to establish the usefulness of the spectral observer technique vis-a-vis a multitude of other algorithms for impedance protection. The cutoff frequency of the anti-aliasing filter was taken to be 720 Hz and the actual impedance of the protected line is 0.4 pu based on 230 kV and 100 MVA base.

We thank Prof. Sachdev and his group for their interesting contributions. However, we would like to point out here that with a sample rate of 720 Hz, we included 3rd and 5th harmonic signals for computation of impedance without any numerical problem. For example, for a sampling rate of 720 Hz, the values of a_1, b_1, and d_1 are $a_1 = 0.866$, $b_1 = 0.5$ and with 3rd and 5th harmonic signals,

$$\xi_3(k+1) = \xi_4(k) + g_3[y(k\Delta t) - \gamma(k)]$$
$$\xi_4(k+1) = \xi_3(k) + g_4[y(k\Delta t) - \gamma(k)]$$
$$\xi_5(k+1) = .866\xi_5(k) - .5\,\xi_6(k) + g_5\,[y(k\Delta t) - \gamma(k)]$$
$$\xi_6(k+1) = 0.5\xi_5(k) + 0.866\xi_6(k) + g_6\,[y(k\Delta t) - \gamma(k)]$$
$$\gamma(k) = \xi_1(k) + \xi_3(k) + \xi_5(k) + \xi_7(k)$$

Thus the inclusion of harmonics in the model is straightforward. Further, the predicted value of y(k) permits a step ahead to be used to determine the model accuracy, and accordingly the harmonic components can be added or deleted.

We are thankful to Profs. Rahman and Jeysurya for their interesting comments. We, along with Prof. Rahman, are preparing another paper on transformer protection using a spectral observer.

We once again express our deep appreciation for the discussers and accordingly we are trying to do further work to improve the spectral observation algorithm.

Manuscript received October 8, 1986.