International Conference on Recent Advances in Mathematical Sciences
Kharagpur, December 2000

25. APPLICATION OF DIFFERENTIAL GEOMETRY FOR A HIGH PERFORMANCE INDUCTION MOTOR DRIVE

K. B. MOHANTY* **and N. K. DE****
DEPT. OF ELECTRICAL ENGG.,
IIT KHARAGPUR, W.B.-721302, INDIA

ABSTRACT: An attempt has been made in the present work to improve the dynamic and steady state performances of the vector controlled induction motor drive. Concepts of differential geometry has been used for input-output linearization of induction motor drive and to decouple the flux and torque loop. State feedback controller has been designed to obtain desired dynamic and steady state responses. Simulation results show that the performance of the drive system with the designed controller is comparable to that with vector controller and the proposed drive system is more flexible.

Keywords: Input-output linearization, Input-output decoupling, State feedback controller, Induction motor drive.

1. INTRODUCTION

Induction motors (IM) fulfill the *de facto* industrial standard, because of their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, their control is a challenging task. The control of IM in field coordinates using vector control (also known as field oriented control) as proposed by Blaschke [1] and Hasse [2], leads to decoupling between the flux and torque, thus, resulting in improved dynamic torque and speed responses. Significant advances have been made in vector control of

---

* Mr. Mohanty is QIP Research Scholar (on study leave from REC, Rourkela) and
** Dr. De is Professor

induction motors since then. A universal field oriented controller has been developed [3] for an IM to achieve decoupling between flux and torque in any arbitrary flux reference frame and the corresponding decoupling network has been designed, both for flux feedback (direct) [1] and flux feedforward (indirect) [2] control.

A disadvantage of the field-oriented controller [1] is that the method assumes that the magnitude of the rotor flux is regulated to a constant value. Therefore, the rotor speed is only asymptotically decoupled from the rotor flux. The concepts of differential geometry [4 --5] have found considerable use in the development of control techniques for multivariable nonlinear systems. Such schemes have resulted in solutions to several problems, including feedback linearization, input-output linearization and decoupling, and disturbance decoupling. Following Krzeminski [6], Marino et. al. [7] developed a voltage command input-output linearization controller, which decouples the rotor flux and speed, based on a nonlinear transformation performed on the state variables. Kim et. al. [8] have reported a current command input-output linearization controller. A new approach is presented in this paper based on [4-5,13] for input-output linearization and decoupling control of induction motor, and also to improve the dynamic torque and speed responses for high performance motion control applications.

In section 2, the induction motor model is reviewed. The model is simplified for alignment of d-axis with the rotor flux. In section 3, the induction motor model is linearized by input-output linearization technique. Linear state feedback controllers are synthesized to obtain good dynamic and steady state response. Simulation results are discussed in section 4.

2. INDUCTION MOTOR MODEL

From the voltage equations of the induction motor in the synchronously rotating d-q axes reference frame, the state space model with stator current and rotor flux components as state variables is :

$$\frac{d}{dt}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} i_s \\ \psi_r \end{bmatrix} + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} v_s \tag{1}$$

where, $i_s = y = [i_{ds} \quad i_{qs}]^T$, $\psi_r = [\psi_{dr} \quad \psi_{qr}]^T$, and $v_s = [v_{ds} \quad v_{qs}]^T$.

$A_{11} = -a_1 I - \omega_e J$, $A_{12} = a_2 I - P a_3 \omega_r J$, $A_{21} = a_5 I$,

$A_{22} = -a_4 I - (\omega_e - P\omega_r) J$, $B_1 = c I$.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, and $J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

$c = L_r/(L_s L_r - L_m^2)$, $a_1 = c R_s + c R_r L_m^2 / L_r^2$, $a_2 = c R_r L_m / L_r^2$,

$a_3 = c L_m / L_r$, $a_4 = R_r / L_r$, $a_5 = R_r L_m / L_r$,

R_s , R_r , L_s , L_r ,L_m : Motor parameters (given in appendix), P : Number of pole pairs,

ω_r : Mechanical rotor angular velocity, ω_e : Fundamental supply frequency,

v_{ds} , v_{qs} : d-q axes stator phase voltages, i_{ds} , i_{qs} : d-q axes stator phase currents,

ψ_{dr} ,ψ_{qr} : d-q axes rotor fluxes.

The torque developed by the motor is :

$$T_e = K_t (\Psi_{dr} i_{qs} - \Psi_{qr} i_{ds}) \tag{2}$$

where, $K_t = 3 P L_m / 2 L_r$.

The conditions required for decoupling control [9] are :

$\Psi_{qr} = 0$ and $\dot{\Psi}_{qr} = 0$.

From (1),

$\dot{\Psi}_{qr} = a_5 i_{qs} - (\omega_e - P\omega_r) \Psi_{dr} - a_4 \Psi_{qr}$.

Decoupling is obtained, when

$$a_5\, i_{qs} = (\omega_e - P\,\omega_r)\,\Psi_{dr}$$

$$\text{or, } \omega_e = P\,\omega_r + a_5\, i_{qs} / \Psi_{dr} \qquad (3)$$

When (3) is satisfied, the dynamic behaviour of the induction motor is :

$$\dot{i}_{ds} = -a_1\, i_{ds} + a_2 \Psi_{dr} + \omega_e\, i_{qs} + c\, v_{ds} \qquad (4)$$

$$\dot{i}_{qs} = -\omega_e\, i_{ds} - a_1\, i_{qs} - P\, a_3\, \omega_r\, \Psi_{dr} + c\, v_{qs} \qquad (5)$$

$$\dot{\Psi}_{dr} = -a_4\, \Psi_{dr} + a_5\, i_{ds} \qquad (6)$$

$$T_e = K_t\, \Psi_{dr}\, i_{qs} \qquad (7)$$

Even in the IM model described by (4-7) nonlinearity and interaction exist.

3. CONTROLLER DESIGN

A. INPUT-OUTPUT LINEARIZATION

The concept behind field oriented control is that rotor flux can be controlled according to (6), with i_{ds} acting as the control input. The q-axis current component i_{qs} serves as an input in order to control the torque (7) as a product of ψ_{dr} and i_{qs}. But transition from field oriented voltage components, v_{ds} and v_{qs} to current components as in (4) and (5) involves leakage time constants and interactions. The interaction between current components and nonlinearity in the overall system is eliminated by using the input-output linearization approach given in [4-8].

Let the developed torque, T_e be chosen as a variable in place of i_{qs} in the induction motor model. Differentiating both sides of (7) and simplifying with appropriate substitutions:

$$\dot{T}_e = -(a_1 + a_4)\, T_e + K_t \Psi_{dr} \left[c\, v_{qs} - P\, \omega_r (i_{ds} + a_3 \Psi_{dr}) \right] \qquad (8)$$

The nonlinear control laws are chosen as

$$u_1 = \omega_e \, i_{qs} + c \, v_{ds} \tag{9}$$

$$u_2 = K_t \, \Psi_{dr} \left[c \, v_{qs} - P \omega_r \, (i_{ds} + a_3 \, \Psi_{dr}) \right] \tag{10}$$

The induction motor model is now decoupled into two linear subsystems :

(1) electrical and (2) mechanical.

(1) Electrical subsystem:

$$\dot{i}_{ds} = -a_1 i_{ds} + a_2 \psi_{dr} + u_1 \tag{11}$$

$$\dot{\psi}_{dr} = -a_4 \psi_{dr} + a_5 i_{ds} \tag{12}$$

(2) Mechanical subsystem:

$$\dot{T}_e = -(a_1 + a_4) \, T_e + u_2 \tag{13}$$

$$\dot{\omega}_r = (T_e - T_l - \beta \, \omega_r) / J \tag{14}$$

The transformed model given above is valid only for $\psi_{dr} \neq 0$. Since the induction motor system described by the above four equations is linear and decoupled, the developed torque (or the speed) and the rotor flux are independently controlled. Linear control theories are used to obtain desired steady state and transient performance. Here, the linearizing control inputs, u_1 and u_2 are derived using State Feedback Controllers (SFC). The block diagram of the electrical subsystem with SFC is shown in Figure 1. The block diagram of the mechanical subsystem with SFC can be drawn in a similar way.

B. DESIGN OF STATE FEEDBACK CONTROLLER (SFC)

In a regulator problem, the output variables are regulated to the set points (references) by means of state feedback. The technique proposed in [10] is employed to derive an augmented model of multivariable system from the linearized state space equation. A stable linear state feedback control law of the form

$$u = -K_p\ x + K_i \int_0^t (y_r - y)dt \tag{15}$$

can be designed for the augmented system by the pole placement technique [11]. The above control law comprises of the feedback of the sates (first term) as well as the integral of the output errors (IOE) (second term) and does not require the knowledge of the disturbance vector. The IOE feedback makes the controller fairly robust by making it insensitive to modeling imperfections and step like disturbances.

For the electrical subsystem, the control law is

$$u_1 = -K_{p1}\ i_{ds} - K_{p2}\ \Psi_{dr} + K_{i1} \int_0^t (\ \Psi_{dr}^* - \psi_{dr}\ )\ dt \tag{16}$$

The control law for the mechanical subsystem is

$$u_2 = -K_{p3}\ T_e - K_{p4}\ \omega_r + K_{i2} \int_0^t (\ \omega_r^* - \omega_r\ )\ dt \tag{17}$$

In the above two control laws, the proportional gains K_{p1} , K_{p2} , K_{p3} , K_{p4} and the integral gains K_{i1} , K_{i2} are determined using the pole placement technique [11-12].

The eigenvalues of the augmented system matrix of the electrical subsystem are –288.55, –10.22 and 0.0. To place the closed loop poles of the electrical subsystem at –288.55, –20 and –20, the gains of the state feedback controller are, K_{p1} = 29.78, K_{p2}= 2130.2 and K_{i1} = 28,922.

The eigenvalues of the augmented matrix of the mechanical subsystem are –298.77, –0.34 and 0.0. To place the closed loop poles of the mechanical subsystem at –298.77, –10 and –8, the gains of the state feedback controller are, K_{p3} = 17.66, K_{p4} = 47.1 and K_{i2} = 210.33. The simulation results of the closed loop with these controller gains are presented in the next section.

4. SIMULATION RESULTS AND DISCUSSIONS

The drive system with the controller has been simulated using MATLAB. One case of simulation result is discussed here. The reference speed is increased from 1000 r/min to 1300 r/min and then decreased to 800 r/min after 1 second. Reference flux linkage and load torque are kept constant at 0.45 V.s and 1 N.m, respectively. The simulation results are shown in Fig. 2. As seen from the speed response (Fig. 2a), speed settles to 5% of reference speed within 0.5 sec and there is no overshoot of speed at all. d-axis stator current (Fig. 2b) and d-axis rotor flux linkage (Fig. 2c) are constants, which proves decoupling of speed and flux. Torque and q-axis stator current undergo similar variations, which shows that torque is proportional to q-axis stator current. Stator voltages (Fig. 2d) are sinusoidal. The fact that, speed response does not have any overshoot, is a great advantage of this controller. Speed response also tracks the command value very fast. The resposes are better than vector controlled IM drive and this is achieved when the motor is fed from a Voltage Source Inverter (VSI). Fast torque response is another advantage of this system.

5. CONCLUSION

A controller for induction motor drive using input-output linearization and decoupling technique is developed. This is an improvement of conventional vector controlled IM drives. State feedback controllers are used to control the dynamic and steady state responses of flux, speed, torque or current, independent of each other. The simulation results show that the dynamics are controlled in about 0.5 second, fast for such systems, and also the voltage and current variations are within limits. There is no overshoot in speed at all. The implementation is underway in the laboratory.

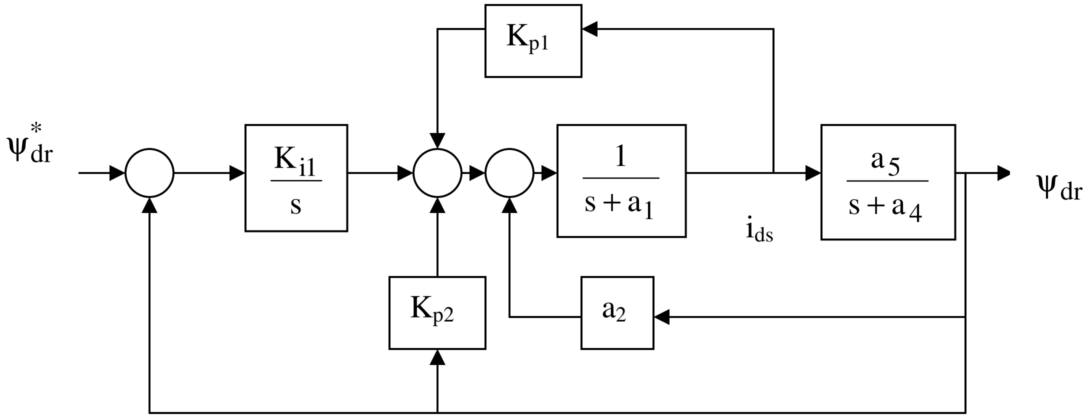

Fig. 1. Block diagram of the electrical subsystem with SFC

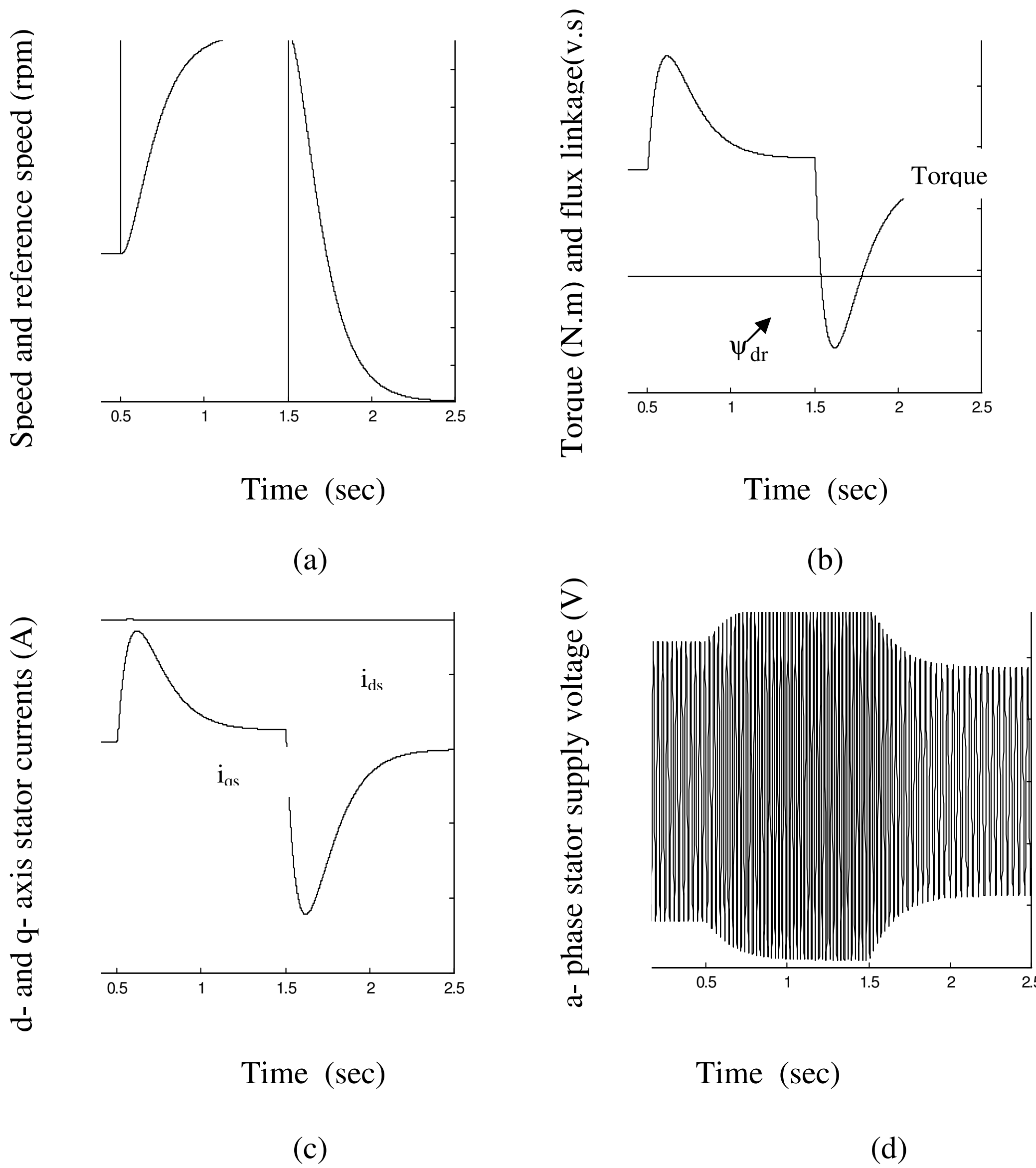

Fig. 2. Step change in speed command (a) Speed response, (b) Torque and flux linkage, (c) d- and q- axis stator current, (d) a- phase stator supply voltage.

6. APPENDIX

Rating and parameters of the induction motor are: 0.75 kW, 3-phase, 220 V, 3 A, 50 Hz, 1440 r/min, Stator resistance, $R_s = 6.37\ \Omega$, Rotor resistance, $R_r = 4.3\ \Omega$

Mutual inductance, $L_m = 0.24$ H, Stator/Rotor leakage inductance $= 0.02$ H

Stator/Rotor self-inductance, L_s, $L_r = 0.26$ H

Moment of inertia (motor & load), $J = 0.01$ kg·m^2

Damping coefficient, $\beta = 0.003$ N·m·s/rad

7. REFERNCES

[1] F. Blaschke, “The principle of field orientation as applied to the new TRANSVEKTOR closed-loop control system for rotating-field machines,” Siemens Review, vol. 39, no. 5, May 1970, pp. 217-220.

[2] K. Hasse, “On the dynamic behavior of induction machines driven by variable frequency and voltage source,” ETZ Arch. Bd. 89, H. 4, 1968, pp. 77-81.

[3] R. W. De Doncker, and D. W. Novotny, “The universal field oriented controller,” IEEE Trans. on Industry Applications, vol. 30, no. 1, February 1994, pp. 92-100.

[4] A. Isidori, A. J. Krener, C. Gori-Giorgi, and S. Monaco, “Nonlinear decoupling via feedback: A differential-geometric approach,” IEEE Trans. on Automatic Control, vol. 26, 1981, pp. 331-345.

[5] T. J. Tarn, A. K. Bejczy, A. Isidori and Y. Chen, “Nonlinear feedback in robot arm control,” Proceedings of 23rd Conference on Decision and Control, December 1984, pp.736-751, CH2093-3/84.

[6] Z. Krzeminski, “Nonlinear control of induction motor,” IFAC 10th World Congress on Automatic Control, vol. 3, Munich, 1987, pp. 349-354.

[7] R. Marino, S. Peresada, and P. Valigi, “Adaptive input-output linearizing control of induction motors,” IEEE Trans. on Automatic Control, vol. 38, no. 2, 1993, pp. 208-221.

[8] G. S. Kim, I. J. Ha and M. S. Ko, “Control of induction motors for both high dynamic performance and high-power efficiency,” IEEE Trans. on Industrial Electronics, vol. 39, no. 4, Aug. 1992, pp. 323-333.

[9] B. K. Bose, *Power Electronics and AC Drives,* Englewood Cliffs, NJ: 1986, p. 272.

[10] H. W. Smith and E. J. Davison, “Design of industrial regulators”, Proc. IEE, vol. 119, no. 8, August 1972, pp. 1210-1216.

[11] W. H Wonham, “On pole assignment in multi input controllable linear systems,” IEEE Trans. on Automatic Control, vol. 12, December 1967, pp. 660-665.

[12] A. K. Chattopadhyay and N. Meher, “Microprocessor implementation of a state feedback control strategy for a current source inverter fed induction motor drive,” IEEE Trans. on Power Electronics, vol. 4 (2), pp 279-288, April, 1989.

[13] K. B. Mohanty and N. K. De, “Nonlinear controller for induction motor drive,” Proc. of IEEE International Conference on Industrial Technology (ICIT), 2000, pp. 382-387.