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Fuzzy Receiver Design for GSM application

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Abstract

This paper presents the application of nonlinear channel equalizers, based on fuzzy IF-THEN rules, for Global System for Mobile (GSM) communication applications. Two types of fuzzy filters namely Type–1 TSK (Takagi – Sugeno – Kang) fuzzy adaptive filter (FAF) and Type–2 TSK FAF are considered for performance comparison. The equalizers proposed can be trained with very short training signal of 26samples as desired by GSM data format. These equalizer performances are also compared with Bayesian equalizers based on maximum a-posteriori criteria and radial basis function (RBF) network Bit error rate (BER) is considered as performance index. The equalizer proposed here outperforms linear equalizers and other forms fuzzy equalizers in terms of BER at given SNR.

I. Introduction

The Mobile cellular wireless system operates under harsh and challenging channel conditions like multipath propagation with fading, Doppler spread and time dispersion or Delay spread. It is very difficult to recover the original information from the corrupted signals affected by inter-symbol interference (ISI), fading with additive white Gaussian noise (AWGN). The purpose of equalization is to compensate for these channel influences so that the original information can be extracted from the corrupted received signal. Adaptive equalization methods are used in mobile communication receivers for compensating these effects. Additionally, equalization needs to be performed efficiently both in terms of computational complexity and time of operation since it works in real time environment. For this reason equalizer forms the single most computationally complex sub-stem in a mobile communication receiver. The equalizer used is desired to achieve minimum bit error rate (BER) performance. Global System for Mobile (GSM) is a globally accepted standard for digital cellular communication and used in many continents. India has adopted GSM as its mobile cellular communication standard. According to the GSM frame structure [1] the equalizers should be trained within 26 numbers of samples in a frame which consist of 114 data bits.

In digital communication systems neural network equalizers have been used in the

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last one and half decades. Most of the equalizers proposed either used the back propagation neural networks or the radial basis function (RBF) based network with Gaussian kernel. RBF equalizers for GSM applications have been proposed [2]. One uses well-established Back Propagation Algorithm and other receiver based on a partially supervised Self Organizing Map in order to perform effective real time learning. Non-Linear equalizers based on Radial Basis Function (RBF) Network had been used in GSM application [2] which proved to be optimal one in terms of BER performance with acceptable computational complexity.

Following the success of fuzzy logic system in different signal processing applications, fuzzy adaptive filters (FAF) were designed [3]. Initial channel equalizers based fuzzy systems demanded high computational complexity. A new form of implementation of Bayesian equalizer using fuzzy filters proposed in [4] which reduced computational complexity issues. The information to be processed by a FAF is often uncertain due to uncertain linguistic knowledge and uncertain numerical values. In fuzzy concepts such as slowly time varying, moderately time varying, or rapidly time varying, experts may not agree on how to represent these linguistic labels using fuzzy membership functions, which causes linguistic uncertainties. In mobile communication, the mappings between input and output data pairs are uncertain due to channel dynamics. This numerical data uncertainty causes type-1 FAF as proposed in [4] and other nonlinear filters to perform poorly. Linguistic and numerical uncertainties require type-2 FAF [5] (the term Type-2 FAF is termed here in this paper) to handle them where antecedent and consequent membership functions are type -2 fuzzy set. This set was introduced by Zadeh [7] as an extension of the concept of an ordinary fuzzy set. A type-2 membership grade can be any set in [0, 1] — *primary membership* and corresponding to each primary membership, there is a *secondary membership*, which can also be [0, 1] that describes the possibilities for the primary membership which allows to handle linguistic uncertainties.

This paper evaluates the performance of fuzzy implementation of Bayesian equalizers, Type-1 FAF reduced computational complexity, Type-2 FAF with RBF equalizer. The paper also compared the performance of linear equalizer with fuzzy filters.

The paper is organized as follows. Following this introduction, Section II provides the derivation of Bayesian equalizer decision function. Section.III discusses Fuzzy equalizer, which is followed by advantages of Fuzzy equalizer. Fuzzy equalizer training is discussed next. Section.VI discusses the results and the paper paper ends with concluding remarks.

II Bayesian Equalizer Decision Function:

The system model for the problem discussed in this paper is presented in Figure.1. The equalizer uses an input vector $r(k) \in \Re^p$, in the p dimensional space. The term p is the equalizer order (i.e. number of taps in equalizer) and the channel order is n (n+1 taps). The equalizer provides decision function $\Im\{r(k)\}$ based on the input vector, which is then passed through a decision device to provide the estimate of transmitted signal $\hat{s}(k-d)$ where d is the delay associated with equalizer decision. The communication system considered is assumed to be a two level PAM system where the transmitted sequence $s(k)$ is drawn from an independent identically distributed (i.i.d.) sequence comprising of {±1} symbols. How ever this can be generalized to any type of signal constellation.

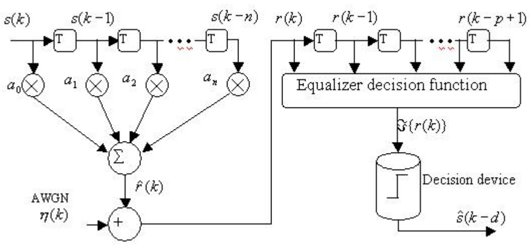

Figure 1. ***Discrete time model of DCS using Transversal Equalizer structure***

The presence of AWGN makes the channel observation vector $r(k)$ a random process having a conditional Gaussian density function centered at each noise free received vector $\hat{r}(k)$. Given this to be the channel state $\hat{r}(k) = c_j$, $1 \leq j \leq N_s$, the conditional probability density distribution of the observed vector is,

$$P(r(k) \mid c_j) = (2\pi\sigma_\eta^2)^{-m/2} \exp\left(-\frac{\|r(k)-c_j\|^2}{2\sigma_\eta^2}\right) \quad \ldots \quad (1)$$

Where $\|.\|$ constitute the Euclidean distance. If Calculating State conditional probability and applying Baye's rule [7] with some mathematical evaluation, we found the decision function of the Bayesian equalizer [4] as

$$\Im\{r(k)\} = \sum_{i=1}^{N_s} w_i \exp\left(-\frac{\|r(k)-c_i\|^2}{2\sigma_\eta^2}\right) \quad \ldots \; (2)$$

Where w_i = +1, if $c_i \in C_d^+$ and w_i = -1, if $c_i \in C_d^-$ and C_d^+, C_d^- represents the positive channel states categories and negative channel state categories respectively.

The Radial Basis Function (RBF) network can implement a mapping f_{rbf} : $r^m \rightarrow$ r by the function,

$$f_{rbf}\,\{x(k)\} = \sum_{i=1}^{N_r} w_i \phi(\|x(k)-\rho_i\|) \quad \cdots \quad (3)$$

Where $x(k) \in r^m$ is the input vector, ϕ(.) is the radial basis function from r^+ to r, w_i, $1 \leq i \leq N_r$ are weights and $\rho_i \in r^m$ are known RBF centers.

It can be realized that the RBF decision function (2.3) by using Gaussian kernel and the Bayesian equalizer decision function of (2.2) are similar. The RBF network can provide a Bayesian decision function by setting RBF centers, ρ_i, to channel states c_i, RBF spread parameter, σ_r^2, to channel noise variance, σ_η^2 and the weights w_i = +1, if $c_i \in C_d^+$ and w_i = -1, if $c_i \in C_d^-$. The function changes for scalar states.

Bayesian decision function is given by

$$f(r(k)= \sum_{i=1}^{n_s} p_i \exp\left(\frac{-\|r(k)-c_i\|^2}{2\sigma_e^2}\right) \quad \ldots \quad (4)$$

where, $n_s = 2^{m+n_h-1}$ are number of channel states with $n_s^+ = n_s^- = n_s / 2$, 'm' is the equalizer order, 'n_h' is the number of taps in the channel and p_i are weights associated with each centers. Rewriting the squared norm of (2.4) as summation and exploiting the properties of exp function making summation into multiplications and changing squared norm $\|.\|^2$ to the absolute distance $|.|^2$, we yield another realization of Bayesian decision function given by a radial basis function network as

$f_{rbf}\{r(k)\}=$

$$\sum_{i=1}^{n_s} p_i \left\{ \prod_{l=0}^{m-1} \exp\left(-\frac{|r(k-l)-c_{il}|^2}{2\sigma_e^{\ 2}} \right) \right\} \quad \dots \ (5)$$

III Fuzzy Implementation (Type 1 FAF)

Wang and Mendel [3] has proposed Type – 1 fuzzy LMS and RLS filters and used them for nonlinear channel equalization. Patra and Mendel has proposed a new realization of fuzzy adaptive filter by decreasing computational complexity and time consumption [4] and used them for fixed linear channel equalization, which we used here for equalization of a mobile channel in GSM environment. Let M = scalar channel states. In this fuzzy filter, setting the membership function centers with scalar channel states, the spread parameter with the channel noise variance and generating the Gaussian membership functions, an equalizer with fuzzy filter is represented by

$$f_k\ (r(k)) = \sum_{i=1}^{n_c} \theta_i \left\{ \prod_{l=0}^{m-1} \psi_l^{\ ij} \right. \qquad (6)$$

Where $\psi_l^{\ ij}$ is the membership function generated by the scalar centers $c_l^{\ ij}$, corresponding to the (j+1) center of the (l + 1) th input scalar by

$$\psi_i^{\ j}(k) = \exp\left\{ -\frac{1}{2} \left(\frac{|r(k-i)-C_j|^2}{\sigma_\eta^{\ 2}} \right) \right\} \qquad \dots \quad (7)$$

Where 1≤ j≤ M and 0 ≤l≤ m-1 and θ_i is a free design parameter of the filter which is adjusted during training. The equalizer input vector can be formed from the time-delayed samples of the received scalar. With this the membership function for input scalar *r (k-1)* will be the delayed membership functions for input *r(k)*. This can be represented as

$$\phi_l^{\ j}(k) = \phi_{l-1}^{\ j}(k-1) \qquad \dots \ (8)$$

Where, 1 ≤l≤ (m – 1) and 0 ≤j≤ M–1. The fuzzy equalizer developed here uses an Fuzzy Basis Function with product inference and Center Of Gravity (COG) defuzifier. Owing to the close relationship of this equalizer with the Bayesian equalizer, this equalizer can also be implemented with an RBF [8] with scalar centers. However, use of a fuzzy system to implement this equalizer provides the possibility of using other forms of inference rules and defuzzification processes. This can provide some of the alternate forms of fuzzy implementation of the Bayesian equalizer, which is discussed in .

For Gaussian membership function generator, output of this generator for any input is 0 ≤ $\phi_l^{\ j}$ ≤ 1 and in product inference case, output of any inference rule will always be less than the smallest membership input to the rule. For this reason, the product inference rule can be approximated by the minimum inference rule by which the decision function will be

$$\Im\{r(k)\} = \sum_{i=1}^{n_s} \left\{ \min_{l=0}^{m-1} \phi_l{}^{ij} \right\} \quad \ldots \qquad (9)$$

Where $\min_{l=0}^{m-1}$ selects the minimum of the inputs to each of the components of the inference block. With this, the computation of the products has been replaced by comparisons, which are easy to implement in hardware. This minimum inference fuzzy equalizer is termed as fuzzy in this thesis.

IV Advantages of fuzzy equalizer

The fuzzy implementation of Bayesian equalizer provides the Bayesian equalizer decision function. Major advantages of such equalizer over the RBF implementation of Bayesian equalizer are lower computational complexity.

Computational Complexity:

Fuzzy implementation of the Bayesian equalizer provides a significant reduction in addition, division and $\exp(x)$ evaluations and the time shift property of the membership function generation provides a considerable reduction in evaluation of functions and division. Evaluation of exp and division functions in a Bayesian equalizer are related to N_S which in turn is exponentially related to the sum of the equalizer and channel order but in the fuzzy equalizer it is related to *M* which is exponentially related to the equalizer order only.

V. Fuzzy equalizer training for mobile communication applications

As described the fuzzy equalizers are developed with the knowledge of channel states, which describes the centers of fuzzy membership functions and the weights, which are free parameters to be updated timely. The knowledge of channel states, which depends upon the channel information, is not known beforehand, so the channel states can be estimated during training period. Considering small training sequence available in GSM frame, the channel model can be identified from the channel outputs available and the known training input information. First the channel is estimated and from the knowledge of channel the states are estimated. The channel is estimated using RLS algorithm during training. With the knowledge of channel, it is straight-forward to find the scalar channel states taking all possible combinations of channel input and the channel states are estimated by the combination of these scalar states [4]. This technique of estimating the channel may not be suitable for channels that are corrupted by non-linearity. Once scalar channel states have been estimated, fuzzy rules can be formed. The equalizer is constructed with the weights of inference rules assigned +1/ -1, depending on whether rule belongs to positive or negative channel states. During the process of detection of samples the channel varies due to fading in a mobile communication system. To compensate for this effect the channel states are continuously modified in a decision directed mode using the estimated samples. The states are updated using LMS algorithm. This provides the equalizer the capability to track the channel variation due to movement of the mobile wrt the base station.

VI. Results and Discussion

The performance of the proposed equalizers was evaluated by computer simulation. During the simulation BER was used as the performance index. This section presents the BER performance of fuzzy equalizers for a variety of parameters. The BER performance of equalizers was computed using Monte-Carlo simulation. During the process 10^5 bits of data were transmitted and BER observed for a variety of AWGN. The BER Vs SNR at receiver input is plotted for performance analysis. All simulations were conducted using PC with MATLAB on WindowsXp operating system on Intel P-IV @ 2.8GHZ HTT processor and 256MB RAM. Uniform random sequences were generated and transmitted through the channel. The channels were affected the ISI, AWGN along with Rayleigh fading with Clarke and Gan's Model [1]. In all cases the data frame as specified by the GSM was used. Only 26 training bits were transmitted and following this 114 information bits were transmitted for detection. This set of 140 bits set considered at the receiver as a frame. During simulation 715 frames were transmitted to allow the transmission of 10^5 numbers of bits.

It may be noted that in all the simulations plots, "Fuzzy" receiver used minimum inference.

Figure.2. presents a typical Rayleigh fading simulated envelope with mobile carrier frequency at 900MHz. The vehicle speed of 120km/hr is considered. This figure describes how the signal level changes with respect to the time elapsed. For this simulation 256 numbers of frequency spacing points were taken. This shows the changes in signal level about its root mean square (rms) values in dB scale for certain time ranges in milliseconds.

In Figure.3, BER performance of a fuzzy receiver was compared with RBF, Bayesian and

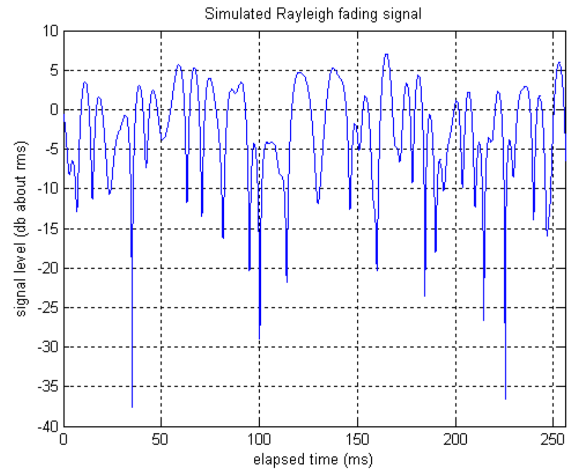

Figure 2. A typical Rayleigh fading envelope

with the linear equalizer (The linear equalizers are trained with LMS and RLS algorithm) algorithms for SNR = 2.5dB to 20dB, using Monte Carlo simulations. Rayleigh fading simulator was used at carrier frequency of 2GHz and mobile speed 13.5 km/hr. Transmitted data rate was conducted at 270.8Kb/sec. Here the equalizer order and the decision delay are 2 and 1 respectively.

Here it is seen that LMS equalizer provides worst performance. The RLS equalizer performs better than LMS since

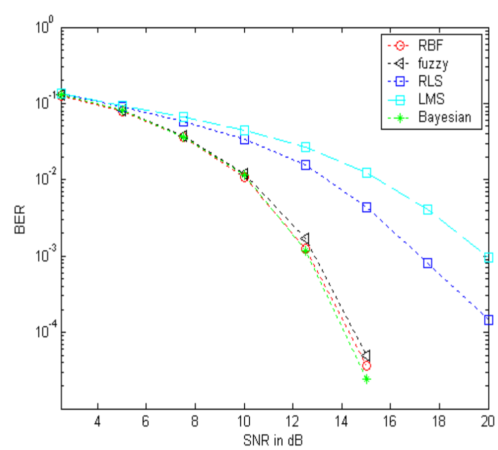

Figure 3 BER plot for $H(z) = 0.5 + 1.0z^{-1}$

the RLS equalizer is able to adjust its parameter very quickly. The Bayesian equalizer and RBF implementation of it provide the optimal performance. The reduced complexity fuzzy equalizer suffers from slight performance degradation compared to optimum equalizer. The fuzzy equalizer provides nearly 5dB performance gain over RLS equalizer and 7dB compared to

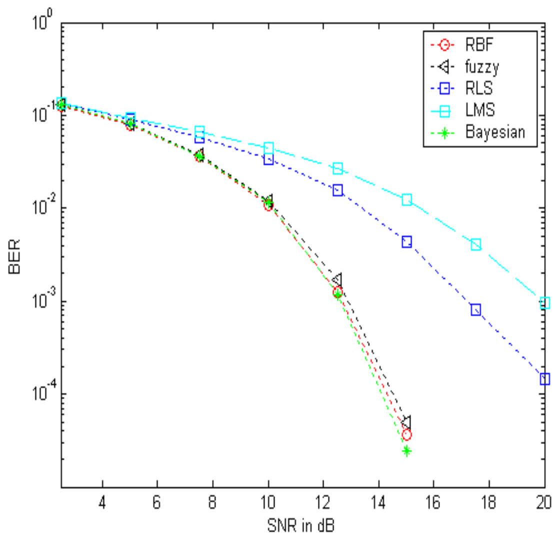

Figure 4. BER with m=3 and d=1 for channel
$\mathrm{H}(z) = 0.26+0.92z^{-1}+0.26z^{-2}$

LMS equalizer at BER of 10^{-3}. In order to investigate, further simulation was conducted and result is presented in Figure.3 for another channel having three multipaths.

In Figure.5., BER performance of a Type-2 Fuzzy Adaptive Filter (FAF) proposed by Mendel [5] was compared with the proposed Type-1 FAF for GSM application. For this simulation all the equalizers are trained for 26 numbers of symbols according to the GSM specification. We observe that the Type-1 FAF proposed here provide nearly 1dB performance gain over Type-2 FAF under GSM environment

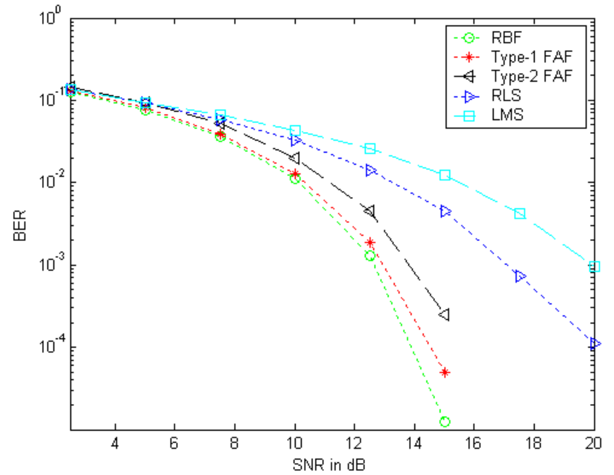

Figure 5. BER for channel with m = 2, d = 1

$H(z) = 0.5 + 1.0z^{-1}$

Mendel [5] was compared with the proposed Type-1 FAF for GSM application. For this simulation all the equalizers are trained for 26 numbers of symbols according to the GSM specification. We observe that the Type-1 FAF proposed here provide nearly 1dB performance gain over Type-2 FAF under GSM environment and nearly 5dB, 7dB compared to RLS and LMS equalizers respectively at BER of 10^{-3}. The Type-2 FAF provides nearly 4dB performance gain over RLS and 6dB over LMS equalizers.

In Figure (5), the performance of proposed Type-1 FAF under different vehicle speeds was evaluated. We observed that the performance of Type-1 FAF degrades by increasing the vehicle speed, which causes the increase in fading by increasing the Doppler frequency shift. It is observed that the performance of Type-1 FAF at vehicle speed of 20km/hr shown better, nearly 2.5dB compared to the vehicle speed at 150km/hr at BER of 10^{-3}.

Conclusion:

The fuzzy equalizer (Type-1 FAF) provides an efficient implementation of the Bayesian equalizer. RBF equalizer (fuzzy implementation of Bayesian equalizer with product inference) and the computationally efficient fuzzy (Type-1) equalizer provide very little performance

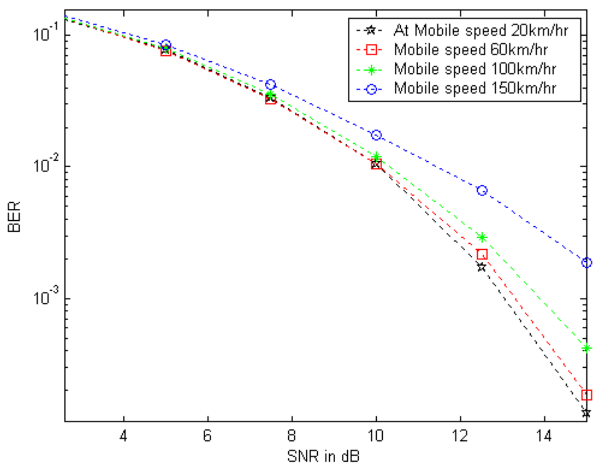

Figure 6. BER performance for Type-1 FAF with m=3, d=1 at different vehicle speeds for channel model $\mathrm{H}(z) = 0.26+0.92z^{-1}+0.26z^{-2}$

difference in terms of BER but the proposed Type-1 FAF shows better performance than Type-2 FAF proposed by Mendel under GSM environment. Type-1 FAF can be trained in 26 training data, which Type-2 could not.

References:

[1] T. S. Rappaport, *Wireless Communications: Principles and Practice.* Pearson Education, 2002.

[2] A. Kantsila, M. Lehtokangas, and J. Saarinen, ”On Radial Basis Function Network Equalization in the GSM System,” *Proceedings of European Symposium on Artificial Neural Networks Bruges* (Belgium), ESANN’2003), pp. 179-184, 23-25 April 2003.

[3] L. X. Wang and J. M. Mendel, ”Fuzzy Adaptive Filters, with Application to Non-Linear Channel Equalization,” *IEEE Transactions of Fuzzy Systems*, vol. 1, pp. 161-170, August 1993.

[4] S. K. Patra and B. Mulgrew, ”Efficient Architecture for Bayesian Equalization Using Fuzzy Filters,” *IEEE Transactions on Circuits and Systems – II*, vol. 45, No. 7, pp. 812-820, July 1998.

[5] Q. Liang and J. M. Mendel, “Equalization of Nonlinear Time-varying Channels Using Type-2 Fuzzy Adaptive Filters,” *IEEE Transactions on Fuzzy Systems*, vol. 8, No. 5, pp. 551-563, October 2000.

[6] L. A. Zadeh, ”The Concept of a Linguistic Variable and its Application to Approximate Reasoning – I,” *Information Sciences*, vol. 8, pp. 199-249, 1975.

[7] R. O. Duda and P. E. Hart, *Pattern Classification and Scene Analysis*. John Wiley and Sons, 1973.

[8] M. J. D. Powell, “Radial basis function for multivariable interpolation: A review,” in Algorithms for Approximation of Functions and Data (J. C. Mason and M. G. Cox, eds.), pp. 143-167, Oxford University press, 1987.