DESIGN OF A FUZZY SLIDING MODE CONTROLLER
FOR A FIELD ORIENTED INDUCTION MOTOR DRIVE

K. B. Mohanty, A. Routray and N. K. De
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur, India-721302
Mr. K. B. Mohanty is on QIP study leave from REC, Rourkela

Abstract: An attempt has been made here to improve the dynamic and steady state performances of the Field Oriented Induction Motor (FOIM) drive. Fuzzy Sliding Mode Controllers (FSMC) and Proportional-cum-Integral (P-I) controllers have been designed separately for both flux loop and speed loop of the FOIM drive system. The performance of the FSMC has been evaluated with respect to the conventional constant gain Proportional-cum-Integral (P-I) controller. The simulation studies demonstrate the superiority of FSMC over the P-I controller.

1. INTRODUCTION

Induction motors (IM) fulfill the de facto industrial standard, because of their simple and robust structure, higher torque-to-weight ratio, higher reliability and ability to operate in hazardous environment. However, their control is a challenging task. One of the classical methods of induction motor control, by now is the field-oriented control as proposed by Blaschke [1]. It leads to decoupling between the flux and torque, thus, resulting in improved dynamic response of torque and speed. For the systems, where model imprecision, parameter fluctuations and noise exist, for them sliding mode control is an appropriate robust control method. The sliding mode control is especially appropriate for the tracking control of robot manipulators and also for motors whose mechanical load change over a wide range. The induction motor drive as a plant is non-linear with imprecise model. Therefore, sliding mode controller is expected to be a better choice. Benchaib et. al. [2] have presented the comparative performance of a sliding mode and an input-output linearizing control scheme for a field oriented induction motor drive. Lin et. al. [3] have developed a robust P-I control scheme with an observer based on model reference adaptive system for a speed-sensorless induction motor drive under direct field oriented control. Shieh and Shyu [4] have applied the Sliding Mode Control philosophy for the torque control with adaptive back stepping. In another interesting application, Park and Lee [5] have combined the theory of input-output linearization and sliding mode control to develop an integrated controller for an induction motor drive under field-oriented control.

It has also been proved that in principle, a fuzzy logic controller (FLC) works like a modified sliding mode controller [6]. In this paper the principle of sliding mode and fuzzy logic are combined together to form a Fuzzy Sliding Mode Controller (FSMC). The need for an advanced and somewhat complex controller can be ascribed to
the poor performance of the conventional constant gain proportional-integral (P-I) controller under varying operating conditions.

2. INDUCTION MOTOR MODEL

From the voltage equations of the induction motor in the synchronously rotating d-q axes reference frame, the state space model with stator current and rotor flux components as state variables is:

\[
\begin{bmatrix}
i_s \\
\psi_f \\
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
i_s \\
\psi_f \\
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
0
\end{bmatrix} v_s
\]

(1)

where, \(i_s = [i_{ds} \quad i_{qs}]^T\), \(\psi_f = [\psi_{dr} \quad \psi_{qr}]^T\), \(v_s = [v_{ds} \quad v_{qs}]^T\),

\(A_{11} = -a_1 I - \omega_c J\),

\(A_{12} = a_2 I - P a_3 \omega_r J\),

\(A_{21} = a_5 I\),

\(A_{22} = -a_4 I - (\omega_c - P \omega_r) J\),

\(B_1 = c I\),

\(I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\),

\(J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\),

\(c = L_r / (L_s L_r - L_m^2)\),

\(a_1 = c R_s + c R_r L_m^2 / L_r^2\),

\(a_2 = c R_r L_m / L_r^2\),

\(a_3 = c L_m / L_r\),

\(a_4 = R_r / L_r\),

\(a_5 = R_r L_m / L_r\).

The torque developed by the motor is:

\(T_e = K_t (\psi_{dr} i_{qs} - \psi_{qr} i_{ds})\)

(2)

where, \(K_t = 3 P L_m / 2 L_r\).

The torque balance equation is:

\(T_e = T_L + J \frac{d\omega_r}{dt} + \beta \omega_r\)

(3)

The conditions required for decoupling control are \(\dot{\psi}_{qr} = 0\) and \(\dot{\psi}_{dr} = 0\).

From (1)

\(\dot{\psi}_{qr} = a_5 i_{qs} - (\omega_c - P \omega_r) \psi_{dr} - a_4 \psi_{qr}\)

(4)

Decoupling is obtained when

\(a_5 i_{qs} = (\omega_c - P \omega_r) \psi_{dr}\)

or, \(\omega_c = P \omega_r + a_5 i_{qs} / \psi_{dr}\)

(5)

When Eq. (5) is satisfied, the dynamic behavior of the induction motor is:

\[
i_{ds} = -a_1 i_{ds} + a_2 \psi_{dr} + \omega_c i_{qs} + c v_{ds}
\]

\(i_{qs} = -\omega_c i_{ds} - a_1 i_{qs} - P a_3 \omega_r \psi_{dr} + c v_{qs}\)

\(\psi_{dr} = -a_4 \psi_{dr} + a_5 i_{ds}\)

(8)

\(T_e = K_t \psi_{dr} i_{qs}\)

(9)

With the presence of a number of factors, the motor model as described by (6-9) turns out to be nonlinear and interactive system and presents a complex control problem. The following sections present the design principle for P-I and fuzzy sliding mode controllers.

3. P-I CONTROLLER DESIGN

Before suggesting more complicated and sophisticated controllers it is necessary to evaluate their performance with respect to conventional P-I controllers. Two P-I controllers are designed separately, for the speed and flux control loops, following the method explained in [7]. The control laws for speed and flux control loops are:

\(v_{qs} = K_{p1} (\omega_{ref} - \omega_r) + K_{i1} \int_0^t (\omega_{ref} - \omega_r) dt\)

(10)

\(v_{ds} = K_{p2} (\psi_{dr} - \psi_{ref}) + K_{i2} \int_0^t (\psi_{dr} - \psi_{dr}) dt\)

(11)

The plant parameters, such as speed, inertia, rotor resistance, inductance and the friction coefficients, are changed within a feasible range to study the root locus and possible instability. The test results for the controller performance have been outlined in section 5.

4. FUZZY SLIDING MODE CONTROLLER DESIGN

In sliding mode controller, the system is controlled in such a way that the error in the system states always moves towards a sliding surface. The sliding surface is defined with the tracking error of the state and its rate of change as variables. The distance of the error trajectory from the sliding surface and its rate of convergence are used to decide the control value. The sign of the control value must change at the intersection of tracking error trajectory with the sliding surface. In this way the error trajectory is forced to move always towards the sliding surface.
Equations (6-9) of induction motor are rewritten in the following way
\[
\dot{X} = F(X) + u + d \tag{12}
\]
where,
\[
X = \begin{bmatrix} \omega_r & \Psi_{dr} & i_{ds} & i_{qs} \end{bmatrix}^T
\]
\[
F(X) = \begin{bmatrix} f_1 & f_2 & f_3 & f_4 \end{bmatrix}^T
\]
\[
u = \begin{bmatrix} 0 & 0 & c & v_{ds} & c & v_{qs} \end{bmatrix}^T
\]
\[
d = [-T_L / J 0 0 0]^T
\]
\[
f_1 = (-\beta \omega_r + K_T \Psi_{dr} i_{qs}) / J
\]
\[
f_2 = -a_4 \Psi_{dr} + a_5 i_{ds}
\]
\[
f_3 = a_2 \Psi_{dr} - a_1 i_{ds} + P \omega_r i_{qs} + a_5 i_{qs}^2 / \Psi_{dr}
\]
\[
f_4 = -P \omega_r (i_{ds} + a_3 \Psi_{dr}) - a_1 i_{qs} - a_5 i_{qs}^2 / \Psi_{dr}
\]

In the present control problem, speed and flux are two state variables, which need to track their respective command values. So two different sliding surfaces are designed with speed error and flux error and their respective rate of changes.

**Speed and Flux Controllers:**

The structures of the speed and the flux controllers are similar and therefore, the discussion of one shall clarify the other.

Let \( e_i = \omega_i - \omega_{ref} \) \tag{13}

Then, the sliding surface is given as
\[
s_i = \lambda_1 e_i + \dot{e}_i \tag{14}
\]
\( \lambda_1 \) is a positive constant, called break frequency of the system.

The condition of sliding mode \([6]\) is
\[
\text{sgn}(s_i) \dot{s}_i \leq -\eta_i \tag{15}
\]
where,
\[
\text{sgn}(s_i) = \begin{cases} 1 & \text{when } s_i > 0 \\ -1 & \text{when } s_i \leq 0 \end{cases}
\]
and \( \eta_i \) is a positive constant for sliding mode to exist.

Simplifying Eq. (15) with proper substitution from Eqs. (12-14), the following equation is obtained.
\[
\text{sgn}(s_i)[(-\dot{f}_1 + K_T \Psi_{dr} f_4 + K_T i_{qs} f_2
\]
\[
+ K_T \Psi_{dr} c v_{qs}) / J + \lambda_1 \dot{e}_1 - \dot{\omega}_\text{ref} \] \leq -\eta_i \tag{16}
\]
Let,
\[
G_1 = (-\dot{f}_1 + K_T \Psi_{dr} f_4 + K_T i_{qs} f_2) / J
\]
and
\[
u_1 = K_T \Psi_{dr} c v_{qs} / J \tag{17}
\]

From the system measurements and/or estimation, let \( G_1 \) be found out as \( \hat{G}_1 \). Then the actual value of \( G_1 = \hat{G}_1 + D_1 \), where, \( D_1 \) represents the uncertainties in the model and measurements. Therefore, Eq. (16) reduces to
\[
\text{sgn}(s_i)[(\hat{G}_1 + D_1 + \lambda_1 \dot{e}_1 - \dot{\omega}_\text{ref}) + \text{sgn}(s_i) u_1 \leq -\eta_i
\]

To achieve the sliding mode of Eq. (15), we choose \( u_1 \) so that
\[
u_1 = (-\hat{G}_1 - \lambda_1 \dot{e}_1) - K_1 \text{sgn}(s_i) \tag{19}
\]

In Eq. (19), the first term is a compensation term and the second term is the controller. \( K_1 \) is a positive constant, which is determined by considering the maximum amount of uncertainty, \( D_1 \) in the estimation process.

\( \lambda_1 \) is determined by considering the sample rate \( (f_{\text{sample}}) \) and the largest time constant \( (\tau_{\text{plant}}) \) of the plant. For quick damping, the value of \( \lambda_1 \) should be large, but should be constrained within.
\[
\lambda_1 < \frac{f_{\text{sample}}}{2(1 + \tau_{\text{plant}} f_{\text{sample}})}
\]

The command value of \( v_{qs} \) is obtained by substituting Eq. (19) in Eq. (17).

In a fuzzy sliding mode controller, the gain, \( K_1 \) of Eq. (19) (sliding mode controller) is determined from a fuzzy rule. The inputs to the fuzzy control block are \( s_1 \) and \( \dot{s}_1 \) (\( s_2 \) and \( \dot{s}_2 \) for flux controller) as shown in Fig. 2. Normalizing gains \( (g_s \text{ and } g_\dot{s}) \) have been used to bring the variation within \( \pm 1 \). Each of these normalized inputs has been fuzzified into three fuzzy sets (N: Negative, Z: Zero, and P: Positive). The membership grades for each of the inputs are shown in Fig. 3. Linear and symmetrical memberships are used for ease of realization in the hardware. Similarly, the fuzzy sets for the output \( K_{in} \) are chosen as: LP: Large Positive, MP: Medium Positive, Z: Zero, MN: Medium Negative and LN: Large Negative. The membership grades for the normalized output have been shown in Fig.
4. The rule base for the fuzzy inference is given in the form of Table I.

From Table I, for output Fuzzy set MN (Medium Negative), there exist two rules, which are:

If \( e_n \) is N and \( r_n \) is Z, then \( K_{ln} \) is MN.

If \( e_n \) is Z and \( r_n \) is N, then \( K_{ln} \) is MN.

Now, given the membership grades of the inputs, the output membership grades are obtained as follows.

\[
\mu_{MN1} = \min[\mu_N(e_n), \mu_Z(r_n)] \quad \text{(Zadeh AND)}
\]

\[
\mu_{MN2} = \min[\mu_Z(e_n), \mu_N(r_n)]
\]

\[
\therefore \mu_{MN} = \max(\mu_{MN1}, \mu_{MN2}) \quad \text{(Zadeh OR)}
\]

Similarly, for the determination of zero membership grades:

\[
\mu_{Z1} = \min[\mu_N(e_n), \mu_P(r_n)]
\]

\[
\mu_{Z2} = \min[\mu_Z(e_n), \mu_Z(r_n)]
\]

\[
\mu_{Z3} = \min[\mu_P(e_n), \mu_N(r_n)]
\]

Now, \( \mu_Z = \max(\mu_{Z1}, \mu_{Z2}, \mu_{Z3}) \).

After obtaining the membership grades of the output the defuzzification is carried out as follows:

\[
\sum_{i=1}^{5} \mu_i K_{ln_i} = \sum_{i=1}^{5} \mu_i K_{ln} (20)
\]

where,

\( \mu_i = i^{th} \) membership value of the output

\( K_{ln_i} = \) the value of the output, where the membership grade for \( i^{th} \) Fuzzy set is 1.0 The actual value of fuzzy gain of the controller, \( K_1 \) is:

\[
K_1 = g_u K_{ln} \quad (21)
\]

where, \( g_u \) = denormalization factor.

The output, \( v_{qs} \) is obtained from the fuzzy sliding mode controller for the speed control loop (Eqs. (19)-(21) and Eq. (17)). The output from the flux control loop is the d-axis command voltage, \( v_{ds} \). These command voltages after necessary transformation, generate the three phase command voltages, \( V_a'(k), V_b'(k) \) and \( V_c'(k) \) (Fig. 1) for the three phase PWM inverter.

5. SIMULATION RESULTS

The drive system is subjected to various changes and disturbances with each of the above controllers. The three phase Induction Motor has the following rating and parameters:

\[ \text{0.75 kW, 220V, 3A, 50 Hz, 1440 rpm.} \]

\[ P = 2, \quad R_s = 6.37 \text{ ohms, } R_r = 4.3 \text{ ohms, } L_s = L_r = 0.26 \text{ H, } L_m = 0.24 \text{ H,} \]

\[ J = 0.0088 \text{ Kg m}^2, \beta = 0.003 \text{ N m s/rad} \]

The conventional P-I controller gains are chosen by the pole placement and then by tuning the gains for the best set of results based on several simulations. Similarly, the parameters of the fuzzy sliding mode controller are found out by optimizing the performance (rise time, overshoot etc) from several simulations. The determination of these parameters is mostly by trial and error and there is no definite criteria for choosing the same.

The following tests have been carried out to determine the drive response.

1. Step decrease in the speed reference (within base speed operation)
2. Speed tracking with flux weakening above base speed

(a) Step Change in Speed

Fig. 5 shows the response of the IM drive, when subjected to a step change in the speed reference from 1000 rpm to 800 rpm. The P-I controller action (Fig. 5a) is instantaneous and exhibits little undershoot. Being a decoupled system, the speed control loop is not supposed to affect the direct axis flux. But the situation is not so in case of drive under P-I controller action. There is about 70% overshoot in the direct axis flux linkages. This results in large variations in the d- and q- axis components of the supply current. It may also lead to large \( \text{di/dt} \) stress on the converter switches.

In case of FSMC (Fig. 5b), the control action is bit slow. But the other state variables including the direct axis flux show small change during the step decrease. Small changes in the d-axis and q-
axis components of the supply current are observed using FSMC.

(b) Speed Tracking

Fig. 6 demonstrates the response of the drive resulting from tracking a given command speed. The command speed is given a step change from 500 rpm to 1000 rpm followed by a linear increase till 2250 rpm. The flux weakening starts automatically after the drive reaches a speed of 1500 rpm. The performance of the P-I controller is not satisfactory as seen from the large spikes in the d- and q-axis components of the supply current (Fig. 6a). The variations are as large as 20 times the rated value.

The superior performance of the FSMC is evident from Fig. 6b. The tracking is satisfactory and there are practically no change in the supply currents in each of these cases. For a step increase of 500 rpm in the reference speed, there is a delay of 0.05 sec for the actual speed to track the reference speed. For a ramp increase of reference speed, thereafter, the actual speed tracks the reference speed without error. Variation in flux linkage is also very small.

6. CONCLUSION

Fuzzy sliding mode controllers and P-I controllers are designed for a field oriented induction motor drive. Simulation results of the field oriented induction motor drive under the influence fuzzy sliding mode controllers and P-I controllers have been compared. The performance of FSMC has been shown to be superior compared to constant gain P-I controller, with the drive system being less oscillatory and also improved dynamic performance.

7. NOMENCLATURE

$R_s, R_r$: stator and rotor resistances (ohms)
$L_s, L_r$: stator and rotor self inductances
$L_m$: magnetizing inductance (H)
$P$: Number of pole pairs,

$\omega_r$: Mechanical rotor angular velocity,
$\omega_e$: Fundamental supply frequency,
$v_{ds}, v_{qs}$: d-q axes stator phase voltages,
i$_{ds}, i_{qs}$: d-q axes stator phase currents,
$\Psi_{dr}, \Psi_{qr}$: d-q axes rotor fluxes (Vs)
$T_L$: Load torque (Nm)
$\beta$: Damping coefficient (Nm s/rad)
$K_1$: Gain of the sliding mode controller
$K_{1n}$: normalized value of $K_1$

8. REFERENCES

Fig. 1 Block diagram of the closed loop field oriented drive system

Fig. 2 Block Diagram of the Fuzzy Logic Controller
Fig. 3 The normalized input membership function

Fig. 4 The normalized output membership function

Table-I
Rule Base

<table>
<thead>
<tr>
<th>$r_n$</th>
<th>$e_n$</th>
<th>P</th>
<th>Z</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>LP</td>
<td>MP</td>
<td>Z</td>
<td></td>
</tr>
<tr>
<td>Z</td>
<td>MP</td>
<td>Z</td>
<td>MN</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>Z</td>
<td>MN</td>
<td>LN</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 5-a  P-I Controller

Fig. 5-b  FSMC

Fig. 5  Step Change in Speed Reference (1000 rpm to 800 rpm)
Fig. 6-a  P-I Controller

Fig. 6-b  FSMC

Fig. 6  Speed Tracking with Flux Weakening