

A CLOSED LOOP OBSERVER FOR ROTOR FLUX ESTIMATION IN INDUCTION MACHINES

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ABSTRACT: Flux estimation in induction machines is presented using observer theory. It is pointed out that estimators presently used in connection with schemes such as field oriented control are typically real time simulation of machine equations, without feedback of any corrective prediction error. Design steps in a closed loop observer with prediction error correction term are presented. It is shown that corrective feedback in closed loop observer speeds up convergence of the flux estimate.

1. INTRODUCTION

Field Oriented control has emerged as an important approach to the control of AC machines, and continues to be discussed and developed in the literature [1]. There are two basic forms of rotor flux field orientation: *direct* field orientation, which relies on direct measurement or estimation of the rotor flux, and *indirect* field orientation, which utilizes an inherent slip relation. Though indirect field orientation essentially uses the command (reference) rotor flux, some recent work using the actual rotor flux has been reported to achieve perfect decoupling.

The implementation of direct field orientation via airgap flux measurement has typically been plagued by the complexities and lack of mechanical robustness associated with intrusive sensors located within machine airgap. Furthermore, a correction is required for the rotor leakage flux if rotor flux field orientation is to be achieved. Estimation rather than measurement of the rotor flux is an alternative approach for both direct and indirect field orientation that has received considerable attention [2-7]. In many popular implementations of field oriented induction machine drives, rotor flux is estimated from the terminal variables such as stator voltage and current, and rotor speed. The task of rotor flux estimation may also be expected to arise in other approaches to control and monitoring of induction machines.

Even though observers in general have been around for several decades [8-9], most of the estimation schemes in use for rotor flux field orientation of induction machine are typically real-time simulations of the dynamic equations governing rotor flux. Technically, the word "Observer" implies an estimator that employs both inputs to the system and feedback control for error correction. In this sense, real-time flux simulators can be termed as "Open loop Observers". Closed loop observers are the estimators using feedback for prediction error correction to improve estimation accuracy and for defining the error correction dynamics. Though present field oriented control schemes based on real-time flux simulators demonstrate high performance, it is believed that there will be situations where improved convergence of the closed loop flux observers itself will be called for. Some of the present flux estimators are, apart from real-time flux simulators, based solely upon Gopinath's minimal order observer theory [8]. But, such observers have also certain shortcomings in the methodology. A method for evaluating the accuracy implications of parameter errors is also lacking in the literature.

In this paper, Section 2 examines the real-time flux simulator based on the equations governing the rotor circuit. The faster convergence of the closed loop flux observers is shown analytically in Section 3 and verified by numerical simulations in Section 4. It should be kept in mind that the basic methodology of observer design is applicable to a much broader class of problems than the particular (and important) one addressed here, and the analysis presented here can be extended to other situations involving estimation and control for electrical machine systems.

2. A REAL TIME FLUX SIMULATOR

Most of the field orientation control schemes, and some other approaches to control and monitoring of induction machine require the rotor flux to be estimated. Most of the existing rotor flux estimation schemes are open loop estimation scheme having *no correction term* derived from the prediction error. This is essentially real-time simulation of the rotor circuit equations. So, they are called real-time flux simulator or open loop flux observer. Consider the idealized two-axis model of a squirrel cage induction machine that is used throughout the literature on field-oriented control. The rotor flux dynamics in this model satisfies

$$\lambda_r' = [(-1/T_r) \mathbf{I} + \omega_r \mathbf{J}] \lambda_r + (M/T_r) \mathbf{i}_s \quad (1)$$

where λ_r and \mathbf{i}_s are two component vectors that constitute the two-axis representations (in stator reference frame) of rotor flux and stator current, respectively. ω_r is the angular velocity of the rotor. T_r is the rotor time constant. M is the mutual inductance between rotor and stator.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \quad (2)$$

The model (1) motivates the following real-time simulation for open loop rotor flux estimation.

$$\hat{\lambda}_r' = [(-1/T_r) \mathbf{I} + \omega_r \mathbf{J}] \hat{\lambda}_r + (M/T_r) \mathbf{i}_s \quad (3)$$

It is assumed that rotor speed and stator current are known quantities, so that the state space model (3) is in principle simply implemented, with analog or digital hardware to perform the required real-time integration of (3). The error in the flux estimate produced by (3) is

$$\mathbf{e} = \hat{\lambda}_r - \lambda_r \quad (4)$$

and is governed by the following state space equation, obtained by subtracting (1) from (3)

$$\mathbf{e}' = [(-1/T_r) \mathbf{I} + \omega_r \mathbf{J}] \mathbf{e} \quad (5)$$

For a given speed waveform ω_r , (5) is a linear system. However, since ω_r is in general a time-varying function, the convergence properties of (5) cannot in general be studied by simply taking the eigenvalues of the matrix in brackets. When the speed ω_r is constant, this matrix becomes constant. Since the eigenvalues of this matrix are $(-1/T_r) \pm j \omega_r$, the two scalar components of \mathbf{e} display an oscillation at the frequency ω_r (the constant rotor speed) that is damped with a time constant T_r (the rotor time constant).

In the case of general, time varying ω_r , the error analysis can be done as follows. Premultiplying both sides of (5) by $2\mathbf{e}^*$, where \mathbf{e}^* denotes transposition of \mathbf{e} , and using the facts that $2\mathbf{e}^*\mathbf{e}' = (\mathbf{e}^*\mathbf{e})'$ and $\mathbf{e}^*\mathbf{J}\mathbf{e} = 0$, we see that

$$(\mathbf{e}^*\mathbf{e})' = -2(\mathbf{e}^*\mathbf{e})/T_r \quad (6)$$

The magnitude of the error thus decays with the time constant of the rotor. This analysis is really the Lyapunov function analysis, with $\mathbf{e}^*\mathbf{e}$ as Lyapunov function.

3. A CLOSED LOOP FLUX OBSERVER

Suppose now it is desired to have a rotor flux estimation that converges faster than the real-time simulator given above. The philosophy underlying observer theory naturally suggests that a corrective signal derived from a prediction error should be added to the estimator in (3). So, this is essentially a closed loop observer. The companion equation to (1), which completes the idealized description of the electromagnetic variables, and which relates stator voltage \mathbf{v}_s (a two component vector) to stator current and rotor flux is

$$\mathbf{v}_s = (M/L_r)\lambda_r' + (\sigma L_s)\mathbf{i}_s' + R_s\mathbf{i}_s \quad (7)$$

where $\sigma = 1 - (M^2/L_s L_r)$ is the leakage coefficient. L_s , L_r and R_s are stator and rotor self inductances, and the stator resistance, respectively. Rotor time constant, $T_r = L_r/R_r$, where R_r is the rotor resistance. If the stator voltage is measured, it can be compared with the stator voltage predicted on the basis of (7), but using $\hat{\lambda}_r'$ instead of λ_r' .

$$\hat{\mathbf{v}}_s = (M/L_r)\hat{\lambda}_r' + (\sigma L_s)\mathbf{i}_s' + R_s\mathbf{i}_s \quad (8)$$

The resulting observer then takes the form

$$\hat{\lambda}_r' = [(-1/T_r)\mathbf{I} + \omega_r\mathbf{J}]\hat{\lambda}_r + (M/T_r)\mathbf{i}_s + \mathbf{G}(\hat{\mathbf{v}}_s - \mathbf{v}_s) \quad (9)$$

where \mathbf{v}_s is measured stator voltage and $\hat{\mathbf{v}}_s$ is the stator voltage computed from (8). \mathbf{G} is a 2×2 matrix of observer gains. A straight forward calculation shows that the system (5) that governed error dynamics of the real-time simulation is now replaced by

$$\mathbf{e}' = [(-1/T_r)\mathbf{I} + \omega_r\mathbf{J}]\mathbf{e} + (M/L_r)\mathbf{G}\mathbf{e}' \quad (10)$$

$$\text{or } \mathbf{e}' = [\mathbf{I} - (M/L_r)\mathbf{G}]^{-1} [(-1/T_r)\mathbf{I} + \omega_r\mathbf{J}]\mathbf{e} \quad (11)$$

It is evident that different choices of the observer gain matrix \mathbf{G} will lead to different error dynamics.

For illustration purpose, let us assume

$$\mathbf{G} = g\mathbf{I} \quad (12)$$

where g is a scalar observer gain parameter. If the rotor speed ω_r is constant, then (11) is a time-invariant linear system, and the eigenvalues that govern it are:

$$(1 - gM/L_r)^{-1} ((-1/T_r) \pm j\omega_r) \quad (13)$$

Thus the eigenvalues of the error dynamics are scaled up by the factor $(1 - gM/L_r)^{-1}$, i.e., the time constant that governs the error decay is scaled down from that of the real-time

simulator by this factor, while the frequency of oscillation in the error decay waveform is scaled up by the same factor. For the more general time varying rotor speed case, we proceed as earlier to find that (6) is replaced by

$$(\mathbf{e}^* \mathbf{e})' = -2(1 - \mathfrak{g} M / L_r)^{-1} (1 / T_r) (\mathbf{e}^* \mathbf{e}) \quad (14)$$

so that the error magnitude now decays with a time constant of

$$(1 - \mathfrak{g} M / L_r) T_r \quad (15)$$

It is evident that \mathfrak{g} can be chosen to make this time constant considerably smaller than T_r . In implementing the closed loop flux observer given by (8) and (9), it is necessary to avoid taking derivatives. For this purpose, let us define the auxiliary variable

$$\mathbf{z} = [\mathbf{I} - (M / L_r) \mathbf{G}] \hat{\lambda}_r - \mathbf{G} (\sigma L_s) \mathbf{i}_s \quad (16)$$

Then \mathbf{z}' is obtained by grouping together all the terms of (9) that contain derivatives (after substituting for $\hat{\mathbf{v}}_s$ from (8)) so that

$$\mathbf{z}' = [(-1/T_r) \mathbf{I} + \omega_r \mathbf{J}] \hat{\lambda}_r + (M / T_r) \mathbf{i}_s + \mathbf{G} (\mathbf{R}_s \mathbf{i}_s - \mathbf{v}_s) \quad (17)$$

Now (16) shows that

$$\hat{\lambda}_r = [\mathbf{I} - (M / L_r) \mathbf{G}]^{-1} [\mathbf{z} + \mathbf{G} (\sigma L_s) \mathbf{i}_s] \quad (18)$$

Expression for $\hat{\lambda}_r$ given by (18) is substituted in (17) to get a state equation for the vector \mathbf{z} , as follows:

$$\mathbf{z}' = \mathbf{F} \mathbf{z} + \mathbf{H} \mathbf{i}_s - \mathbf{G} \mathbf{v}_s \quad (19)$$

where

$$\mathbf{F} = [(-1/T_r) \mathbf{I} + \omega_r \mathbf{J}] [\mathbf{I} - (M / L_r) \mathbf{G}]^{-1} \quad (20)$$

$$\mathbf{H} = \mathbf{F} \mathbf{G} (\sigma L_s) + (M / T_r) \mathbf{I} + \mathbf{G} \mathbf{R}_s \quad (21)$$

The differential equation (19) is solved forward from the initial condition $\mathbf{z}(\mathbf{0})$ obtained from (16), through the choice of $\hat{\lambda}_r(\mathbf{0})$. Then to find $\hat{\lambda}_r$ from \mathbf{z} , only (18) is used. Thus the observer is implemented without differentiating any signal.

3.1 Extension of The Closed loop Observer

The particular gain in (12) was chosen for ease of illustration. However, the case of a more general gain is

$$\mathbf{G} = \mathfrak{g}_1 \mathbf{I} + \mathfrak{g}_2 \mathbf{J} \quad (22)$$

If the rotor speed ω_r is constant, the corresponding error dynamic system is again a time invariant linear system. By proper choice of \mathfrak{g}_1 and \mathfrak{g}_2 , the eigenvalues of the error dynamic system can be placed at any specified pair of conjugate locations. For example, if it is desired to have the eigenvalues of the error dynamic system at $(-a \pm j b)$ (i.e., the error components should display an oscillation at the frequency b rad/sec and damped with a time constant of $1/a$ sec), then the scalar gains \mathfrak{g}_1 and \mathfrak{g}_2 in the observer gain matrix must be:

$$g_1 = \frac{L_r (a - (1/T_r)) (1/T_r) + (b - \omega_r) \omega_r}{M (\omega_r^2 + (1/T_r^2))} \quad (23)$$

$$g_2 = \frac{L_r (a - (1/T_r)) \omega_r + (b - \omega_r) (1/T_r)}{M (\omega_r^2 + (1/T_r^2))} \quad (24)$$

When the rotor speed ω_r is varying slowly, g_1 and g_2 can also be varied as given above to control the variation of error dynamics.

4. SIMULATION RESULTS AND DISCUSSIONS

Above observer design theory is verified by numerical simulations. The induction motor whose rating and parameters are given in Table 1, having a rotor time constant of 0.06 sec is considered for simulation study. The trace in Fig. 1 (a) shows the speed waveform after the motor is started from standstill with a 220 V, 50 Hz, 3 – Phase supply. The trace in (b) is the actual rotor flux component along axis 1. The rotor flux component along axis 2, is very similar to that along axis 1, being approximately phase sifted by 90 degrees. So, rotor flux along axis 2 is not shown. The trace in Fig. 1 (c) is the estimated rotor flux using conventional real time simulator given by eqn (3). The trace in (d) shows the flux estimate produced by the closed loop observer with prediction error correction, given by eqns (8) and (9), with the choice $\mathbf{G} = (L_r / 2 M) \mathbf{I}$ to give an error time constant of $T_r / 2$, according to (14). Fig. 1 (e) plots the error in the conventional real time simulator, and (f) shows error with the closed loop observer. Comparing the plots in (e) and (f), it can be verified that in the later case time constant of error decay is half while frequency of oscillation is twice that in former case. Fig. 2 shows the traces of flux estimation error with another closed loop observer whose gain is given by eqns (22) to (24). The fixed eigenvalues of the error dynamic system are located at $(-80 \pm j120)$. The trace of error when $\mathbf{z}(\mathbf{0}) = [0.2; 0.2]$ is shown in Fig. 2 (a), and when $\mathbf{z}(\mathbf{0}) = [1.0; 1.0]$ is shown in Fig. 2 (b). It is seen from Fig. 2 that the plot of error is of same nature in both the cases, irrespective of initial estimation error. This is because the eigenvalues of the error dynamic system are the same in both the cases.

Table – 1 Rating and Parameters of the Induction Motor

Three phase, 50 Hz, 0.75 kW, 220V, 3A, 1440 rpm
Stator and rotor resistances: $R_s = 6.37 \Omega$, $R_r = 4.3 \Omega$
Stator and rotor self inductances: $L_s = L_r = 0.26 \text{ H}$
Mutual inductance between stator and rotor: $M = 0.24 \text{ H}$
Moment of Inertia of rotor and load: $J = 0.0088 \text{ Kg} \cdot \text{m}^2$
Viscous friction coefficient: $\beta = 0.003 \text{ N} \cdot \text{m} \cdot \text{s/rad}$

5. CONCLUSIONS

Real-time simulator for rotor flux estimation has been discussed. Design methodology for closed loop observer based on prediction error feedback has been presented. It is shown analytically as well as by numerical simulation that the closed loop observer leads to faster convergence. It is also shown that with closed loop observer, the gain matrix can be designed to give the desired error dynamics. There are several interesting directions in which the present study can be extended. Among these are the development of adaptive observers, and nonlinear observers for estimation of flux as well as rotor speed.

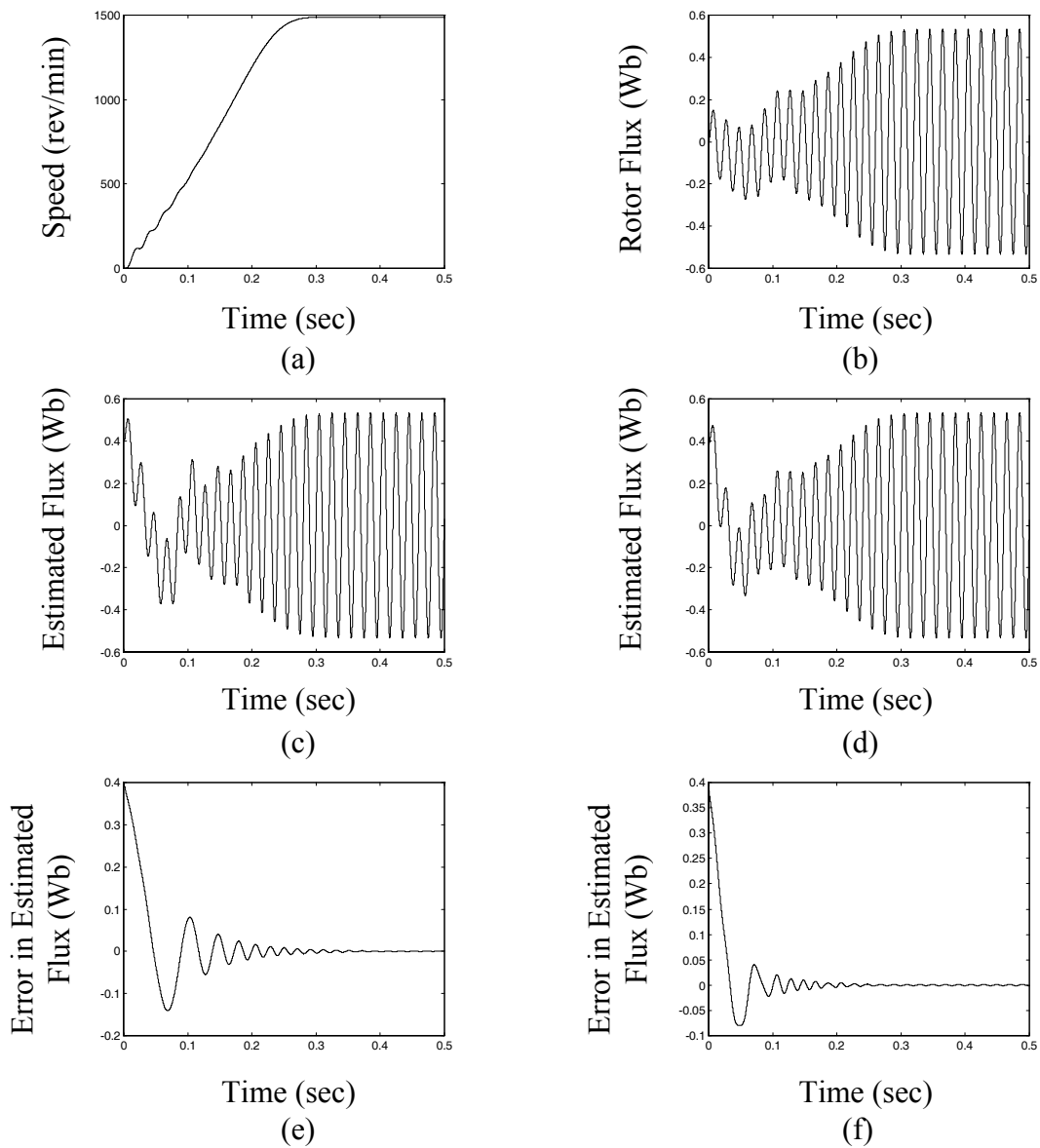


Fig. 1 (a) Rotor speed starting from standstill, (b) Rotor flux along axis 1, (c) Flux estimate based on real time simulator, (d) Flux estimate of a closed loop observer making error decay time constant half of the rotor time constant, (e) estimation error for real-time simulator, (f) estimation error with the closed loop observer

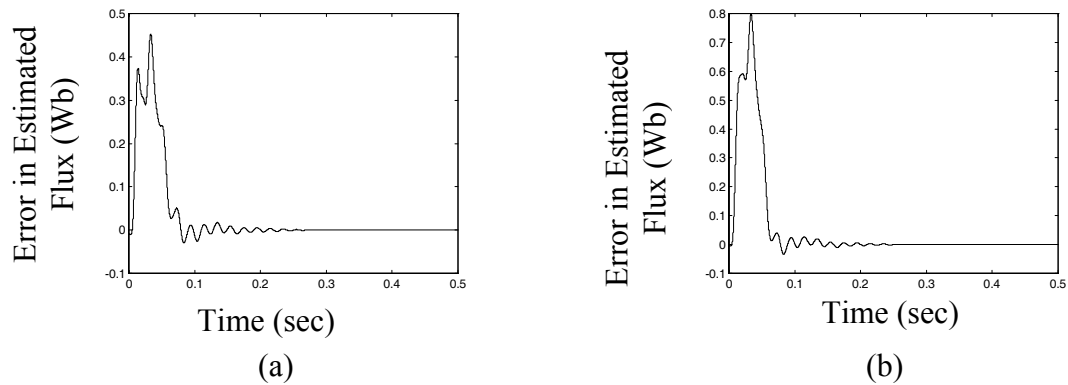


Fig. 2 Estimation error with closed loop observer having fixed eigenvalues of the error dynamic matrix (a) for $\lambda(\mathbf{0}) = [0.2; 0.2]$, (b) for $\lambda(\mathbf{0}) = [1.0; 1.0]$

REFERENCES

1. D. W. Novotny, and R. D. Lorenz (eds.), "Introduction to Field Orientation and High Performance AC Drives", Tutorial book from 1985 & 1986, IEEE-IAS Annual Meetings.
2. Y. Hori, V. Cotter, and Y. Kaya, "A novel induction machine flux observer and its application to a high performance ac drive system", 10th IFAC World Congress on Automatic Control, Munich, Germany, July 1987.
3. G. C. Verghese, and S. R. Sanders, "Observers for flux estimation in induction machines", *IEEE Trans. on Indust. Elect.*, vol. 35, no. 1, Feb. 1988, pp. 85-94.
4. P. L. Jansen, and R. D. Lorenz, "A physically insightful approach to the design and accuracy assessment of flux observers for field oriented induction machine drives", *IEEE Trans. on Ind. Appl.*, vol. IA-30, no. 1, Jan./Feb. 1994, pp. 101-110.
5. H. Kubota, and K. Matsuse, "Flux observer of induction motor with parameter adaption for wide speed range motor drives", *Proc. IPEC – Tokyo, 1990*, pp. 1213-1218.
6. Y. Hori, and T. Umeno, "Flux observer based field orientation (FOFO) controller for high performance torque control", *Proc. IPEC – Tokyo, 1990*, pp. 1219-1226.
7. K. B. Mohanty, "Study of Different Controllers and Implementation for an Inverter Fed Induction Motor Drive", Ph. D. Thesis, IIT Kharagpur, May 2001.
8. B. Gopinath, "On the control of linear multiple input-output systems", *The Bell System Technical Journal*, vol. 50, no. 3, March 1971, pp. 1063-1081.
9. D. G. Luenberger, "An introduction to observers", *IEEE Trans. on Automatic Control*, vol. 16, no. 6, Dec. 1971, pp. 596-602.