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Coordinated Motion Control of a Mobile Manipulator

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Abstract: *The Coordination of the motion planning of the manipulator end point and the mobile base is a crucial task for a mobile robot control. The mobile manipulator is mounted on the mobile platform where the end point of the manipulator is guided to follow an arbitrarily chosen path .The mobile platform may move in such a manner so that the manipulator end may track the arbitrary trajectories. This paper presents a planning and control algorithm for the mobile platform so that the manipulator can track the arbitrary line with minimum error. Simulation studies are given for the verification of the algorithm at the end. The result shows the effectiveness of the motion algorithm.*

1. Introduction:

The Coordination of the motion planning of the manipulator end point and the mobile base is a crucial task for a mobile robot control. If the motion of the manipulator end point is unknown a priori, e.g., driven by a visual sensor or guided by a human operator, the path planning has to be made locally and in real time rather than globally and off line. This paper presents a control algorithm for the platform in the above case, which takes the measured joint displacement of the manipulator as the input for motion planning and controls the platform to bring the manipulator into a preferred operating region. By using this algorithm, the mobile platform will be able to "understand the intention of its manipulator and respond accordingly." Since the mobile platform is subject to nonholonmic constraints, the control algorithm is developed using nonholonomic system theory.

Wheeled mobile robot is a typical example of mechanical systems with nonholonomic constraints. Although navigation and planning of mobile robots have been investigated extensively over the past decade, the work on dynamic control of mobile robots with nonholonomic constraints is much more recent.

We consider mechanical systems that are subject to m velocity level equality type of nonholonomic constraints characterized by

$$B(q)\dot{q} = 0 \qquad (1)$$

where q is the n-dimensional generalized coordinates, B(q) is an m X n dimensional matrix with m X n. Since the constraints are assumed to be nonholonomic, (1) is not integrable. It will be assumed that these constraints are independent. In another words, B(q) has rank m. It is noted that most nonholonomic constraints encountered in mechanical systems, including rolling constraints, are in the form of (1).

Using the Lagrange multiplier rule, the equations of motion of nonholonomically constrained systems are governed by

$$M(q)\ddot{q} + V(q,\dot{q}) + G(q) = E(q)u + B^T(q)\lambda_n \qquad (2)$$

where M(q) is the n x n dimensional positive definite inertia matrix, $V(q,\dot{q})$ is the n-dimensional velocity-dependent force vector, G(q) is the the gravitational force vector, u is the r-dimensional vector of actuator force/torque, E(q) is the n x r dimensional matrix mapping the actuator space into the generalized coordinate space, and _n is an m-dimensional vector of Lagrange multipliers. It has been established that nonholonomic systems described by the motion equation (2) and constraint equation (1) have a canonical state space representation [1], [2]. The state vector consists of the generalized coordinates q and a set of pseudovelocities

$v(t) = [v_1 \quad v_2 \quad \ldots \quad v_{n-m}]$, where n is the number of the generalized coordinates and m is the number of the constraints.

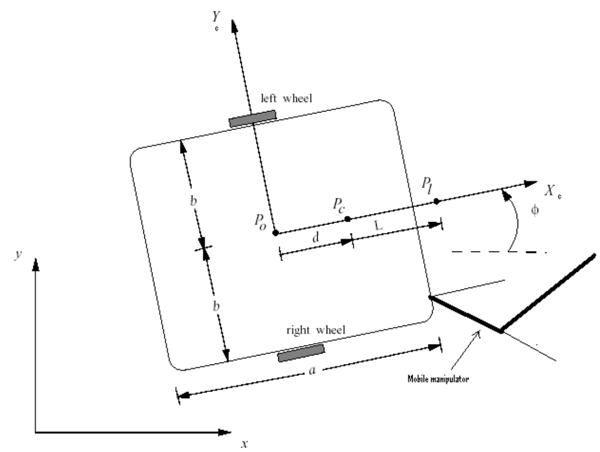

Fig.1. Model of the Mobile Manipulator

2. Statement of the problem

A. Constraint Equation:

According to the above section we could derive the constraint equation for the mobile platform. The platform has two driving wheels (the center ones) and four passive supporting wheels (the comer ones). The two driving wheels are independently driven by the dc motors. The following notation will be used in the derivation of the constraint equations and dynamic equation.

P_0 : the intersection of the axis of symmetry with the driving wheels axis;

P_C : the centre of mass of the platform ;

P_b : the location of the manipulator of the platform;

P_r : the reference point to be followed by the mobile platform;

d: the distance from P_0 to P_C ;

b: the distance between the driving wheels and the axis of symmetry;

r : the radius of each driving wheels and the axis of symmetry;

m_c : the mass of the platform without the driving wheels and the rotors of the dc motors;

m_w : the mass of each driving wheel plus the rotor of its motor;

I_c : the moment of inertia of the platform without the driving wheels and the rotors of the motors about a vertical axis through P_0

I_w : the moment of inertia of each wheel and the motor rotor about the wheel axis .

I_m : the moment of inertia of each wheel and the motor rotor about the wheel diameter.

There are three constraints :

The first one is the platform must move in the direction of the axis of symmetry, i.e.,

$$\dot{y}_c \cos\phi - \dot{x}_c \sin\phi - d\dot{\phi} = 0 \qquad (3)$$

where (x_c , y_c) are the coordinates of the center of mass P_c in the world coordinate system, and ϕ is the heading angle of the platform measured from the X axis of the world coordinates. The other two constraints are the rolling constraints, i.e., the driving wheels do not slip,

$$\dot{x}_c \cos\phi + \dot{y}_c \sin\phi + b\dot{\phi} = r\dot{\theta}_r \qquad (4)$$

$$\dot{x}_c \cos\phi + \dot{y}_c \sin\phi - b\dot{\phi} = r\dot{\theta}_l \qquad (5)$$

where $\theta_r and \theta_r$ are the angular displacement of the right and left wheels, respectively.

Letting $q = (x_{c,} y_c, \phi, \theta_r, \theta_l)$, the three constraints can be written in the form of

$$A(q)\dot{q} = 0 \tag{6}$$

which is according to the (1), where

$$A(q) = \begin{bmatrix} -\sin\phi & \cos\phi & -d & 0 & 0 \\ -\cos\phi & -\sin\phi & -b & r & 0 \\ -\cos\phi & -\sin\phi & b & 0 & r \end{bmatrix} \tag{7}$$

It can be shown that among the three constraints, two of them are nonholonomic and the other one is holonomic[3]. In principle, one can always eliminate variable using holonomic constraints when deriving dynamic equations. Nevertheless, the elimination may be cumbersome in practice. To show the generality of the control method which is able to incorporate both holonomic and nonholonomic constraints, we treat both kinds of constraints in the same way; that is , we do not eliminate variables by using holonomic constraints.

B. *Dynamic Equations*

We now derive the dynamic equations for the mobile platform. The Lagrage equations of motion of the platform with the Lagrange multiplier $\lambda_1, \lambda_2, \lambda_3$

are given by

$$m\ddot{x}_c - m_c d(\ddot{\phi}\sin\phi + \dot{\phi}^2\cos\phi) - \lambda_1 \sin\phi - (\lambda_2 + \lambda_3)\cos\phi = 0 \tag{8}$$

$$m\ddot{y}_c + m_c d(\ddot{\phi}\cos\phi - \dot{\phi}^2\sin\phi) + \lambda_1 \cos\phi - (\lambda_2 + \lambda_3)\sin\phi = 0 \tag{9}$$

$$-m_c d(\ddot{x}_c \sin\phi - \ddot{y}_c \cos\phi) + I\ddot{\phi} - d\lambda_1 + b(\lambda_3 - \lambda_2) = 0 \tag{10}$$

$$I_w \ddot{\theta}_r + \lambda_2 r = \tau_r \tag{11}$$

$$I_w \ddot{\theta}_l + \lambda_3 r = \tau_l$$

where $$m = m_c + 2m_w \tag{12}$$

$$I = I_c + 2m_w(d^2 + b^2) + 2I_m$$

and τ_l and τ_r are the torques acting on the wheel axis generated by the right and left motors, respectively. These five equations of motion can be written in the vector form as

$$M(q)\ddot{q} + V(q,\dot{q}) = E(q)\tau - A^T(q)\lambda \ . \tag{13}$$

This equation is a form of (2) which ignore the gravity element. The matrix $A^T(q)$ has been defined in (7), and the matrices $M(q)$,$V(q,\dot{q})$ and $E(q)$ are given by

$$M(q) = \begin{bmatrix} m & 0 & -m_c d\sin\phi & 0 & 0 \\ 0 & m & m_c d\cos\phi & 0 & 0 \\ -m_c d\sin\phi & m_c d\cos\phi & I & 0 & 0 \\ 0 & 0 & 0 & I_w & 0 \\ 0 & 0 & 0 & 0 & I_w \end{bmatrix}$$

$$V(q,\dot{q}) = \begin{bmatrix} -m_c d\dot{\phi}^2\cos\phi \\ -m_c d\dot{\phi}^2\sin\phi \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad E(q) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Then, we will represent the motion equation (13) and the constraint equation (6) in state space by properly choosing a state vector. To do so, we define a 5 X 2 dimensional matrix $S(q)$ such that $A(q)S(q) = 0$. It is straightforward to verify that the following matrix has the required property

$$S(q) = [s_1(q) \quad s_2(q)] = \begin{bmatrix} c(b\cos\phi - d\sin\phi) & c(b\cos\phi + d\sin\phi) \\ c(b\sin\phi + d\cos\phi) & c(b\sin\phi - d\cos\phi) \\ c & -c \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

where the constant $c = (r/2b)$. From the constraint equation (6), $\dot{q}$ is in the null space of $A(q)$. Because the two columns of $S(q)$ are in the null space of $A(q)$ and are linearly independent, it is possible to express $\dot{q}$ as a linear combination of the two columns of $S(q)$, that is,

$$\dot{q} = S(q)\upsilon \tag{14}$$

The rational behind (14) is to introduce a set of independent velocity variables, υ.

Owing to the choice of $S(q)$ matrix, we have

$$\upsilon = \begin{bmatrix} \upsilon_1 \\ \upsilon_2 \end{bmatrix} = \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix}$$

Differentiating equation (14), substituting the expression for $\ddot{q}$ into (13) and premultiplying it by S^T, we have

$$S^T(MS\,\dot{\upsilon}(t) + M\dot{S}\upsilon(t) + V) = \tau \tag{15}$$

Using the state-space vector $x = [q^T \quad \upsilon^T]^T = [x_{c,} y_c, \phi, \theta_r, \theta_l, \dot{\theta}_r \quad \dot{\theta}_l]^T$, we will be able to represent the constraint and motion equations of the mobile platform in state space.

$$\dot{x} = \begin{bmatrix} S\upsilon \\ f_2 \end{bmatrix} + \begin{bmatrix} 0 \\ (S^T MS)^{-1} \end{bmatrix}\tau \tag{16}$$

where $f_2 = (S^T MS)^{-1}(-S^T MS\upsilon - S^T V)$.

This state equation can be further simplified to

$$\dot{x} = \begin{bmatrix} S\upsilon \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} u \tag{17}$$

by applying the following nonlinear feedback

$$\tau = S^T MS(u - f_2) \tag{18}$$

3. Simulation and Control Algorithm

A. Control Algorithm

According to (17) we could assume a new input u which could linearize the equation (16).

The problem is what is the u 's form. We could differentiate the output

$$y = h(q) = [x_r \quad y_r]^T \tag{19}$$

and with (14) we could have

$$\dot{y} = (\partial h(q)/\partial q)\dot{q} = J_h(S\upsilon) = (J_h S)\upsilon = \Phi\upsilon \tag{20}$$

Φ is called decoupling matrix.

$$\Phi = J_h(q)S(q) = \begin{bmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{bmatrix} \tag{21}$$

$$\Phi_{11} = c((b - y_r^c)\cos\phi - (d + x_r^c)\sin\phi)$$
$$\Phi_{12} = c((b + y_r^c)\cos\phi + (d + x_r^c)\sin\phi)$$
$$\Phi_{21} = c((b - y_r^c)\sin\phi + (d + x_r^c)\cos\phi)$$
$$\Phi_{22} = c((b + y_r^c)\sin\phi - (d + x_r^c)\cos\phi)$$

Then we differentiate the (20) again, we will have

$$\ddot{y} = \dot{\Phi}\upsilon + \Phi\dot{\upsilon}$$

substitute $\ddot{y}$ with v which is the linearized feedback

$$v = \dot{\Phi}\upsilon + \Phi\dot{\upsilon} = \dot{\Phi}\upsilon + \Phi u$$

$$\Rightarrow u = \Phi^{-1}(v - \dot{\Phi}\upsilon) \tag{22}$$

so we could use the desired path y^d to feed back the error $e = y^d - y$

$$\ddot{y} = v = \ddot{y}^d + K_d(\dot{y}^d - \dot{y}) + K_p(y^d - y) \tag{23}$$

from (23) we know the v and then we can get the u and then $\dot{x}$, integral $\dot{x}$ From this we can resolve this problem.

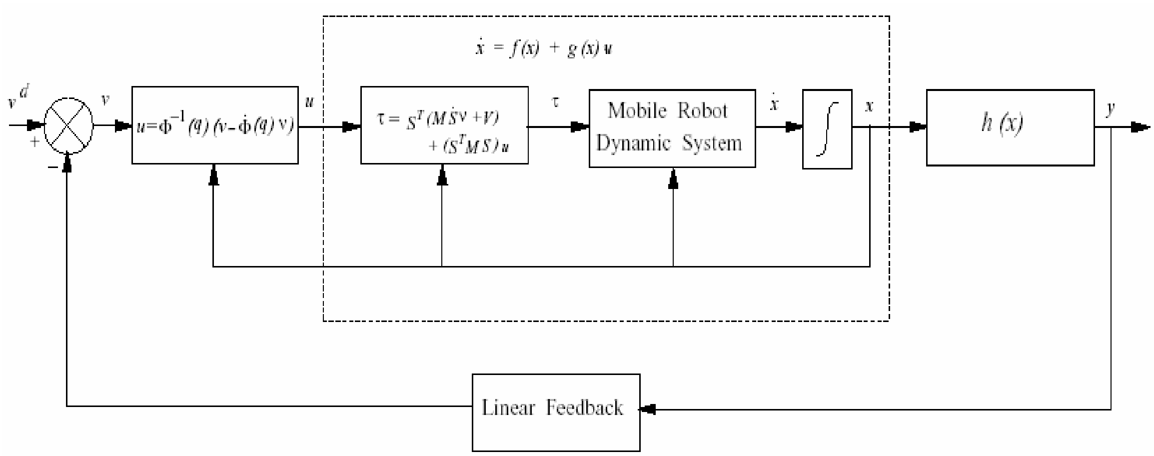

Fig.2: Block diagram of the mobile manipulator

B. Simulation Result discussion

The mobile platform is initially directed toward positive X axis. Two different paths used for the simulation are shown in figure.3.

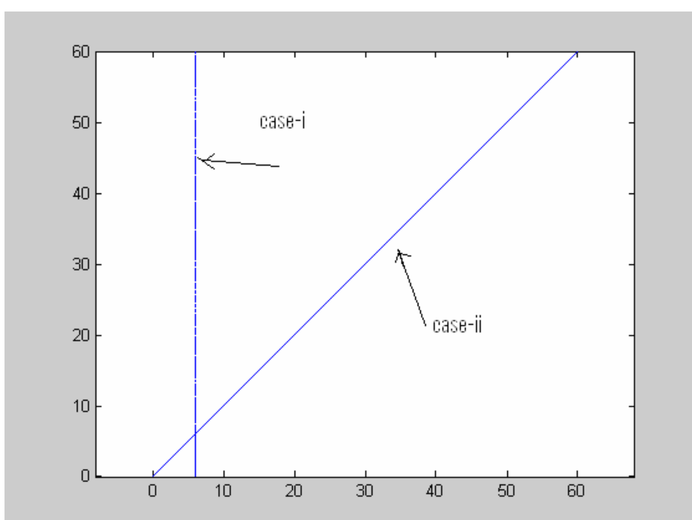

Fig.3: Desired Trajectory of the mobile manipulator

Case i) Straight line perpendicular to the X axis or the initial forward direction of the mobile platform. Figure.3 shows the trajectory of point Pc, in which the litter guy represents the mobile platform. Note that the desired trajectory is given for the reference point Pr, Po has no desired trajectory. We also note the actual trajectory obtained for the reference point coincides with the given desired trajectory.

The velocity of point Po are shown in shown in fig.4 indicates that the mobile platform moved backward (fig.5) for a short period of time at the very beginning to achieve the needed heading angle. Note that the motion of the platform, or more precisely the trajectory the point Po, is not planned. Therefore, the exhibited backward motion is not explicitly planned and is a consequence of the control algorithm. The presence of such backward motion depends on the direction of a desired trajectory, the desired velocity, and the location of the reference point.

Fig 6. shows a little degradation of manipulability measure corresponding to early maneuver by the mobile platform. Overall, we you can see the variable state from fig.7.

Case ii) Straight line -45^0 to the X axis or the initial forward direction of the mobile platform. We see the variable state from fig. 8. Comparison with the case- i is given in the figure 7.

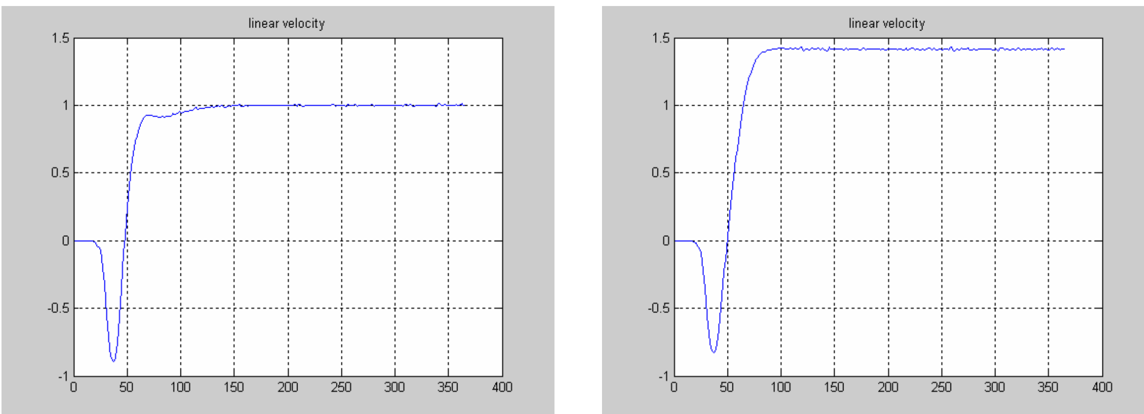

Fig.4. Linear Velocity of Point Po (Case-I and Case- ii)

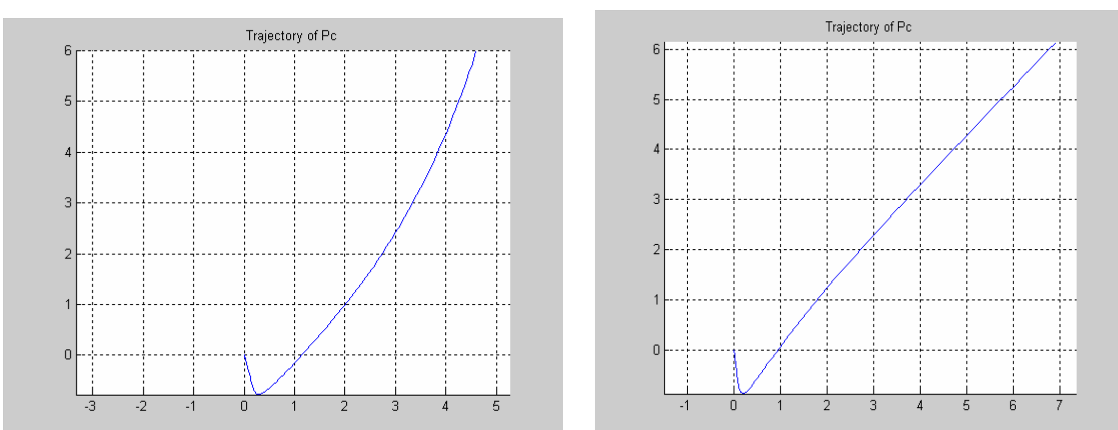

Fig.5. Trajectory of Point Pc (Case-i and Case- ii)

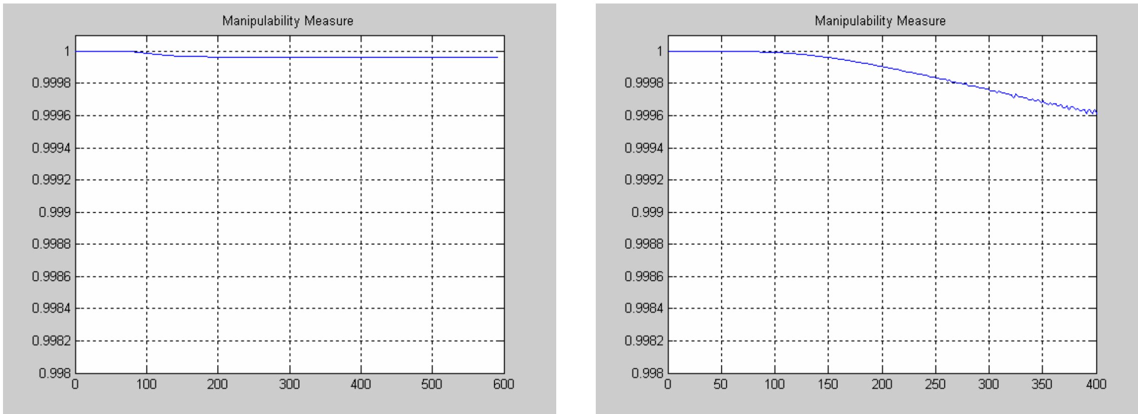

Fig.6. Manipulability Measurement (Case-i and Case- ii)

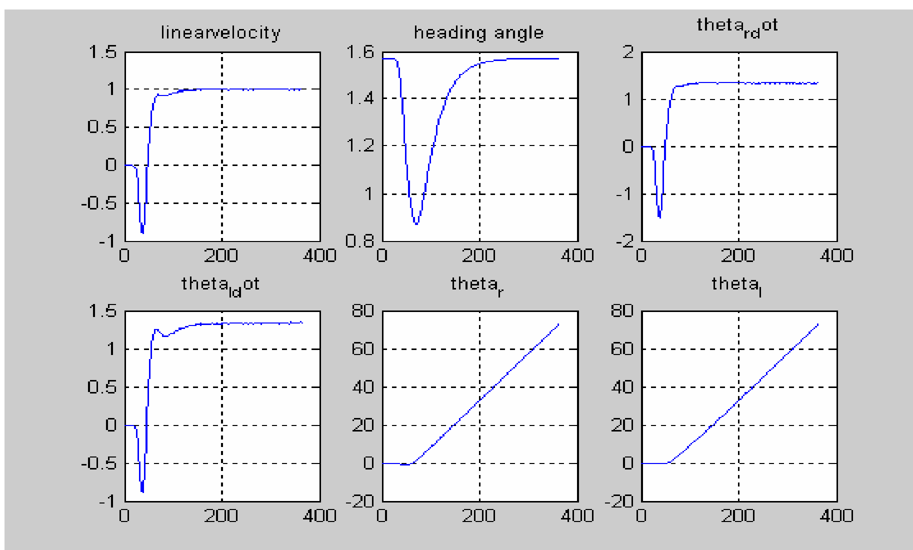

Fig. 7 variable States of Case-i

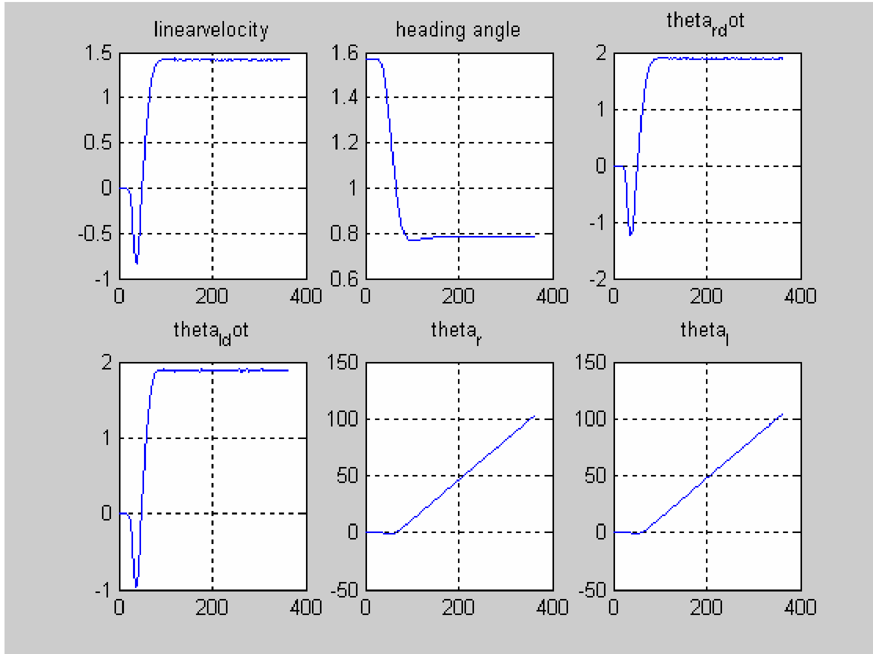

Fig. 8. variable States of Case-ii

4. Conclusion

The design criterion was to control the mobile platform so that the manipulator is maintained at a configuration which maximizes the manipulability measure. The control algorithm is designed b using feedback linearization. Since the mobile platform is subject to nonholonomic constraints, the dynamic system governing the motion of the mobile platform is not input-state linearizable. Thus a nonlinear feedback is deployed to achieve input-output linearization. The output equations are chosen to be co-ordinates of the manipulator when it is at the configuration with the maximum manipulability measure. We verified the effectiveness of our method by simulations on two representative trajectories. For future work ,we will implement this algorithm on the real mobile manipulator.

5. References:

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