

A DOUBLE MARKOV RANDOM FIELD MODEL FOR COLOR IMAGE SEGMENTATION

S. Panda, P.K Nanda and Rahul Dey

Image Processing and Computer Vision Lab., Department of Electrical Engineering
National Institute of Technology Rourkela-769008, Orissa, India.
sucheta_panda@rediffmail.com, pknanda_d13@rediffmail.com, rahuldey@rediffmail.com

ABSTRACT

In this paper, color image segmentation problem is cast as a pixel labeling problem in stochastic framework. The observed color image is assumed to be the degraded version of the image pixel label process. RGB color model is employed to model the color. A new Double Markov Random Field (DMRF) model is proposed to model the intraplane label process and also the interplane label process. The pixel labels are estimated using Maximum a Posteriori (MAP) criterion. A hybrid algorithm is proposed to obtain the MAP estimates and the algorithm is found to converge faster than that of Simulated Annealing (SA) algorithm. The performance of the proposed model is found to be superior to that of using Markov Random Field (MRF) model as the intraplane model. The proposed model yielded satisfactory results for different real images.

KEY WORDS

Color image, color model, Segmentation, Simulated Annealing, and MRF model.

1. INTRODUCTION

Color image segmentation is a key step for many computer vision problems such as object recognition, visual tracking, fault detection etc. In color image segmentation, choice of proper color model is one of the important factors for segmentation and different color models such as RGB, HSV, YIQ, Ohta (I_1, I_2, I_3), CIE (XYZ, Luv, Lab) are used to represent different colors ([1], [2], [3]). None of the above color models possesses the unifying property of representing the intrinsic characteristics of all possible real images. Very recently the notion of fusing color models has been used [4] for efficient representation of color.

Besides color model, image model does play a crucial role for color image segmentation. Stochastic models, particularly Markov Random Field (MRF) models, have been extensively used as the image model for image restoration and segmentation problems ([5], [6], [7], [8], [9], [10]). Often, in stochastic framework, the color image segmentation problem is formulated as a pixel labeling problem and the pixel labels are estimated using the Maximum a Posteriori (MAP) criterion. By and large, the MAP estimates of the pixel labels are obtained using Simulated Annealing (SA) algorithm ([11], [12]). MRF model has also been successfully used as the image model while addressing the problem of color image segmentation in supervised and unsupervised framework ([8], [13]). Hidden Markov Random Field (HMRF) has also been employed

for color image segmentation in unsupervised framework [14]. In order to take into account the dependencies of the color planes as well as the interactions across the scales, a Wavelet Domain Hidden Markov Model (WDHMM) [15] has been proposed which yielded promising results for color texture. Recently, Gaussian Mixture Models (GMM) have been proposed to model color textures on various feature spaces for image segmentation ([16], [17]).

In this paper, a new image model called Double MRF model (DMRF) is proposed as the image model. RGB color model is used to model the color and the proposed new model not only takes care of the spatial distributions of each color plane but also take care of the inter-color-plane interactions. The interplane interactions of RGB model are also modeled as MRF model and thus the color image is represented by a Double MRF model. The segmentation problem is formulated as a pixel labeling problem. The pixel labels are estimated using MAP estimation criterion. Simulated Annealing (SA) algorithm is used to obtain the MAP estimates and it is observed that SA incurs substantial amount of computational burden. In order to reduce the computational burden and to accelerate the convergence, a hybrid algorithm is proposed to obtain the MAP estimates. The proposed algorithm is found to converge to the solution much faster than that of using SA algorithm. The DMRF model in RGB plane yielded segmentation results with less misclassification error than that of using single MRF model in each RGB planes. The model could be successfully tested for a wide variety of real images. However, for the sake of illustration, results are presented for three real images.

2. THE DOUBLE MRF MODEL

Capturing salient spatial properties of an image lead to the development of image models. MRF theory provides a convenient and consistent way to model context dependent entities for e.g. image pixels and correlated features [9]. Though the MRF model takes into account the local spatial interactions, it has its limitations in modeling natural scenes of distinct regions. It is to be noted that there is high correlation among the color components of RGB model, and hence this color space is not suitable for image segmentation. Instead, in our formulation, we have decorrelated the color components and introduced an interaction process to improve the segmentation. The interplane interaction process reinforces partial correlation among different color components.

We assume all images to be defined on discrete rectangular lattice of size $M \times M$. Let Z denotes the

label process corresponding to the segmented image and z is the realization of the label process i.e. the segmented image. The observed image X is also a random field that is assumed to be the degraded version of the label process Z . The degradation process is assumed to be Gaussian process. The label process Z is assumed to be MRF. Another random field N , that is assumed to be MRF, is defined to take care of the inter-color-plane interactions. The label process Z and interaction process N are modeled as the proposed Double MRF model. It is known that if Z is assumed MRF, then the prior probability distribution $P(Z = z)$ is Gibb's distributed be expressed as $P(Z = z | \theta) = \frac{1}{Z'} e^{-U(z, \theta)}$, where $Z' = \sum_z e^{-U(z, \theta)}$ is the partition function, θ denotes the clique parameter vector, the exponential term $U(z, \theta)$ is called the energy function, $U(z, \theta) = \sum_{c \in C} V_c(z, \theta)$ with $V_c(z, \theta)$ being referred as the clique potential function. Here, we consider the RGB color model and in each plane, we consider the observed image to be label process and a degraded process consisting of a label process and the degraded process. The degraded process in each plane (say for example R plane) is a Gaussian process.

The pixel in each plane (say for e.g. R-plane) is assumed to have interaction with every other planes (G and B planes). The interaction process of each plane is shown in Fig. 1(a) and 1(b). For the sake of illustration the interaction of one pixel in G plane with pixels of R plane for a first order neighborhood system is shown in Fig 1(b). Thus, the label process in RGB plane consists of two MRF models; (i) Z corresponding to intraplane interactions, and (ii) N the interplane interactions. Thus the Double MRF model representing the observed image X is given by $X = (Z, N)$. The Gibbs equivalent of both interplane and intraplane interactions is given by,

$$Z = \frac{1}{Z'} \exp \frac{U(z)}{T} \quad (1)$$

$$N = \frac{1}{N'} \exp \frac{U(n)}{T} \quad (2)$$

Where Z' and N' denote the partition function, T is the temperature which controls the sharpness of the distribution. $U(z)$ and $U(n)$ denote the energy function and can be written as $U(z) = \sum_{c \in C} V_c(z)$ and $U(n) = \sum_{c \in C} V_c(n)$. The model parameters for both Z and N processes are selected on an *ad hoc* manner. The weak membrane model is considered for intra as well as interplane clique potential functions. For example, the weak membrane model for interplane interaction process for two color components for example G and R (as shown in Fig.1 (b)) can be expressed as $V_c(n^1) = \alpha \left\{ (n_{i',j'}^G - n_{i-1,j}^R)^2 + (n_{i',j'}^G - n_{i,j-1}^R)^2 + (n_{i',j'}^G - n_{i,j+1}^R)^2 + (n_{i',j'}^G - n_{i+1,j}^R)^2 \right\}$, where α is the MRF bonding parameter. Superscripts R and G corresponds to R-plane and G-plane respectively.

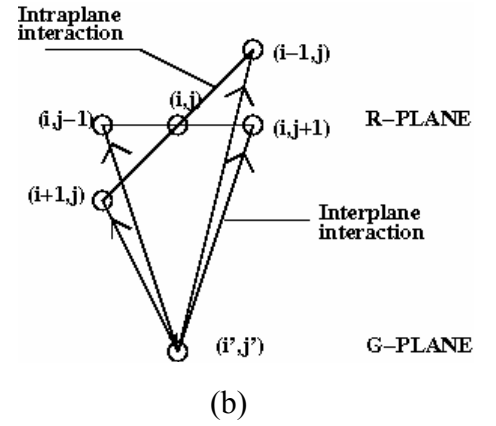
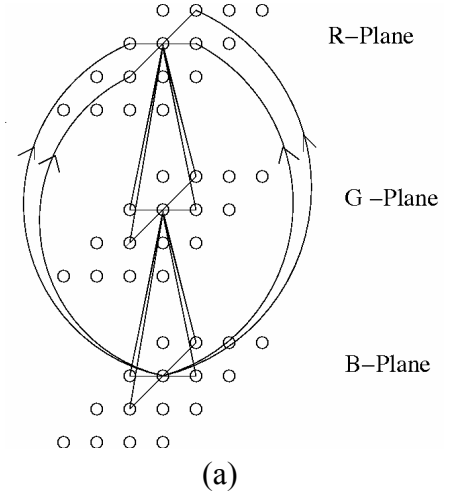


Fig.1. (a) RGB Plane Interaction (b) Interaction of one pixel of G-plane with R-plane

2.1. IMAGE SEGMENTATION

The problem is formulated as the pixel labeling problem. Each pixel can assume a label from the set of labels $\{0-L\}$. Let $Z_{i,j}$ denote the random variable for each pixel, where Z denotes the label process and let N denote the random field of the interplane interaction process. The labels \hat{z} and \hat{n} are obtained by maximizing the posterior probability $P(Z=z, N=n | X=x, \hat{\theta})$. Hence, the optimality criterion considered is

$$(\hat{z}, \hat{n}) = \arg \max_{(z,n)} P(Z = z, N = n | X = x, \hat{\theta}) \quad (3)$$

Where, X denotes the random field corresponding to the observed image and θ is the associated parameter vector. Assuming the intraplane interaction process (Z) and interplane interaction processes (N) are statistically independent, (3) can be expressed as,

$$(\hat{z}, \hat{n}) = \arg \max_{(z,n)} P(Z = z | X = x, \hat{\theta}) P(N = n | X = x, \hat{\theta}) \quad (4)$$

Where, N denotes the random field to take care of interplane (R-G-B) interactions. We assume N to be

stastically independent of the observed image random field X . Therefore, $P(N=n|X=x, \theta)$ can be expressed as the prior probability distribution $P(N=n)$ and since N is assumed to be MRF, $P(N=n)$ can be expressed as

$$P(N=n) = \frac{1}{N'} \exp \frac{-U(n)}{T} \quad \text{Where, } U(n) = \sum_{c \in C} V_c(n)$$

$V_c(n)$ for two color components (for example G and R) can be expressed as $V_c(n^1) = \alpha \left\{ (n_{i,j}^G - n_{i-1,j}^R)^2 + (n_{i,j}^G - n_{i,j-1}^R)^2 + (n_{i,j}^G - n_{i,j+1}^R)^2 + (n_{i,j}^G - n_{i+1,j}^R)^2 \right\}$. Since z is unknown,

using Bayes's theorem, $P(Z=z | X=x, \hat{\theta})$ can be written as

$$P(Z=z | X=x, \hat{\theta}) = \frac{P(X=x | Z=z, \theta) P(Z=z)}{P(X=x | \theta)} \quad (5)$$

$P(X=x | \theta)$ is a constant quantity and $P(Z=z)$ is the *a priori* probability distribution of the labels. The observed image process is also assumed to be the degraded version of the unknown label process and the degradation process is assumed to be Gaussian. Hence $P(X=x|Z=z, \theta)$ can be written as $P(X=x|Z=z, \theta) = P(X=z+w|Z, \theta) = P(W=x-z|Z, \theta)$. W is a Gaussian process and there are three spectral components present in a color image. As the three spectral components are assumed to be decorrelated, we obtain

$$P(W=x-z | Z, \theta) = \frac{1}{\sqrt{(2\pi)^3 \sigma^3}} e^{-\frac{1}{2\sigma^2}(x-z)^2} \quad (6)$$

Substituting (6) and the prior distribution of the interplane process $P(N=n)$ in (4) we obtain,

$$(\hat{z}, \hat{n}) = \arg \max_{(z,n)} \frac{1}{\sqrt{(2\pi)^3 \sigma^3}} e^{-\left[\sum_{i=1}^3 \frac{1}{2\sigma^2} (x^i - z^i)^2 + \sum_{c \in C} V_c(z^{(1)}, z^{(2)}, z^{(3)}) \right]} \frac{1}{N'} e^{-\sum_{c \in C} V_c(n^{(1)}, n^{(2)}, n^{(3)})} \quad (7)$$

The above maximization can be written as

$$(\hat{z}, \hat{n}) = \arg \min_{(z,n)} \left[\sum_{i=1}^3 \frac{1}{2\sigma^2} (x^i - z^i)^2 + \sum_{c \in C} V_c(z^{(1)}, z^{(2)}, z^{(3)}) + \sum_{c \in C} V_c(n^{(1)}, n^{(2)}, n^{(3)}) \right] \quad (8)$$

Where \hat{n} denote the estimated labels in the previous iteration. Hence, minimization of (8) with respect to z and n reduces to minimizing (8) with respect to z only. Therefore (8) can be modified as follows,

$$\hat{z} = \arg \min_{(z)} \left[\sum_{i=1}^3 \frac{1}{2\sigma^2} (x^i - z^i)^2 + \sum_{c \in C} V_c(z^{(1)}, z^{(2)}, z^{(3)}) + \sum_{c \in C} V_c(n^{(1)}, n^{(2)}, n^{(3)}) \right] \quad (9)$$

The color image has three spectral components x^i, z^i $\{i=1, 2, 3\}$ and V_c is the clique potential function for all the three spectral components, C is the set of all the cliques. In particular, we consider the following energy function

$$U(z, h, v) = \sum_{i,j} \alpha \left[\|z_{i,j} - z_{i,j-1}\|^2 (1 - v_{i,j}) + \|z_{i,j} - z_{i,j-1}\|^2 (1 - h_{i,j}) \right] + \beta [v_{i,j} + h_{i,j}] + \sum_{c \in C} V_c(n^{(1)}, n^{(2)}, n^{(3)}) \quad (10)$$

$[\alpha, \beta]^T$ are unknown parameters that are selected on *ad hoc* basis,

Where $V_c(n^1) = \alpha \left\{ (n_{i,j}^G - n_{i-1,j}^R)^2 + (n_{i,j}^G - n_{i,j-1}^R)^2 + (n_{i,j}^G - n_{i,j+1}^R)^2 + (n_{i,j}^G - n_{i+1,j}^R)^2 \right\}$ and

$$\|z_{i,j}\|^2 = (z_{i,j}^1)^2 + (z_{i,j}^2)^2 + (z_{i,j}^3)^2.$$

The vertical line field $v_{i,j}=1$ if, $f_v(z_{i,j}, z_{i,j-1}) > \text{thresh}$,

where $f_v(z_{i,j}, z_{i,j-1}) = \frac{1}{3} \sum_{q=1}^3 |z_{i,j}^q - z_{i,j-1}^q|$ and the

horizontal line field $h_{i,j}=1$ if, $f_h(z_{i,j}, z_{i-1,j}) > \text{thresh}$,

where $f_h(z_{i,j}, z_{i-1,j}) = \frac{1}{3} \sum_{q=1}^3 |z_{i,j}^q - z_{i-1,j}^q|$. The

posterior energy function is given by,

$$U_p(z, h, v) = \frac{\sum_{i=1}^3 (x^i - z^i)^2}{2\sigma^2} + U(z, h, v) \quad (11)$$

The proposed hybrid algorithm and the SA algorithm are used to obtain the MAP estimate of equation (9).

2.2. HYBRID ALGORITHM

It is observed that SA algorithm takes substantial amount of time for convergence. This algorithm also helps to come out of the local minima and converge to the global optimal solution. This feature could be attributed to the acceptance criterion (acceptance with a probability). We have exploited this feature, that is the proposed hybrid algorithm uses the notion of acceptance criterion to come out of the local minima. Subsequently, it is assumed that solution is locally available and hence a local convergent based strategy is adopted for quick convergence. We have used the Iterated Conditional Mode (ICM) algorithm [7] as the locally convergent algorithm. A specific number of iterations of SA algorithm are executed to achieve the near optimal solution. The number of iterations is fixed by trial and error. This avoids the undesirable time taken by SA when the solution is close to the optimal solution. The steps of proposed hybrid algorithm are enumerated as below:

1. Initialize the temperature T_{in} .
2. Compute the energy U of the configuration.
3. Perturb the system slightly with suitable Gaussian disturbance.
4. Compute the new energy U' of the perturbed system and evaluate the change in energy $\Delta U = U' - U$.
5. If $\Delta U < 0$, accept the perturbed system as the new configuration Else accept the perturbed system

as the new configuration with a probability $\exp(-\Delta U)/t$ where t the temperature of cooling schedule.

6. Decrease the temperature t according to the cooling schedule.

7. Repeat steps 2-7 till some pre specified number of epochs.

8. Compute the energy U of the configuration.

9. Perturb the system slightly with suitable Gaussian disturbance.

10. Compute the new energy U' of the perturbed system and evaluate the change in energy $\Delta U = U' - U$.

11. If $\Delta U < 0$, accept the perturbed system as the new configuration, otherwise retain the original configuration.

12. Repeat steps 8-12, till the stopping criterion is met. The stopping criterion is the energy ($U < \text{threshold}$).

2.3. SIMULATION

Three real images as shown in Figures 3, 4 and 5 have been considered for simulation. In each case, the RGB color model is used. The Double MRF model consists of; one on intraplane to take care of spatial interactions and the other to take care interplane interactions. Associated model parameters in both the cases are selected on trial and error basis. Line fields have been incorporated in the clique potential functions of intra color planes with a view to preserve edges where as line fields are not introduced in the clique potential function of inter plane interactions. The intraplane model parameters α , β and σ (as considered for simulation) are tabulated in Table 1 and the interplane parameter α is taken to be unity. The MAP estimates of pixel labels and hence the segmented image is obtained by the proposed hybrid algorithm. In simulation, we have taken the initial temperature parameter of hybrid algorithm as $T_{in} = 0.38$ and the logarithmic cooling schedule all images. The hybrid algorithm is found to converge to the solution much faster than that of using SA adhered for algorithm. As observed from Fig.2, hybrid algorithm converges at around 200 iterations as opposed to SA algorithm converging at 2000 iterations. Similar observations are also made for other images. The ground truth image corresponding to Fig.3 (a) is drawn manually and is shown in Fig.3 (b). Fig.3(c) and Fig. 3(d) correspond accurate segmented images to the segmentation results obtained using Double MRF model using SA and hybrid algorithm respectively. The rock and sea image could be well segmented in both the cases. This is corroborated from the percentage of misclassification error given in of Table 2. The misclassification error is 8.82% in case of DMRF with hybrid algorithm and 9.03% with SA algorithm. Fig.3 (e) and (f) shows the segmentation results obtained using MRF model without the interplane interactions. As observed, rock portion in both the cases has been misclassified and the percentage of misclassification is as high as 27.79%.

Similar observations are also made for the house image as shown in Fig.4. The ground truth is shown in Fig.4 (b). As seen from Fig.4(c) and (d), there are some misclassified pixels on the grass portion and rest of the

image is segmented properly. From Table 2, it is observed that there is more misclassification while using MRF model and the percentage of misclassification is more than that of using DMRF model.

TABLE I
PARAMETERS FOR IMAGES OF DIFFERENT CLASSES

Images	Intraplane parameters		
	α	β	σ
Fig.3	0.09	2.93	0.1
Fig.4	0.01	2.9	2.5111
Fig.5	0.60	2.3	1.42

TABLE II
PERCENTAGE (%age) OF MISCLASSIFICATION ERROR FOR DIFFERENT IMAGES

Images	MRF Hybrid	MRF SA	DMRF Hybrid	DMRF SA
	Fig.3	27.97	27.79	8.82
Fig.4	8.69	8.69	6.95	7.05
Fig.5	33.17	33.17	2.15	2.15

The effect of using DMRF is more prominent in case of the another real image as shown in Fig.5. The corresponding ground truth image is shown in Fig.5 (b). As seen from Fig. 5(c) and 5(d), proper segmentation results could be obtained using DMRF and the percentage of misclassification is around 2%. The misclassification error is 33% while using MRF model. In all the three cases, the proposed hybrid algorithm converged faster than that of using SA. In case of three real images, the DMRF model yielded better results than that of using MRF model.

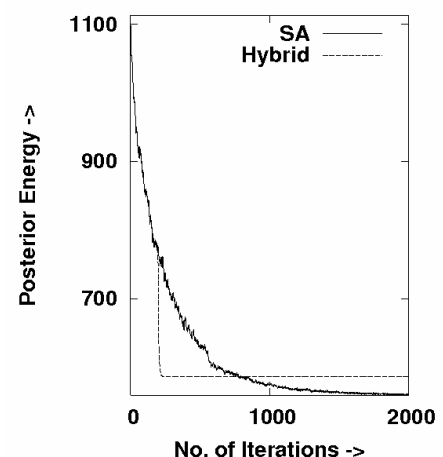
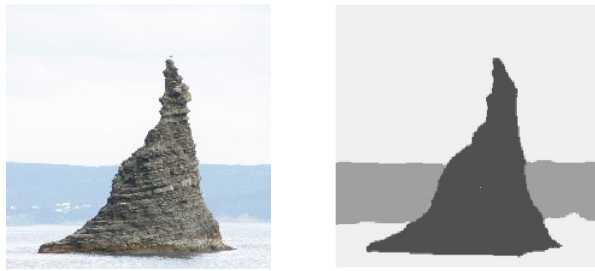
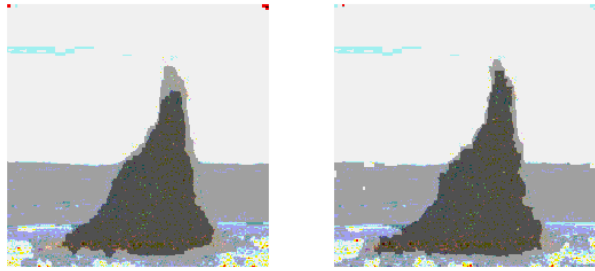


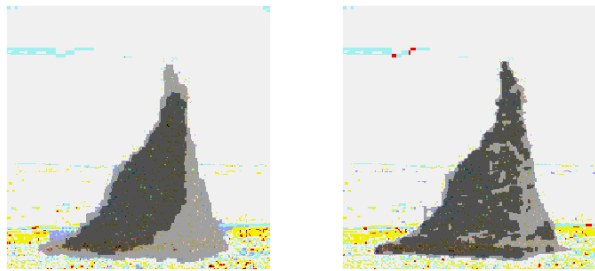
Fig.2 Convergence curve of “Bell-island” image using SA and Hybrid algorithm.



(a) (b)



(c) (d)



(e) (f)

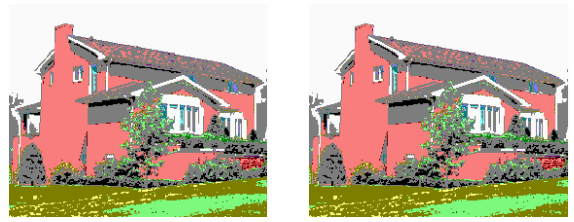
Fig.3 (a) Original “Bell-island” image (220x220) (b) Ground truth (c) DMRF with SA (d) DMRF with Hybrid (e) MRF with SA (f) MRF with Hybrid.



(a) (b)

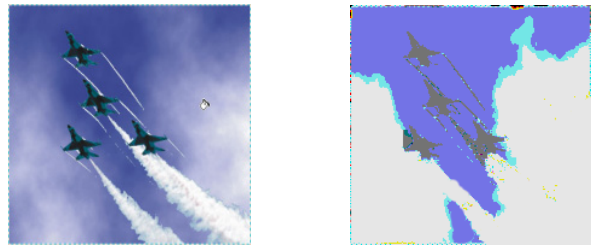


(c) (d)

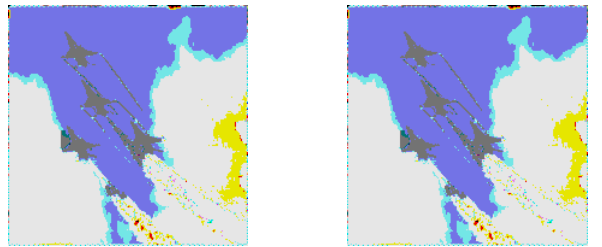


(e) (f)

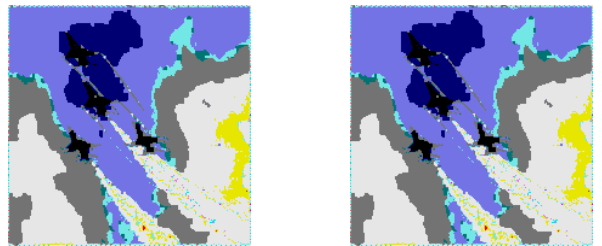
Fig.4 (a) Original “House” image (210x256) (b) Ground truth (c) DMRF with SA (d) DMRF with Hybrid (e) MRF with SA (f) MRF with Hybrid.



(a) (b)



(c) (d)



(e) (f)

Fig.5 (a) Original “Plane” image (200x200) (b) Ground truth (c) DMRF with SA (d) DMRF with Hybrid (e) MRF with SA (f) MRF with Hybrid.

3. CONCLUSIONS

In this paper, a new Double MRF model is proposed for image segmentation using RGB model. The intraplane process takes care of the spatial intrinsic properties in each plane. The parameters of the intraplane process are selected on a trial and error basis with a view to achieve minimum misclassification error. The parameter of the interplane process is set at unity. However, this also

can be varied to obtain proper segmentation for other images. The MAP estimates of the pixel labels are obtained using the proposed hybrid algorithm that converges around 10 times faster than that of using SA algorithm. Current work includes the estimation of model parameters together with the pixel labels.

ACKNOWLEDGEMENTS

This work is supported by the MHRD sponsored R&D project titled "Real time Signal and Image Processing using Soft Computing Approach".

REFERENCES

- [1].H.D.Cheng, X.H.Jiang, Y.Sun, J.Wang, Color Image Segmentation: Advances and prospects. *Pattern Recognition*, vol.34, pp.2259-2281, 2001.
- [2].L. Lucchese, and S. K. Mitra, Color Image Segmentation: A state of art survey. *Image Processing vision and pattern recognition, proc. of the Indian national science academy*, vol. 67, No. 2, March 2001, pp. 207-221.
- [3].M.Tkalcic and J.F Tasic, Colour spaces - perceptual, historical and applicational background. *EUROCON, Computer as a Tool, the IEEE Region 8*, vol-1, pp-304-308, 2003.
- [4].H.Stokman and T.Gevers, Selection and Fusion of Color Models for Image Feature Detection. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol.29 (3), pp.371-381, 2007.
- [5]N.R. Pal and S.K. Pal, "A Review on Image Segmentation," *Pattern Recognition*, vol. 13, pp. 3-16, 1981.
- [6]. S. Geman, D. Geman, Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence* vol. 6, pp. 721-741, 1984.
- [7].J. Besag, on the statistical analysis of dirty pictures, *J. Roy. Statistical. Soc. B.* 62 (1986) 259-302.
- [8]. Z.Kato, T.C.Pong, S.G.Qiang, Multicue MRF Image segmentation: Combining Texture and Color Features. *IEEE.comp.Society ICPR*, 2002.
- [9]. S.Z.Li, *Markov Random Field modeling in computer vision* (Springer, Berlin, 1995).
- [10].Ciro D'Elia, G. poggi, and G. Scarpa. A Tree Structured Markov Random Field Model for Bayesian Image Segmentation. *IEEE transactions on Image Processing*, vol. 12, No.. 10., Oct 2003.
- [11]. S.A.Kirkpatrick and C.Gelatt Jr. and M Vecchi, Optimization by Simulated Annealing. *Science*, vol-220, pp-671-680, 1983.
- [12]. E. Aarts, J. Korst. *Simulated Annealing and Boltzmann Machines: A stochastic approach to combinatorial optimization and neural computing*. (Wiley-Interscience, Jan 1991.)
- [13]. P.K Nanda, MRF model learning and its application to image restoration and segmentation. *PhD dissertation, Electrical Engg. Department, I.I.T. Bombay, 1995.*
- [14]. F.Destrepes, M.Max and A.J.Francois, A stochastic method for bayesian estimation of Hidden Markov Random Field models with application to a color model. *IEEE Transactions on Image Processing*, vol-14, pp-1096-1108, 2005.
- [15]. Q.Xu, J.Yang, S.Ding, Color texture analysis using the wavelet-based hidden Markov model. *Pattern.Recog.Letter*.vol.26, pp.1710-1719, 2005.
- [16]. Z.Kato. T.C.Pong,A Markov random field image segmentation model for color textured images. *Image.vision.comp*, vol.24, pp.1103-1114, 2006.
- [17].H.Permuter, J.Francois, I.Jermyn, A study of Gaussian mixture models of color and texture features for image classification and segmentation. *Pattern.Recog*, vol.39, pp.695-706, 2006.