

A K-Medoids-LSTM based Technique for Electromechanical Modes identification for Synchronphasor Applications

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Abstract— In this proposed work an efficient K-MedoidsLSTM based technique that takes into account of the degraded Phasor measurement unit (PMU) data for the estimation of poorly damped modes for wide area monitoring in smart grid is presented. This technique is designed in such a way that the detrimental effect of data missing and outliers which are created due to congestion in communication network, malfunction of PMUs or Phasor data concentrators (PDCs), and malicious attacks on mode estimation are mitigated. Here, the detection and removal of outliers are treated by applying K-Medoid algorithm, thereby the Long Short-term Memory (LSTM) is exploited for missing data imputation. Finally, total-least square-estimation-of-signal-parameters via rotational invariance technique (TLS-ESPRIT) is applied for mode estimation. The effectiveness and robustness of the proposed approach is validated by conducting statistical analysis study on synthetic signal through Monte Carlo simulation and compared with other recently developed techniques. This technique is also validated on Two area data and real probing data obtained from Western Electricity Co-ordinating Council (WECC).

Keywords—PMU, K-Medoids-LSTM, TLS-ESPRIT, Modes Estimation

I. INTRODUCTION

In modern days highly interconnected Power systems are effectively sharing the increase in load demand. Because of these complex interconnected networks, power system engineers faces many challenges in detection and monitoring of poorly damped low frequency oscillation. Stability is an important factor for the interconnection among large scale power grids. Using Supervisory Control and Data Accusation (SCADA) arrangement, it is a tough task to estimate the low frequency inter-area oscillations, so to overcome that problem wide area measurement system (WAMS) came in to picture[1]. In WAMS, it has become easier to monitor and control the power system online. The dynamic time-stamped measurements of currents, voltages and angle differences across the transmission line are provided by the global positioning system (GPS) in PMU. Further, these collected PMU data are given to the control office via an adequate communication channel and the system's dynamic behaviors can be determined [2].

Low Frequency Oscillation (LFO) creates many issues in the protection and control of power system. Because of these oscillations several blackouts have been encountered. In order to detect low frequency mode, several online detection techniques such as Kalman Filter [3], sparsity[4], variable projection [5] and PRONY algorithm [6], Fast Fourier Transform (FFT) [7] have developed recently. An iterative

approach has been used in the Kalman filter, so it is numerically unstable. Variable projection algorithm includes orthogonal projection, which extracts model parameters from the signal space. FFT technique is faster, simpler, and cost effective with less reactive to noise, but still it has issues with frequency resolution for fewer sample points and the damping of the modes are not directly accessible. In case of ESPRIT algorithm it creates an auto correlation matrix from the observed data. ESPRIT method is highly noise immune than PRONY.

On-site PMUs typically experience different degrees of data quality issues because of communication congestion, hardware failures, transmission delays and other issues. Data loss severely affects data quality which can lead to reduced system observability, deteriorates state estimation, parameter identification and even threatens grid security [1]. Hence, different approaches have been suggested for the removal of outliers and compensation of missing measurements. The Weighted K-nearest Neighbour (WKNN) and Bagged Averaging of Multiple Linear Regression (BAMLR) have been suitably implemented for the reconstruction of degraded PMU data in [8] and [9], respectively. But in both cases, the authors have not considered the existence of outliers in the signal. In [8], a WKNN-TLS-ESPRIT has been discussed, which is a two-stage technique for mode estimation. Where first WKNN technique is implemented for missing data imputation and thereafter the reconstructed signal is passed through TLS-ESPRIT for the estimation of modes. This technique is not very much suitable as the presence of outliers is not discussed. The presence of a few numbers of outliers in the signal data is tolerable, but the larger number of outliers affects the estimation of the mode.

In this proposed approach an efficient K-Medoids-LSTM based technique has been discussed to deal with the degraded PMU signal. Initially, the K-medoids has been implemented for the detection and removal of outliers and, thereby the LSTM for the imputation of missing values. LSTM identifies the pattern of the data set and based on the pattern, the missing values are predicted. At last, For mode estimation the reconstructed signal is passed through the Modified TLSESPRIT, which works efficiently in different noise levels and PMU reporting rates. The rest of the paper is sorted as follows: The methodology involved in the proposed KMedoids-LSTM based technique is presented in Section II, thereafter the block diagram of

proposed technique is presented in Section III. Lastly, result analysis and conclusion are drawn in Section IV and Section V respectively.

II. METHODOLOGY OF PROPOSED K-MEDOIDS-LSTM TECHNIQUE

A. K-MEDOIDS-LSTM Algorithm

During estimation of modes through TLS-ESPRIT presence of missing data and outliers can lead to inaccurate estimation of modes. Hence, before sending the signal to TLSESPRIT algorithm, the signal is treated for outliers and missing values by using K-MEDOIDS-LSTM algorithm.

1) K-Medoids for Outliers

K-MEDOIDS is one of the unsupervised clustering method in machine learning algorithm [10]. K-Medoid is a partitioning technique of clustering, Which separate Or Cluster 'n' objects in a data set into k clusters. A medoid can be defined as the object of a cluster, whose average dissimilarity to all the objects in the cluster is minimal, it is a

most centrally located point in the given data set. a. Steps involved in K-Medoids Clustering

Step 1 k clusters are initialized in the given data space D. Step 2 k objects are chosen randomly from n objects in data and a n objects to k clusters are assigned such that each object is assigned to one and only one cluster. Hence, now the initial medoid for each cluster is chosen.

Step 3 For all remaining non-medoid objects, the Cost is computed(distance as computed via Manhattan distance method).

$$\text{Manhattan Distance} = \text{mod}(x_1 - x_2) + \text{mod}(y_1 - y_2) \quad (1)$$

Step 4 Now, each remaining non-medoid object to that cluster are assigned whose medoid distance to that object is minimum as compared to other clusters medoid. Step 5 Total cost i.e. The total sum distance of all the nonmedoid objects from their respective cluster centroids is computed and assign it to d_j .

Step 6 A non-medoid object i is randomly selected.

Step 7 Now, the object i with medoid j is swapped temporarily and step 5 is repeated to recalculate total cost and assign it to d_i .

Step 8 If $d_i < d_j$ then the temporary swap in step 7 is made permanent to form the new set of k medoid. Else temporary swap done in step 7 is undone.

Step 9 Repeat Step 4 to Step 8 Until no change occurs.

2) LSTM for Missing Value

LSTM is an advanced version of Recurrent Neural Network (RNN) [11]. Different challenges of recurrent cell in RNN can fail to learn from complex dependencies, LSTM effectively works on this problem. The LSTM cells process from superior learning performance while compared to RNN, it fully uses connected layers. Recurrent units in the hidden layer process sequence data by passing the data into the hidden state from a previous timestep and combining it with current input and pass it through activation function[12]. a. steps related to missing value imputation:

Step 1 In the first step, the whole dataset is splitted into two datasets: training data set and testing dataset. The training dataset will be divided into five columns where in each column known values will be there, from there four columns will be treated as predictor columns and one

column as target column. Like that in the testing dataset from the known predictor column unknown values of the target column will be predicted by LSTM network.

Step 2 LSTM network provide cell state C_t based on three gates (forget gate, input gate and output gate. Input x_t at step t and hidden state h_{t-1} calculate which information needs to forget from cell state C_{t-1} via forget gate layer. Forget gate gives number between 0 and 1 for each cell state vector C_{t-1} . Outputs which are totally forgotten represents by 0 and when output define steps it describes 1. Forget gate output equation is

$$f_t = \sigma(W_f \times [h_{t-1}, x_t] + b_f) \quad (2)$$

Step 3 Input gate defines which cell unit needs to be updated. The equation for input gate is

$$i_t = \sigma(W_i \times [h_{t-1}, x_t] + b_i) \quad (3)$$

Step 4 Now, cell state need to be established based on input

x_t and hidden input h_{t-1} . Computation of cell state \bar{C} can be done by

$$\bar{C}_t = \tanh(W_c \times [h_{t-1}, x_t] + b_c) \quad (4)$$

Next, Cell states need to update

Step 5 Cell state C_t can be updated by equation

$$C_t = f_t * h_{t-1} + (i_t * \bar{C}) \quad (5)$$

Step 6 Variables of input state x_t and hidden state C_{t-1} need to pass through output layer gate, which is considered as

$$O_t = \sigma(W_o[h_{t-1}, x_t]) + b_o \quad (6)$$

Output of next layer h_t calculated from production of activation function tanh of new cell state C_t and output gate layer value

$$h_t = O_t * \tanh(C_t) \quad (7)$$

Where, W_i, W_f, W_c and W_o define as weight matrices and bias vectors are defined as $b_i, b_f, b_c,$ and b_o . Symbol * implies multiplication of each element. Activation functions are defined as σ .

Step 7 Finally the latest cell state (C_{t+1}) and the hidden state (h_{t+1}) go back into the recurrent unit and process repeats at timestep t+1. Loop will be continued till end of sequence.

B. TLS-ESPRIT for Modes identification

The reconstructed data obtained from K-Medoids-LSTM algorithms are inserted in TLS-ESPRIT [13] to create a robust auto-correlation matrix to provide the robust estimation of model parameters. The simulated real time signal is represented as

$$s(n) = \sum_{k=1}^{m_k} A_k e^{-\sigma_k n T_p} \cos(2\pi f_k n T_p + \theta_k) \quad (8)$$

Where, T_p sampling time-period; A_k is amplitude; f_k is frequency and θ_k is phase angle and σ_k is the damping factor.

III. BLOCK DIAGRAM FOR K-MEDOIDS-LSTM TECHNIQUE

The block diagram in Fig.1 shows step-by-step process for the implementation of the proposed K-Medoids-LSTM

technique. The PMU technology sends the time stamp data according to one single clock which is coming from GPS satellite system. The data from PMU are collected at PDC via the wire link. While sending data to PDC some outliers and missing values are created in the data due to communication gap, and PMU failure etc. This deteriorated data is passed through K-Medoids-LSTM algorithm to get complete data. Ultimately the Modes are estimated by implementing the modified TLS-ESPRIT. The estimated modes are analyzed in the control center for stability study.

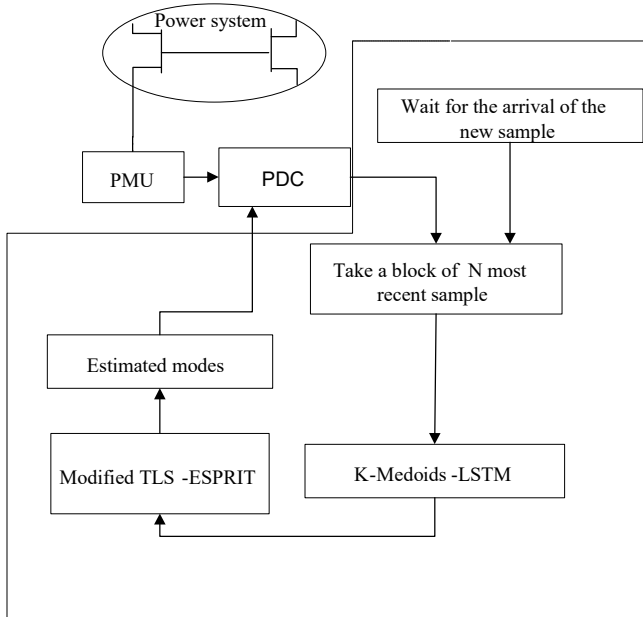


Fig. 1. Block Diagram for Proposed Algorithm

IV. RESULTS AND DISCUSSION

To validate the proposed technique, this technique is tested on two degraded synthetic signal corresponding to local and interarea modes of oscillations, on oscillatory data obtained from two area four generator systems and ultimately on real probing data obtained at WECC. The performance of the proposed technique over other techniques is assessed from the statistical study carried out by running 10000 independent monte carlo cycles at different noise level. For the simulation test carried out for synthetic signal and two area system, the sample window of 251 samples and sampling frequency of 12.5Hz are chosen, whereas for WECC system, the sample window of 8.55s length and 7.5Hz sampling frequency is chosen for mode estimation.

A. Modes Estimation Of Signal Oscillating In Inter Area Mode

The synthetic signal having amplitude 1, frequency 0.8Hz and attenuation factor -0.04 is considered for simulation. The outliers and missing values are added in the synthetic signal as displayed in Fig. 2. The mode estimation is done by the proposed technique, along with the other techniques at different noise level. The mean and variance of frequency and damping factor obtained from the simulation are presented in Table. I. Regarding the frequency of estimation, the proposed method gives a better result (0.8025 Hz), whereas the WKNNTLS-ESPRIT and BAMLMP method gives 0.8071 Hz and 0.8047 Hz respectively with higher variance. The damping factor

estimation degraded highly for WKNN-TLS-ESPRIT (0.8071) and BAMLMP (-0.0537) based method whereas the

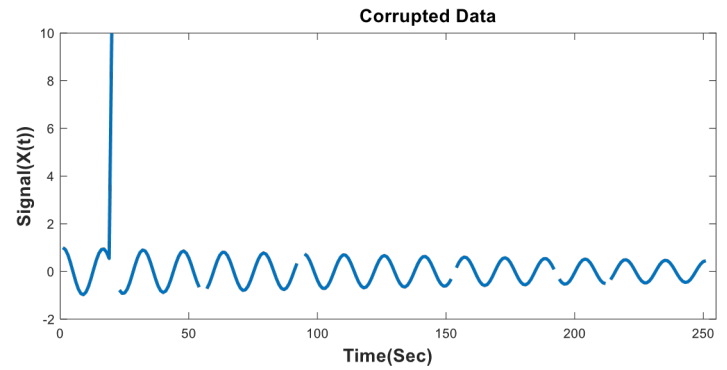
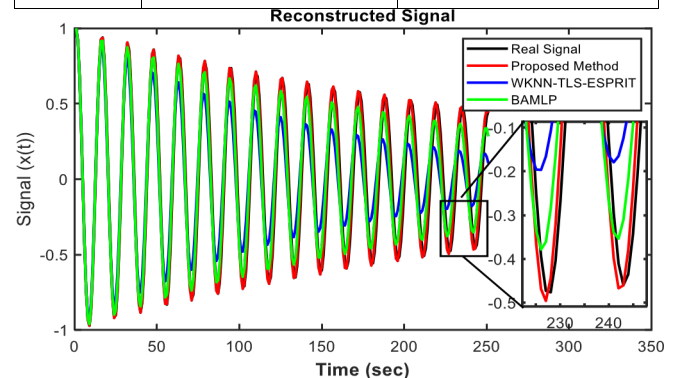


Fig. 2. Simulated corrupted signal having inter area modes of oscillation

TABLE I. ESTIMATED MENAS(μ) AND VARIANCE(s^2) FOR TEST SIGNAL WITH INTER AREA MODES OF OSCILLATION

True Frequency =0.8 Hz and True Damping=-0.04				
BAMLMP				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	0.8047	1.20×10^{-7}	-0.0537	4.95×10^{-6}
30	0.8047	1.18×10^{-8}	-0.0537	4.94×10^{-7}
40	0.8047	1.21×10^{-9}	-0.0537	4.87×10^{-8}
WKNN-TLS-ESPRIT				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	0.8071	3.92×10^{-7}	-0.0895	7.64×10^{-6}
30	0.8071	3.89×10^{-8}	-0.0895	7.67×10^{-7}
40	0.8071	3.83×10^{-9}	-0.0895	7.83×10^{-8}
Proposed Method				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	0.8025	4.08×10^{-8}	-0.0387	1.68×10^{-6}
30	0.8025	4.08×10^{-9}	-0.0387	1.68×10^{-7}
40	0.8025	4.08×10^{-10}	-0.0387	1.68×10^{-8}



proposed method gives the most accurate results (-0.0387).

Fig. 3. Reconstructed signal obtained by simulating modes obtained at

20 dB SNR

30	1.4004	1.86×10^{-9}	-0.0925	6.16×10^{-7}
40	1.4003	1.83×10^{-10}	-0.0925	6.01×10^{-8}

The estimated modes data obtained at SNR value 20 dB are taken for signal reconstruction and shown in Fig.3. Fig. 4 shows the distribution of the signal's frequency and attenuation factor for various methods at SNR=20dB for the inter-area mode. So, from the statistical data presented in Table I and the distribution plot in Fig.4 proves the effectiveness and robustness of the proposed technique over other techniques mode estimation.

B. Signals for Test Matching to local Area Mode

The synthetic signal having the frequency matching to local modes of oscillation (1.4 Hz) is considered for the proposed comparative study and the signal is degraded as like the interarea modes. The degraded data is passed through different technique for signal reconstruction and modes estimation. The statistical study is carried out by taking 10000 cycles of monte calro simulation and displayed in Table II. The frequency estimated by the proposed method is nearly 1.4003 Hz whereas WKNN-TLS-ESPRIT and BAMLP based method could able to estimate 1.3991Hz and 1.3985Hz respectively. For damping factor estimation the proposed technique gives very accurate estimation of -

0.0925 with lesser variation than WKNN-TLS-ESPRIT(0.0689) and BAMLP (-0.0359) based techniques. It is observed from Table II that this proposed technique can be able to estimate the modes more accurately compared to the other two techniques. Although the noise level increase, the proposed technique gives superior results with lesser variation than others.

Fig. 5 shows the distribution of frequency and attenuation factor plots for all three techniques at 20dB SNR. It can be seen from the plot that this technique gives very accurate damping and frequency values with lesser variation, proving the potential of the proposed technique.

TABLE II. ESTIMATED MENAS(μ) AND VARIANCE(S^2) FOR TEST SIGNAL WITH LOCAL AREA MODE OF OSCILLATION

True Frequency=1.4 Hz and True Damping=-0.09				
BAMLP				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	1.3985	1.40×10^{-7}	-0.0359	7.18×10^{-6}
30	1.3985	1.38×10^{-8}	-0.0359	7.12×10^{-7}
40	1.3985	1.40×10^{-9}	-0.0359	7.24×10^{-8}
WKNN-TLS-ESPRIT				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	1.3991	5.71×10^{-7}	-0.0689	8.40×10^{-6}
30	1.3991	5.81×10^{-8}	-0.0689	8.30×10^{-7}
40	1.3991	5.83×10^{-9}	-0.0689	8.30×10^{-8}
Proposed Method				
	Frequency(Hz)		Damping	
SNR (dB)	Mean	Variance	Mean	Variance
20	1.4003	1.88×10^{-8}	-0.0925	6.08×10^{-6}

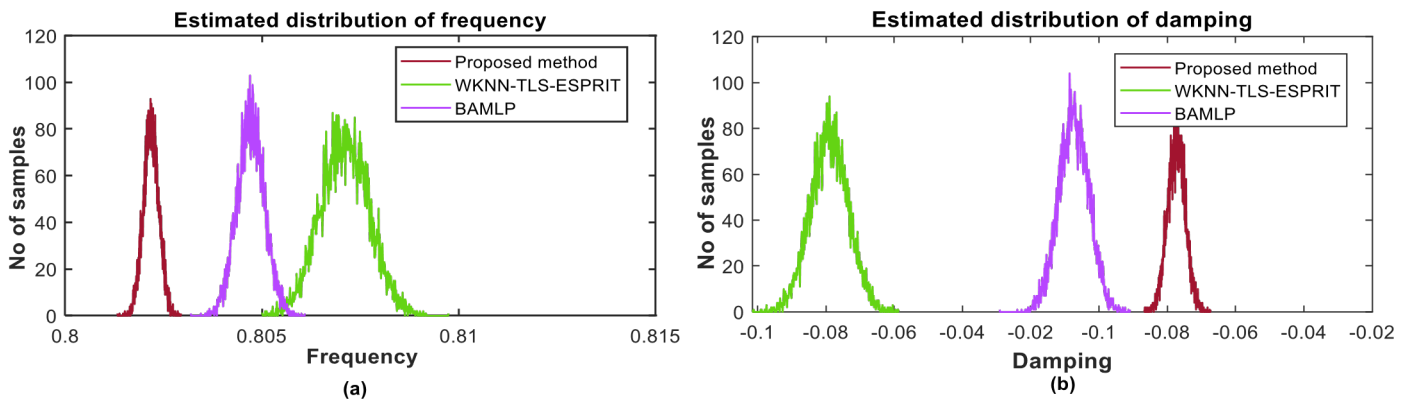


Fig. 4. Inter Area Plot Representing Mean and Variance for Mode Frequency and Damping at SNR = 20 dB

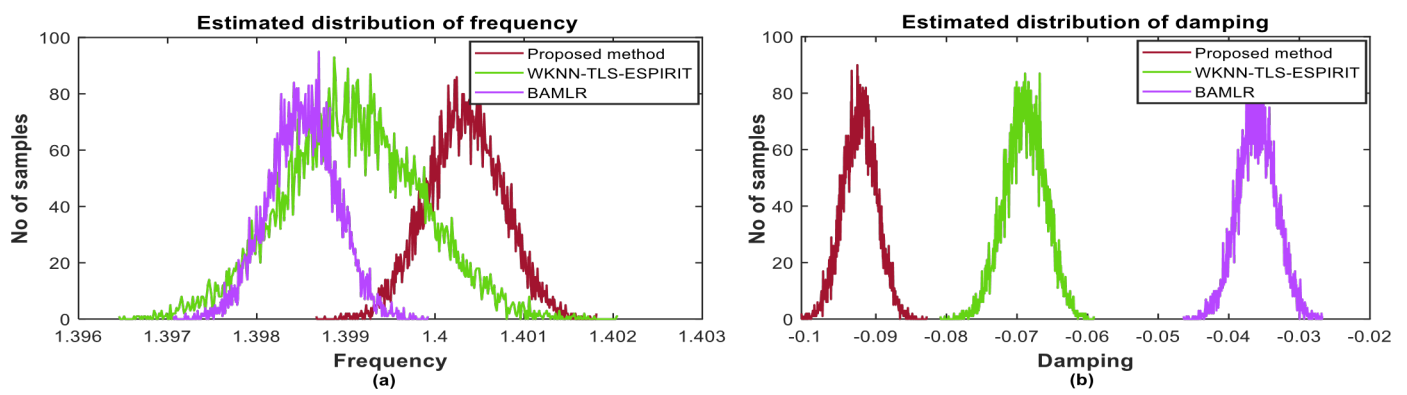


Fig. 5. Local Area Plot Representing Mean and Variance for Mode Frequency and Damping

C. Mode estimation of signal obtained from two area data

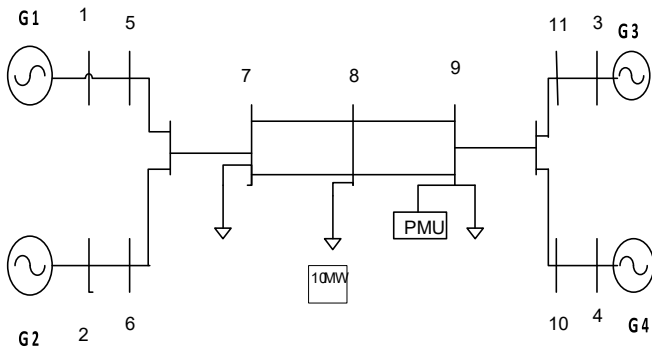


Fig. 6. Two Area Data Represented in Single Line Diagram

The proposed technique is also implemented on two area four generator system as shown in Fig. 6 [13]. Area one (formed by G_1 and G_2) is oscillating with respect to area two (formed by G_3 and G_4), and both areas are connected by tie lines connecting bus 7 and bus 9 [14]. The oscillatory power data obtained by PMU connected at bus 9 due to the isolation of the 10 MW load connected at bus 8. The data obtained at PMU are again inserted by missing values and outliers, and the mode estimation is done at noise level 20 dB. The statistical analysis is carried out by taking 10000 monte carlo simulation for different techniques and the comparable mean data of frequency and damping are presented in Table III.

It can be seen in Table III that the frequency and damping estimated by the proposed technique are (0.5375 Hz, -0.2456), (1.1745 Hz, -0.2243) and (1.2134, -0.2365) respectively, which is close to the true value as discussed in [15]. Whereas in other techniques, the frequency and attenuation factor

TABLE III. ESPIMATION OF MODES FOR TWO AREA DATA

True Value					
Mode 1		Mode 2		Mode 3	
Damping	frequency (Hz)	Damping	frequency (Hz)	Damping	frequency (Hz)
-0.25	0.5372	-0.25	1.1939	-0.25	1.2047
BAMLP					
Mode 1		Mode 2		Mode 3	
Damping	frequency (Hz)	Damping	frequency (Hz)	Damping	frequency (Hz)
-3.6473	1.1732	-4.6023	1.6341	-4.3421	1.7354
WKNN-TLS-ESPRIT					
Mode 1		Mode 2		Mode 3	
Damping	frequency (Hz)	Damping	frequency (Hz)	Damping	frequency (Hz)
-3.4534	1.1654	-3.8456	1.8234	-4.0856	1.6721
Proposed Method					
Mode 1		Mode 2		Mode 3	
Damping	frequency (Hz)	Damping	frequency (Hz)	Damping	frequency (Hz)
-0.2456	0.5375	-0.2243	1.1745	-0.2365	1.2134

estimation degraded heavily. So the comparative study shown in Table III shows the effectiveness and robustness of the proposed technique for mode estimation for degraded PMU signal.

D. Modes estimation using real test signal from WECC

The proposed technique is also tested on comprehensive probing data obtained from PMU connected to the WECC system, which was retrieved on September 14, 2005[5] as shown in Fig.6. According to [18], the predicted mode frequency was noted as 0.318 Hz with an 8.3% damping. Analysis of the probing data for both window 1 and window 2 was done for proposed method and was compared with WKNN and BAMLP algorithms. The statistical data obtained from the proposed method and other methods at SNR 20 dB are presented in Table IV.

It can be observed from the table that, the damping ratio estimated by BAMLP and WKNN-TLS-ESPRIT for both Window 1 (7.7506% and 6.8993%) and Window 2 (4.2956%, and 4.9642%) are highly degraded. So there is a chance of wrong estimation of modes and data loss occurs. Whereas the percentage damping obtained at both the window for the proposed method are 8.2334% and 8.2364%. Which are nearer to the true value of damping estimated in [13]. From Table IV it is observed that the accuracy of modes estimation for the proposed method is much higher than the BAMLP and WKNN-TLS-ESPRIT algorithms.

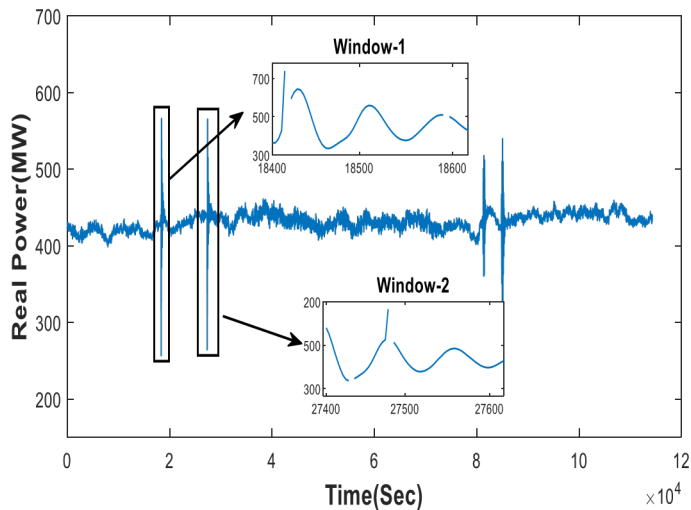


Fig. 7. Posing Data Recorded by PMU Linked to WECC

TABLE IV. ESPIMATION OF FREQUENCY AND PERCENTAGE DAMPING FOR PROBING DATA FROM WECC

WINDOW	MODES	BAMLP	WKNN-TLSESPRIT	K-MEANSANN
1	FREQ.(Hz)	0.3325	0.3307	0.3138
	DAMP.%	7.7506%	6.8993%	8.2334%
2	FREQ.(Hz)	0.3325	0.3315	0.3195
	DAMP.%	4.2956%	4.9642%	8.2364%

V. CONCLUSION

This paper explores a K-Medoids-LSTM based technique that takes into account the degraded PMU signals for identification of poorly damped modes in power system is presented. Here the K-Medoids is implemented for the detection and removal of outliers, thereby LSTM is explored for missing data imputation. Thereafter the reconstructed signal is passed through the modified TLS-ESPRIT for modes estimation, which has proven to be efficient in higher noisy conditions and PMU reporting rate. The robustness of the proposed K-Medoids-LSTM based technique is demonstrated by conducting statistical study and compared with other techniques. The effectiveness of the proposed technique is also validated on two area data and real probing data obtained from WECC. From the simulation study, it can be concluded that the proposed K-Medoids-LSTM technique preserves the signal characteristics and is best suitable to handle the degraded PMU signal for real-time wide-area monitoring system.

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