A MULTIRESOLUTION APPROACH TO COLOR IMAGE RESTORATION AND PARAMETER ESTIMATION USING HOMOTOPY CONTINUATION METHOD

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ABSTRACT

In this paper, we address the problem of color image restoration. Here, we model the image as a Markov Random Field (MRF) and propose a restoration algorithm in a multiresolution framework. The incorporation of multiresolution technique significantly reduces the computational complexity of the restoration algorithm. The energy function at each resolution being non-convex, is minimized using the simulated annealing algorithm. The parameters which describe the MRF model at each resolution are computed a priori using the Homotopy Continuation Method. Simulation results are presented to validate the proposed scheme.

1. INTRODUCTION

Image estimation from images degraded by noise and image capturing nonlinearities is one of the important early vision problems addressed richly in literature for monochrome images. Color images carry information in addition to the intensity (which is the only cue embedded in a monochrome image) in the form of hue and saturation. The extra cues help in distinguishing objects in an image which may not be possible in a monochrome image. Recent studies focus on devising various strategies and methodologies in the context of color image processing (for example see [1]-[7]).

In this paper, we assume the color image to be modeled by a Markov Random Field (MRF). Our degradation model incorporates additive white Gaussian noise. Here we address the problem of color image restoration in a multiresolution framework. The parameters (clique parameters μ and γ) describing the model are assumed to be unknown and hence need to be estimated. The parameters involved in the model are estimated using a pseudolikelihood function and the homotopy continuation method along the lines of [11] for a monochrome image. The color image restoration problem is posed as a maximum *aposteriori* (MAP) estimation problem in a multiresolution framework. Simulated Annealing algorithm [10] is used to obtain the MAP estimate at each resolution.

There are several color coordinate systems (for example RGB, HSV, YIQ, Ohta's I_1, I_2, I_3) used to represent a color image. Based on various experiments performed with color differencing schemes [5], [9], for color image restoration we conclude that the RGB and I_1, I_2, I_3 (Ohta et al [4]) color coordinate systems are best suited for color image restoration. Moreover, I_1 or hue provide the best cue for the inclusion of a line field. In this paper, due to lack of space, we present experimental results with the RGB color coordinate system, and line fields based on I_1.

The main contributions of the paper can be summarized as: (1) Use of single line field for the apriori energy function in the MRF model for the color image. We find that it is not necessary to use three line fields corresponding to the three color coordinates as is often done in literature. (2) Development of an estimator for the parameters of the color image MRF model with line fields at each resolution. (3) A multiresolution framework for color image restoration.

The organization of the paper is as follows: Section 2 describes the problem formulation dealing with color image restoration and parameter estimation. Section 3 describes the scheme adopted, followed by a section on experimental results. We conclude in Section 5.

2. PROBLEM FORMULATION

2.1. Color Image Restoration

Let $X^{\Omega}_{i,j}$, $Y^{\Omega}_{i,j}$, $W^{\Omega}_{i,j}$ be the image to be restored, the observed image and the noise field respectively at resolution Ω and defined on a square lattice of size 2^{Ω}, where Ω is a non-negative integer. Let $Y^{\Omega}_{i,j}=X^{\Omega}_{i,j}+W^{\Omega}_{i,j}$ for $0 \leq i,j \leq 2^{\Omega}$ be the degradation model. We can write the degradation model in the vector form by stacking the rows of the image in lexicographical order as $Y^{\Omega}=X^{\Omega}+W^{\Omega}$. Here, the color image X^{Ω} consists of three components, $X^{\Omega}=[\ {}^1X^{\Omega},\ {}^2X^{\Omega},\ {}^3X^{\Omega}]^T$, corresponding to the three components of some color coordinate system, for example ${}^1X^{\Omega}$ could correspond to red, ${}^2X^{\Omega}$ to green and ${}^3X^{\Omega}$ to blue in the RGB color coordinate system. Each pixel ${}^qX^{\Omega}_{i,j}$ $(q = 1,2,3)$ takes

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a value from a finite set ${}^qG^\Omega \in [0, 256]$. We make the following assumptions in this paper

(a) ${}^pW^\Omega_{i,j}$ for $1 \leq p \leq 3$ is a white Gaussian sequence with zero mean and variance $(\sigma^\Omega)^2$

(b) ${}^pW^\Omega_{i,j}$ is statistically independent of ${}^qX^\Omega_{k,l}$, for all (i,j) and (k,l) belonging to $2^\Omega \times 2^\Omega$, where $1 \leq p, q \leq 3$.

The noise free image X^Ω is modeled as a MRF, with $P(X^\Omega = x^\Omega) = \frac{1}{Z^\Omega} e^{-U(x^\Omega, \phi^\Omega)}$ where, $U(x^\Omega, \phi^\Omega) = \sum_{c \in C} V_c(x^\Omega, \phi^\Omega)$, C represents the set of cliques, ϕ^Ω the set of clique parameters, and x^Ω is a realization of X^Ω. $Z^\Omega \triangleq \sum_{x^\Omega} e^{-U(x^\Omega, \phi^\Omega)}$ is the partition function. In particular we consider the energy function

$$U(x^\Omega, h^\Omega, v^\Omega) = \sum_{i,j} \{\mu^\Omega [\|x^\Omega_{i,j} - x^\Omega_{i,j-1}\|^2 (1 - v^\Omega_{i,j}) + \|x^\Omega_{i,j} - x^\Omega_{i-1,j}\|^2 \ (1 - h^\Omega_{i,j})] + \gamma^\Omega [v^\Omega_{i,j} + h^\Omega_{i,j}]\} \quad (1)$$

where $\|x^\Omega_{i,j}\|^2 = ({}^1x^\Omega_{i,j})^2 + ({}^2x^\Omega_{i,j})^2 + ({}^3x^\Omega_{i,j})^2$, and $v^\Omega_{i,j} = 1$ if $f_v(x^\Omega_{i,j}, x^\Omega_{i,j-1}) > thresh$. $h^\Omega_{i,j}$ is analogously defined. In our simulation work we use

$$\begin{aligned} f_v(x^\Omega_{i,j}, x^\Omega_{i,j-1}) &= \frac{1}{3} \left| \sum_{q=1}^{3} {}^qx^\Omega_{i,j} - {}^qx^\Omega_{i,j-1} \right| \\ f_h(x^\Omega_{i,j}, x^\Omega_{i-1,j}) &= \frac{1}{3} \left| \sum_{q=1}^{3} {}^qx^\Omega_{i,j} - {}^qx^\Omega_{i-1,j} \right| \end{aligned} \quad (2)$$

This amounts to deciding the presence or absence of the line field on the color coordinate $I_1 = (R + G + B)/3$.

The posterior energy function will be

$$U_p(x^\Omega, h^\Omega, v^\Omega) = U(x^\Omega, h^\Omega, v^\Omega) + \frac{\frac{1}{3}\|y^\Omega - x^\Omega\|^2}{2(\sigma^\Omega)^2} \quad (3)$$

$[\mu^\Omega, \gamma^\Omega]^T$ are the unknown parameters that have to be estimated, $\|y^\Omega - x^\Omega\|^2 \triangleq \sum_{q=1}^3 \sum_{i,j} ({}^qy^\Omega_{i,j} - {}^qx^\Omega_{i,j})^2$.

If the parameters are known, restoration is achieved by minimizing $[U_p(x^\Omega, h^\Omega, v^\Omega)]$ with respect to $x^\Omega, h^\Omega, v^\Omega$.

2.2. Parameter Estimation

Our parameter estimation formulation for the apriori model parameters is based on the following optimality criterion

$$\max_{\phi^\Omega} P(X^\Omega = x^\Omega \mid \phi^\Omega) \quad (4)$$

given x^Ω. The probability (4) is expressed as

$$P(X^\Omega = x^\Omega \mid \phi^\Omega) = \frac{e^{-U(x^\Omega, \phi^\Omega)}}{\sum_{\xi^\Omega} e^{-U(\xi^\Omega, \phi^\Omega)}} \quad (5)$$

In (5) the summation is over all possible realizations of X^Ω. From a computational standpoint, handling (5) would be practically impossible. One can view (5) as a likelihood function to be maximized for estimating ϕ^Ω. To overcome the computational complexity and make the parameter estimation problem tractable, we approximate (5) using the pseudolikelihood function, (analogous to Besag [8]) as

$$\prod_{i,j} P(X^\Omega_{i,j} = x^\Omega_{i,j} \mid X^\Omega_{m,n} = x^\Omega_{m,n}, \phi^\Omega) \triangleq \hat{P}(X^\Omega = x^\Omega \mid, \phi^\Omega) \quad (6)$$

where $(m,n) \in \eta_{i,j}$ and $\eta_{i,j}$ represents the neighborhood of the site (i,j). Furthermore, it can be shown along the lines of [11] that (6) can be written as

$$\prod_{i,j} \left[\frac{e^{-\sum_{C:(i,j)\in C} V_c(x^\Omega_{i,j}, \phi^\Omega)}}{\sum_{x^\Omega_{i,j} \in \mathcal{G}} \left\{ e^{-\sum_{C:(i,j)\in C} V_c(x^\Omega_{i,j}, \phi^\Omega)} \right\}} \right]$$

The notation $x^\Omega_{i,j} \in \mathcal{G}$ means $[{}^1x^\Omega_{i,j}, {}^2x^\Omega_{i,j}, {}^3x^\Omega_{i,j}] \in [{}^1G^\Omega, {}^2G^\Omega, {}^3G^\Omega]$, where ${}^qG^\Omega$ is the set of intensity levels for the color coordinate q. Now the parameter estimation problem can be recast as follows

$$\max_{\phi^\Omega} \hat{P}(X^\Omega = x^\Omega \mid \phi^\Omega) \quad (7)$$

The numerical update equation derived for a homotopy map [11] is

$$\phi^\Omega_{k+1} = \phi^\Omega_k - \Delta\phi^\Omega \quad (8)$$

$\Delta\phi^\Omega_k \triangleq \lambda H^{-1}_{\phi^\Omega}(\phi^\Omega_k, \lambda^\Omega_{k+1}, \phi^\Omega_{k-1}) \frac{\partial H_{\phi^\Omega}}{\partial \lambda^\Omega}(\phi^\Omega_k, \lambda^\Omega_{k+1}, \phi^\Omega_{k-1})$, where $H_{\phi^\Omega}(\phi^\Omega_k, \lambda^\Omega_{k+1}, \phi^\Omega_{k-1})$ is the Jacobian of the selected homotopy map and $0 \leq \lambda \leq 1$ is the homotopy parameter (please refer to [11] for more details).

3. PARAMETER ESTIMATION AND RESTORATION IN MULTIRESOLUTION FRAMEWORK

3.1. The Scheme

The basic belief is that the MRF parameters remain *essentially* invariant for a class of images; for example the face image. This belief stems form the extensive empirical study of the MRF model for image restoration. Based on this belief our scheme is as follows:

1. We assume that one or more training images are available from the class of images under consideration. Using this training image we generate images at different resolutions Ω. Model parameter vector ϕ^Ω is generated for each resolution Ω (for example $\Omega = 4$ then 5, 6, and 7), using the supervised scheme of Section 2.2 (equations (7) and (8)). At each resolution we consider only a block of size equal to the size of the coarsest (say $\Omega = 4$) image to determine the parameter vector ϕ^Ω. The chosen block should be *sufficiently rich in information*. The reason for selecting a small block in a large image (16×16 block in a 128×128 image) is based on the following: (a) MRF model is homogeneous and isotropic, (b)

empirical studies showed that a 16×16 block was good enough to give a reasonable estimate of the MRF model parameters, and (c) the observation that the MRF model is robust; within a range of parameters the model performance does not vary much. Of course, outside this range the performance detoriates drastically.

In case more than one training image is available, we average the parameters for each image to obtain the final estimate of the parameters.

2. Now we consider the noisy image Y^{Ω} at the highest resolution. We assume that the SNR at the highest resolution is given. Using Y^{Ω} at this resolution we generate the images at different resolutions; the number of multiresolution levels being the same as that of Step 1. Since SNR at the finest resolution is known we know the σ^{Ω} at the finest resolution. Since going from a higher resolution to a lower resolution using the Gaussian pyramid is a low pass filtering operation, we expect the SNR at coarser resolution to increase, and the corresponding σ^{Ω} to decrease. As a rule of thumb we use $\sigma^{\Omega-1} = \sigma^{\Omega}/2$.

   Now we estimate the image at the coarsest resolution by minimizing (3) and using the estimated parameters ϕ^{Ω} from Step 1, and σ^{Ω}. The restored color image at this resolution is passed onto the next finer resolution $(\Omega + 1)$ using the quadtree interpolation scheme. We next estimate the image at resolution $(\Omega+1)$ by minimizing (3). We repeat this process until the image is restored at the finest resolution.

4. EXPERIMENTAL RESULTS

We use the Face-1 image given in Figure 1 for parameter estimation[1]. Face-1 is modeled using the MRF model (1) with line fields determination based on I_1, as depicted in (1)-(2). Corresponding to Face-1, multiresolution images at resolutions $\Omega = 6, 5, 4$ were constructed using the approach of [13]. The estimated parameters are shown in Table 1.

Figure 2 shows the Face-2 image and Figure 3 the corresponding noisy image. $\sigma^7 = 24.8$ was used to generate the noisy image; this corresponds to a SNR of 13.73 dB. In all the cases minimization of (3) is carried out using the simulated annealing algorithm. For the purpose of comparison, we first carry out restoration at the finest resolution only (referred to as *monoresolution*), using the parameters corresponding to image size 128 in Table 1, and $\sigma^7 = 24.8$. The result is displayed in Figure 4. Figure 5 is obtained by using different parameters (Table 1) at different resolutions and $\sigma^{\Omega-1} = \sigma^{\Omega}/2$. Since parameter estimation is computationally expensive, as an experiment, we kept the model parameters γ^{Ω} and μ^{Ω} to be fixed at the values estimated for the finest resolution, while σ^{Ω} was varied as before, namely, $\sigma^{\Omega-1} = \sigma^{\Omega}/2$. Once again a multiresolution image restoration was carried out, the result of which is displayed in Figure 6. The computation time taken to obtain Figure 4 is atleast $7\frac{1}{2}$ times more than that required to obtain Figure 5 or 6 using the multiresolution approach. The SNR for the restored images are tabulated in Table 2. We see that the multiresolution approach gives an equally good restored image and with much less computation. Also, our belief stated in the beginning of Section 3 is validated; though only one simulation case is presented, we have found this belief to hold true for many other cases tried by us.

[1] Here the images are displayed in gray scale, however, the CD version of the paper has images in color

Image Size	γ^{Ω}	μ^{Ω}
16	4.49×10^{-3}	11.35
32	2.85×10^{-3}	12.53
64	4.76×10^{-3}	14.704
128	3.594×10^{-3}	18.255

Table 1: Parameters used at different resolutions

	SNR (in dB)	Iterations/Ω
Figure 4	19.1855	2000
Figure 5	19.1250	250
Figure 6	19.1286	250

Table 2: Restored image SNR and the number of iterations

5. CONCLUSION

In this paper we have presented a supervised scheme for color MRF model parameter estimation and image restoration in a multiresolution framework. Currently we are developing an unsupervised scheme for this problem, also in a multiresolution framework, with the incorporation of blurring in the image degradation model along the lines of [12].

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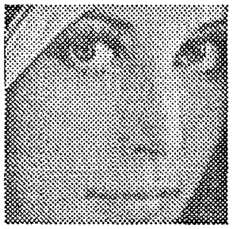

Figure 1. Face-1

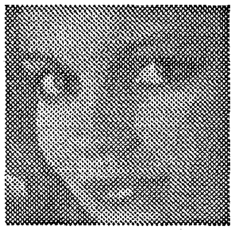

Figure 2. Face-2

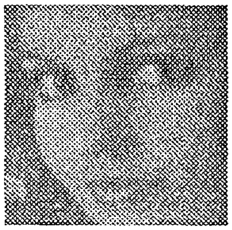

Figure 3. Face-2 (Noisy)

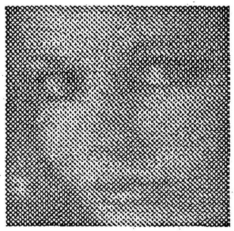

Figure 4. Monoresolution

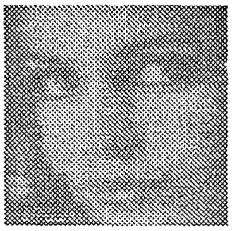

Figure 5. Multiresolution (Different Parameters)

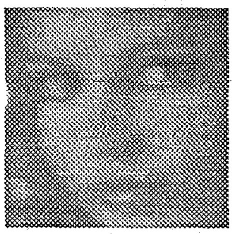

Figure 6. Multiresolution (Same Paremeters)