A MODULAR INTEGRATION AND MULTIRESOLUTION FRAMEWORK FOR IMAGE RESTORATION

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ABSTRACT

We present a framework based on modular integration and multiresolution for restoring images. We model the image as a Markov random field (MRF) and propose a restoration algorithm. In essence, the problem of image restoration requires learning of the MRF model and noise parameters which are used to restore degraded images. In the developed scheme, there exists interaction between the model learning module and the image restoration module. A method based on homotopy continuation is used for unsupervised model learning and the restoration is achieved through the minimization of an energy function.

1. INTRODUCTION

Image estimation from images degraded by noise and image capturing nonlinearities is an important early vision problem addressed richly in literature [1]. The goal of image restoration is to recover the original 2D image, $\mathcal{A}$, from the degraded observation $\mathcal{Y} = \mathcal{B}\mathcal{A} + \mathcal{W}$. In this paper, we develop a novel framework based on modular integration and multiresolution to restore degraded images. We assume the image to be a Markov random field (MRF) and restrict the degradation model to be an additive noise model, namely $\mathcal{Y} = \mathcal{A} + \mathcal{W}$. The emphasis here is in formulating the problem of image restoration in the framework of modular integration and multiresolution and to demonstrate the usefulness and applicability of the developed framework. The parameters associated with image and noise models are estimated using a conditional pseudolikelihood function. The homotopy continuation method along the lines of [2] is used to learn the parameters. The image restoration problem is posed as a maximum apostoriori (MAP) estimation problem in a multiresolution framework. Simulated annealing algorithm [3] is used to obtain the MAP estimate at each resolution.

The rest of the paper is arranged as follows: The problem of image restoration which is equivalent to the problem of parameter estimation and image restoration is formulated in Section 2. We describe the restoration scheme in Section 3 followed by experimental results to validate the use of the proposed framework in Section 4. We conclude in Section 5.

2. PROBLEM FORMULATION

The problem of image restoration is formulated in the framework of modular integration and multiresolution. Integration or synergism of modules is a technique where various modules get together to perform the given task better than when working individually with only feedforward interaction. Synergistic integration of various modules has been used effectively in computer vision. Multiresolution is an efficient and effective way of representing data. The data at each resolution is the output of a bandpass filter with some center frequency (usually the center frequency of the filters are octave apart). The use of multiresolution is also motivated when the computational complexity of any vision task is large and in these situations the notion of multiresolution can be used effectively to reduce the computational complexity [4, 5].

The task of image restoration would involve (i) choosing a suitable degradation model and (ii) modeling the image. In this paper, an additive noise degradation model, $\mathcal{Y} = \mathcal{A}+\mathcal{W}$ is assumed, where $\mathcal{Y}$ is the observed image, $\mathcal{A}$ is the image to be restored and $\mathcal{W}$ is the independent additive Gaussian noise each of size $2^{\Omega} \times 2^{\Omega}$. The image is modeled as a Markov random field (MRF). Now, the problem of image restoration becomes one of: (i) estimating the parameter associated with (a) the image being modeled as MRF, (b) the noise variance associated with the additive noise model and (ii) restoring the image using these parameters. The image restoration problem can be stated as:

Given the observed image $\mathcal{Y}^k$ at resolution k and the degradation model $\mathcal{Y}^k = \mathcal{A}^k + \mathcal{W}^k$. Find the optimum parameter and restored image pair (θ_*^k, a_*^k) such that

$$(\theta_*^k, a_*^k) = arg \max_{\theta^k, a^k} \mathcal{P}\left[\mathcal{A}^k = a^k | \mathcal{Y}^k = y^k, \tilde{a}^k, \theta^k\right] \tag{1}$$

Both θ^k and a^k need to be estimated to satisfy the optimality criterion of (1). It is difficult to find the optimum pair (θ_*^k, a_*^k) [6], and hence the problem is tackled by splitting the problem into two subproblems, namely,

(i) image restoration:

$$a_*^k = arg \max_{a^k} \mathcal{P}\left[\mathcal{A}^k = a^k | \mathcal{Y}^k = y^k, \tilde{a}^k, \theta_*^k\right] \tag{2}$$

(ii) parameter estimation:

$$\theta_*^k = arg \max_{\theta^k} \mathcal{P}\left[\mathcal{A}^k = a^k | \mathcal{Y}^k = y^k, \theta^k\right] \tag{3}$$

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The splitting of the problem and then recursively estimating the required attributes was suggested by Wendell and Horter [7] and is called *partial optimal solution.* Here, the problem of image restoration (2) is posed as a maximum a posteriori (MAP) estimation problem and the constructed energy function is minimized using the simulated annealing algorithm. The parameter estimation problem (3) is solved using the homotopy continuation method along the lines of [2].

2.1. Image Restoration

Let $\mathcal{A}_{i,j}^k$, $\mathcal{Y}_{i,j}^k$, $\mathcal{W}_{i,j}^k$ be the actual image to be restored, the observed image and the noise field respectively at resolution k, defined on a square lattice of size $2^k \times 2^k$. Let $\mathcal{Y}_{i,j}^k = \mathcal{A}_{i,j}^k + \mathcal{W}_{i,j}^k$ for $0 \le i,j \le 2^k - 1$ be the degradation model. Each pixel $\mathcal{A}_{i,j}^k$ takes a value from a finite set $G \in [0, 256]$. We make the following assumptions: (i) $\mathcal{W}_{i,j}^k$ is a white Gaussian sequence with zero mean and *unknown* variance $(\sigma^2)^k$ (ii) $\mathcal{W}_{i,j}^k$ is statistically independent of $\mathcal{A}_{l,m}^k$, for all (i,j) and (l,m) belonging to $2^k \times 2^k$. $\mathcal{A}^k$ is a MRF and hence using the MRF-Gibbs equivalence relationship, we can express the a priori probability density function of $\mathcal{A}^k$ as a Gibbs distribution [8],

$$\mathcal{P}[\mathcal{A}^k = a^k] = \frac{1}{Z^k} \exp^{-U(a^k, \phi^k)}$$

where, $Z^k \triangleq \sum_{a^k} \exp^{-U(a^k,\phi^k)}$ is the partition function, ϕ^k is the set of clique parameters, a^k is a realization of $\mathcal{A}^k$, and $U(a^k, \phi^k)$ is the energy function

$$U(a^k, \phi^k) = \sum_{c \in C} V_c(a^k, \phi^k),$$

$\mathcal{C}$ represents the set of all possible cliques, V_c is the clique potential which maps the local interactions of the elements of the clique c to the energy contributed by the clique towards the total energy. This, in fact, encodes the a priori knowledge about the spatial dependence of the pixel with the neighboring pixels. In particular, we consider the energy function

$$\begin{aligned} U(a^k, \phi^k) &= \sum_{i,j} \{\mu^k \left[\|a_{i,j}^k - a_{i,j-1}^k\|^2 (1 - v_{i,j}^k) \right. \\ &+ \left. \|a_{i,j}^k - a_{i-1,j}^k\|^2 \ (1 - h_{i,j}^k) \right] \\ &+ \gamma^k \left[v_{i,j}^k + h_{i,j}^k \right] \} \end{aligned} \tag{4}$$

where, $\phi^k \triangleq \left[\mu^k, \gamma^k\right]^T$ represents the clique parameters, and $\|\cdot\|$ represents the usual Euclidean norm. In (4), $v_{i,j}^k$ and $h_{i,j}^k$ are the vertical and horizontal line fields defined as:

$$v_{i,j}^k = \begin{cases} 1 & \text{if} \quad f_v(a_{i,j}^k, a_{i,j-1}^k) > thresh \\ 0 & \text{otherwise} \end{cases}$$

$$h_{i,j}^k = \begin{cases} 1 & \text{if} \quad f_h(a_{i,j}^k, a_{i-1,j}^k) > thresh \\ 0 & \text{otherwise} \end{cases}$$

In our simulations we use,

$$\begin{aligned} f_v(a_{i,j}^k, a_{i,j-1}^k) &= \left| a_{i,j}^k - a_{i,j-1}^k \right| \text{ and,} \\ f_h(a_{i,j}^k, a_{i-1,j}^k) &= \left| a_{i,j}^k - a_{i-1,j}^k \right| \end{aligned} \tag{5}$$

Using the Bayes rule, the assumption that the noise is Gaussian distributed and the fact that noise is independent of the image, the posterior energy function can be shown to be [8, 9]

$$U_p(a^k, \theta^k) = U(a^k, \phi^k) + \frac{\|y^k - a^k\|^2}{2(\sigma^2)^k} \tag{6}$$

where, $\|y^k - a^k\|^2 \triangleq \sum_{i,j} (y_{i,j}^k - a_{i,j}^k)^2$ and $\theta^k \triangleq [\mu^k, \gamma^k, (\sigma^2)^k]^T$. The unknown parameters θ^k, need to be estimated. If the parameters are known, restoration is achieved by minimizing $U_p(a^k, \theta^k)$, namely

$$a_*^k = \min_{a^k} U_p(a^k, \theta^k) \tag{7}$$

where a_*^k is the restored image.

2.2. Parameter Estimation

The posteriori energy function (6) is a function of the clique parameters ϕ^k and the noise variance $(\sigma^2)^k$. The choice of the parameters are *crucial* for the construction and hence the minimization of the energy function. The parameter estimation problem can be stated as:

$$\theta_*^k = arg \ \max_{\theta^k} \mathcal{P}\left[\mathcal{A}^k = a_*^k | \mathcal{Y}^k = y^k, \theta^k\right] \tag{8}$$

where, a_*^k is the optimal restored image obtained at the resolution k. The conditional probability can be expressed as $\mathcal{P}\left[\mathcal{A}^k = a_*^k \mid \mathcal{Y}^k = y^k, \theta^k\right] =$

$$\frac{\frac{1}{(2\pi(\sigma^2)^k)^{\frac{(2^k)^2}{2}}} \exp^{\frac{-\|y^k - a_*^k\|^2}{2(\sigma^2)^k}} \frac{1}{Z^k} \exp^{-U(a_*^k, \phi^k)}}{\mathcal{P}\left[\mathcal{Y}^k = y^k \mid \theta^k\right]}$$

It can be shown that [10],

$$\begin{aligned} \mathcal{P}\left[\mathcal{Y}^k = y^k \mid \theta^k\right] &= \sum_{\xi^k} \mathcal{P}\left[\mathcal{Y}^k = y^k, \mathcal{A}^k = \xi^k \mid \theta^k\right] \\ &= \sum_{\xi^k} \mathcal{P}\left[\mathcal{Y}^k = y^k \mid \mathcal{A}^k, \theta^k\right] \mathcal{P}\left[\mathcal{A}^k = \xi^k \mid \theta^k\right] \end{aligned}$$

$$= \sum_{\xi^k} \frac{1}{(2\pi(\sigma^2)^k)^{\frac{(2^k)^2}{2}}} \exp^{\frac{-\|y^k - \xi^k\|^2}{2(\sigma^2)^k}} \frac{1}{Z^k} \exp^{-U(\xi^k, \phi^k)} \tag{9}$$

which implies

$$\mathcal{P}\left[\mathcal{A}^k = a_*^k \mid \mathcal{Y}^k = y^k, \theta^k\right] = \frac{\exp^{\frac{-\|y^k - a_*^k\|^2}{2(\sigma^2)^k}} \exp^{-U(a_*^k, \phi^k)}}{\sum_{\xi^k} \exp^{\frac{-\|y^k - \xi^k\|^2}{2(\sigma^2)^k}} \exp^{-U(\xi^k, \phi^k)}} \tag{10}$$

In (10) the summation is over all possible realizations of $\mathcal{A}^k$. Thus, from a computational standpoint, handling (10) would be practically impossible, because this requires $(G)^{(2^k)^2}$ computations, where G (typically 256) represents the number of possible gray values and 2^k (typically 256) represents the size of the square image. One can view (10) as a *conditional likelihood function* to be maximized for estimating θ^k. To overcome the computational complexity and to make the parameter estimation problem tractable,

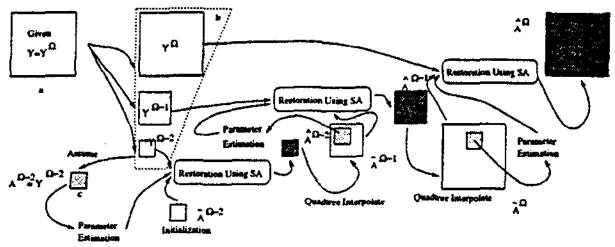

Figure 1. The parameter estimation and restoration.

(10) is approximated using a *conditional pseudolikelihood function*, (analogous to Besag [11]) as

$$\mathcal{P}\left[\mathcal{A}^k = a^k_* | \mathcal{Y}^k = y^k, \theta^k\right] \approx \prod_{i,j} \mathcal{P}\left[\mathcal{A}^k_{i,j} = a^k_{i,j} \mid \mathcal{A}^k_{m,n} = a^k_{m,n}, (m,n) \in \eta_{i,j}, \mathcal{Y}^k = y^k, \theta^k\right] \triangleq \hat{\mathcal{P}}\left[\mathcal{A}^k = a^k_* \mid \mathcal{Y}^k = y^k, \theta^k\right] \quad (11)$$

where, $\eta_{i,j}$ represents the neighborhood of the site (i,j). The conditional probability

$$\mathcal{P}\left[\mathcal{A}^k_{i,j} = a^k_{*i,j} \mid \mathcal{A}^k_{m,n} = a^k_{*m,n}, (m,n) \in \eta_{i,j}, \mathcal{Y}^k = y^k, \theta^k\right]$$

can be expressed as ([9]),

$$\hat{\mathcal{P}}\left[\mathcal{A}^k = a^k_* \mid \mathcal{Y}^k = y^k, \theta^k\right] = \prod_{i,j}\left[\frac{\exp\frac{-(y^k_{i,j} - a^k_{*i,j})^2}{2(\sigma^2)^k} \exp - \sum_{C:(i,j)\in c} V_c(a^k_{*i,j}, \phi^k)}{\sum_{\xi^k_{i,j}\in G}\left\{\exp\frac{-(y^k_{i,j} - \xi^k_{i,j})^2}{2(\sigma^2)^k} \exp - \sum_{C:(i,j)\in c} V_c(\xi^k_{i,j}, \phi^k)\right\}}\right] \quad (12)$$

The notation $C : (i,j) \in c$ denotes the set of all possible i,j pixel locations that fall into the clique $c \in \mathcal{C}$. The summation in the denominator of (12) has a computationally complexity of $G(2^k)^2$, because $\xi^k_{i,j}$ takes all possible values from the set G. This is orders of magnitude less than that required for solving (10), compare $G(2^k)^2$ with $G^{(2^k)^2}$. The numerical update equation for a homotopy map can be shown to be

$$\theta^k_{m+1} = \theta^k_m - \Delta\theta^k_m \quad (13)$$

where, $\Delta\theta^k_m \triangleq \lambda J^{-1}_{\theta^k_m}\left\{h(\theta^k_m, \lambda^k_{m+1}, \theta^k_{m-1})\right\} \frac{\partial}{\partial\lambda^k} J\left\{h(\theta^k_m, \lambda^k_{m+1}, \theta^k_{m-1})\right\}$, and $J_{\theta^k_m}$ is the Jacobian of the selected homotopy map and $0 \leq \lambda \leq 1$ is the homotopy parameter.

3. THE PARAMETER ESTIMATION AND IMAGE RESTORATION SCHEME

The proposed scheme of parameter estimation and image restoration is pictorially depicted in Figure 1. The description of the scheme in this section is based on Figure 1. Given: $\mathcal{Y}^\Omega$ the observed image of size $2^\Omega \times 2^\Omega$ (Figure 1 component a) and the degradation model $\mathcal{Y}^\Omega = \mathcal{A}^\Omega + \mathcal{W}^\Omega$. Estimate $\hat{\mathcal{A}}^\Omega$

Step 0: **Initialization**

1. Construct $\mathcal{Y}^{\Omega-1}, \mathcal{Y}^{\Omega-2}, \cdots, \mathcal{Y}^{\Omega-N}$ using the Gaussian pyramid scheme of Burt and Adelson [12] (Figure 1 component b, for $N = 2$).
2. Assume that at the coarsest resolution $(\Omega - N)$, $\mathcal{A}^{\Omega-N} = \mathcal{Y}^{\Omega-N}$ (Figure 1 component c). The basis for this assumption arises from the observation that the Gaussian pyramid construction is essentially a low pass filtering and subsampling scheme and if we go sufficiently down the pyramid, then the high frequency noise would be filtered out. In practice, it is found that a SNR of ≈ 15 dB at a resolution $\Omega - k$ increases to ≈ 35 dB at resolution $\Omega - k + 3$.

Step I: **Parameter estimation**

1. At the coarsest resolution $(\Omega - N)$: Let $\mathcal{A}^{\Omega-N} = \mathcal{Y}^{\Omega-N}$ and estimate the parameters $\phi^{\Omega-N}$ ($(\sigma^2)^{\Omega-N} \rightarrow$ assumed) by minimizing (12) using the homotopy continuation method (Section 2.2).
2. At resolution $(\Omega - k)$, $(k > N)$: Choose a portion of the image $\mathcal{A}^{\Omega-k}$ which is *sufficiently rich in information* equal to the size of the coarsest image, namely, $2^{\Omega-N} \times 2^{\Omega-N}$. Estimate the parameters $\left[\phi^{\Omega-k}, (\sigma^2)^{\Omega-k}\right]$ by minimizing (12) using the homotopy continuation method (Section 2.2).

Step II: **Image restoration**

1. At the coarsest resolution $(\Omega - N)$: Initialize $\mathcal{A}^{\Omega-N}$ to zero and minimize the posteriori energy function (7) using the simulated annealing algorithm [3] (Section 2.1).
2. At resolution $(\Omega - k)$, $(k > N)$: Initialize $\mathcal{A}^{\Omega-k} = \tilde{\mathcal{A}}^{\Omega-k}$ and minimize the posteriori energy function (7) using the simulated annealing algorithm (Section 2.1).

Step III: ***Coarse to fine resolution***

1. If not working at finest resolution Ω
   (a) $\mathcal{A}^{\Omega-k}$  $\longrightarrow \tilde{\mathcal{A}}^{\Omega-k+1}$
   (b) $k \longrightarrow k - 1$; go back to Step I.
2. At the finest resolution Ω
   (a) Output the restored image $\hat{\mathcal{A}}^\Omega$

4. EXPERIMENTAL RESULTS

Experiments were carried out on real images to validate the proposed scheme of modular integration and multiresolution and test its usefulness for image restoration[1]. The size of each image is 256×256 and was either obtained from various databases on the net. The images were degraded using additive Gaussian noise with 0 mean and some variance. In all our experiments we went through five resolution levels. The size of the coarsest resolution was 16×16. At each resolution the noise and the clique parameters were estimated and the estimated parameters were used for image restoration in the manner described in Section 3. Burt and Adelsons [12] procedure to construct Gaussian pyramid was used to construct images at different resolution (for example, Figures 2(c-g)).

Figure 2 shows the restoration at each resolution using the scheme proposed in this paper. Figure 2a is the original image and Figure 2b is the degraded image (SNR=5.609 dB). Table 1 gives the noise and the model parameters estimated at each resolution and also the SNR of the degraded

[1] Due to lack of space we give only two examples.

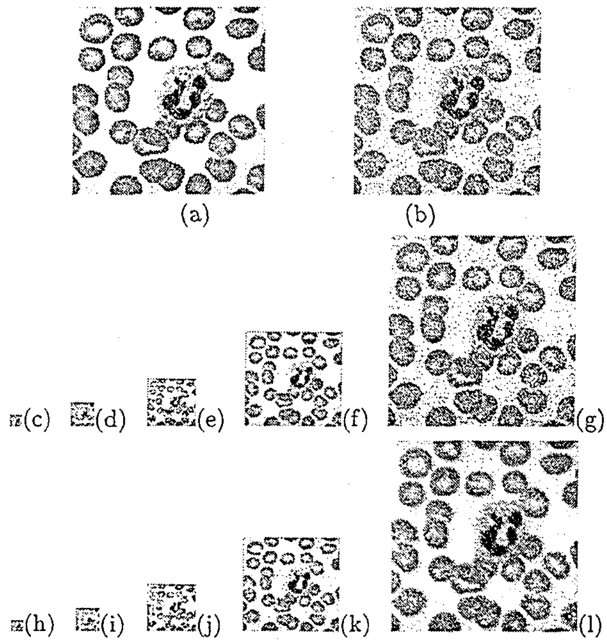

Figure 2. (a) Original image of the blood cells, (b) Noisy image (SNR=5.609 dB), (c-g) Image pyramid at 5 resolutions and (h-l) restored image at each resolution.

	σ^2	μ	γ	SNR (dB) (Noisy)	SNR (dB) (Restored)
16	0.02	0.0038	2.048	14.613	14.613
32	38.66	0.0101	2.677	13.803	7.332
64	39.4	0.0214	3.01	13.617	7.273
128	33.03	0.0285	3.321	12.368	8.885
256	42.23	0.025	3.53	5.609	8.281

Table 1. Parameters used at each resolution for image restoration (Figure 2).

and the restored image at each resolution. The image pyramid using the scheme proposed in [12] is shown in Figures 2(c-g). The restored images at the corresponding resolutions are shown in 2(h-l). As seen in Table 1 there is no SNR improvement at the coarsest resolution and infact the SNR becomes *bad* at intermediate resolution but at the finest resolution there is an overall 2.6 dB improvement in the SNR. In the rest of the experimental results that we present in this paper we only give the restored image at the finest resolution.

The second set of results are for a lab image and is shown in Figure 3. The parameters estimated and used at each resolution are given in Table 2. Figure 3(a) is the original

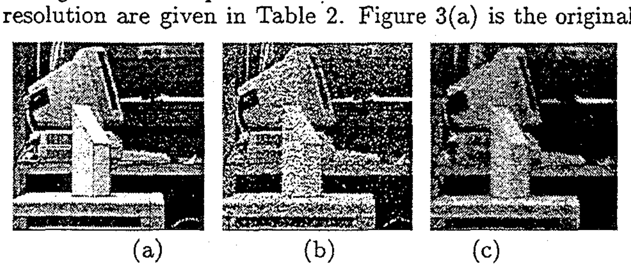

Figure 3. (a) Original image, (b) Noisy image (SNR=5.857 dB) and (c) Restored image (SNR= 7.929 dB).

Parameters (→) Resolution (↓)	σ^2	μ	γ
16	0.02	0.0158	1.52
32	34.67	0.0234	4.29
64	38.56	0.0281	4.945
128	39.24	0.0243	4.899
256	49.53	0.0299	4.998

Table 2. Parameters used at each resolution for image restoration (Figure 3).

image and Figure 3(b) is the degraded image (SNR= 5.857 dB). The final restored image is shown in Figure 3(c) and the SNR is 7.929 dB. The SNR improvement is 2.072 dB at the finest resolution.

5. CONCLUSION

In this paper we have developed a generalized framework based on modular integration and multiresolution for solving image processing and vision tasks. We have demonstrated the usefulness of the proposed framework by applying it to the problem of image restoration.

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