

Unsupervised Image Segmentation using Tabu Search and Hidden Markov Random Field Model

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Abstract

We propose a Tabu search based Expectation Maximization (EM) algorithm for image segmentation in an unsupervised frame work. Hidden Markov Random Field (HMRF) model is used to model the images. The observed image is considered to be a realization of Gaussian Hidden Markov Random Field (GHMRF) model. The segmentation problem is formulated as a pixel labeling problem. The GHMRF model parameters as well as the image labels are assumed to be unknown. This incomplete data problem is solved using the notions of expectation maximization. The expectation step obtains the MAP estimate of the image labels, assuming the availability of parameter estimates. This is achieved by the proposed Tabu Search Algorithm. The estimated image labels are used to obtain the estimates of parameters in the maximization step. Eventually, the EM algorithm converges to the desired labelization. Our algorithm does not require the proper initial estimates of the parameters. Simulation results are presented for three and four class synthetic images.

1. Introduction

The problem of Image Segmentation has attracted the attention of the researchers for quite some time. The problem has been addressed in both deterministic and stochastic framework. In stochastic domain, Markov Random Field model has been extensively used as the image models [1, 2, 3, 4]. The segmentation problem is also addressed as supervised and unsupervised image segmentation [1, 2]. In unsupervised domain, one can still categorize as partially unsupervised or complete unsupervised problem. When the number of class, the image labels and the image model parameters are assumed to be unknown, the problem is viewed as a

complete unsupervised problem. If the number of classes are assumed to be known, but pixel labels and model parameters are assumed to be unknown, the problem is categorized as partial unsupervised one [1,3]. In this article, we address the image segmentation problem in complete unsupervised sense.

Expectation Maximization [EM] algorithm is a potential tool to handle incomplete data problems [5]. MRF model together with EM algorithm has been successfully employed to segment Brain Magnetic Resonance (MR) images [6]. In [6], the image estimates are the Maximum a Posterior (MAP) estimates.

Hidden Markov Model (HMM) have been widely used for speech recognition [7]. In the same line, a Hidden Markov Random Field (HMRF) has been proposed by Zhang et. al [8], to model the observed image. This model which is a stochastic process generated by a MRF, whose state sequence can not be observed through a field of observations. The importance of HMRF model derives from the MRF theory, in which the spatial information of an image is encoded through contextual constraints of neighbouring pixels.

Gaussian Hidden Markov Random Field (GHMRF) Model has been employed in [8] to model the observed image. The image labels are obtained using the MAP criterion and the model parameters are the maximum likelihood (ML) Estimates. This EM algorithm with GHMRF model has been successfully applied to Brain MR Image segmentation [8]. The MAP estimates of the image are obtained by the Iterated Conditional Model (ICM) algorithm [1]. Hence, the performance of the algorithm greatly depends upon the initial choice of the model parameters. The initial parameters were obtained from the histogram of the noisy image.

In this paper, we propose Tabu EM algorithm which does not depend upon proper initial choice of the model

parameters. Tabu search is an adaptive procedure with the ability to make use of many methods such as linear programming algorithms and specialized heuristics, has the potentiality to circumvent the local optimality [9,10,11]. Tabu search based algorithms have been successfully employed for image restoration [4] and image segmentation problem [12]. In our proposed algorithm, we have employed a hybrid Tabu algorithm. We address the problem of unsupervised image segmentation using EM algorithm. The observed image is modeled as GHMRF model and the GHMRF model parameters as well as image labels are assumed to be unknown. The number of labels are also assumed to be unknown. In the expectation step, the image labels are estimated using MAP estimation criterion. The MAP estimates of the labels are obtained by our proposed hybrid Tabu Search algorithm with an arbitrary value of image model parameters. Using these estimated image labels, the estimates of the model parameters are obtained as the ML estimates in the maximization step of the algorithm. Thus, the image labels and the model parameters are estimated recursively and eventually convergence to stable labelization. The proposed Tabu EM algorithm does not need the proper choice of initial parameters. Global convergent property of our algorithm can be attributed to notion of Tabu search in the algorithm. The algorithm has been successfully tested with three and four class synthetic noisy images.

2. Hidden Markov Random Field Model (HMRF)

Hidden Markov Models (HMM) have been applied to the problem of speech recognition [7]. HMMs are defined as stochastic processes generated by a Markov chain whose state sequence can not be observed directly, but only through a sequence of observations. Each observation is assumed to be a stochastic function of state sequence. A special case of a HMM is considered, where the underlying stochastic process is a MRF instead of a Markov chain and therefore, not restricted to one dimension. This special case is referred to as Hidden Markov Random Field (HMRF) model [8].

Let the images are assumed to be defined on a discrete rectangular lattice $S = (N \times N)$. Let X denote the random field associated with the labels of the original image. The label process X is assumed to be MRF with respect to a neighbourhood system η and is described by its local characteristics.

$$\begin{aligned} P(X_{ij} = x_{ij} | X_{kl} = x_{kl}, kl \in S, (k,l) \neq (i,j)) \\ = P(X_{ij} = x_{ij} | X_{kl} = x_{kl}, k, l \in \eta) \end{aligned}$$

Since X is a MRF, or equivalently Gibbs distributed, the joint distribution can be expressed as

$$P(X = x | \phi) = \frac{1}{Z'} e^{-U(x, \phi)} , \text{ where } Z = \sum_x e^{-U(x, \phi)}$$

the partition function, ϕ denote the clique parameter vector, $U(x, \phi)$ is the energy function and is of the form $U(X, \phi) = \sum_{c(i,j) \in c} V_c(x, \phi)$, $V_c(x, \phi)$ is the clique potential. Y is the observed random Field. For any realization x , the random variables Y_i are conditional independent

$$P(Y | X) = \prod_{i \in S} P(y_i | x_i) \quad (1)$$

The joint probability of (X, Y) can be expressed as

$$P(Y, X) = P(Y | X)P(X) = P(X) \prod_{i \in S} P(y_i | x_i)$$

According to the local characteristics of MRF, the joint probability distribution of pair (X_i, Y_i) given the neighbourhood configuration of X_{η_i} is

$$P(y_i, x_i | x_{\eta_i}) = P(y_i | x_i)P(x_i | x_{\eta_i}) \quad (2)$$

Thus, the marginal probability distribution of Y_i dependent on θ and X_{η_i}

$$\begin{aligned} P(y_i | x_{\eta_i}, \theta) &= \sum_{l \in L} P(y_i, l | x_{\eta_i}, \theta) \\ &= \sum_{l \in L} P(y_i, l, \theta)P(l | x_{\eta_i}) \end{aligned} \quad (3)$$

where $\theta = \{\theta_l, l \in L\}$. (3) is the Hidden Markov Random Field model, l denotes the number of labels.

With Gaussian distribution (3) can be expressed as

$$P(y_i | x_{\eta_i}, \theta) = \sum_{l \in L} g(y_i, \theta_l)P(l | x_{\eta_i}) \quad (4)$$

(4) is referred to as the Gaussian Hidden Markov Random Field (GHMRF) model. $g(y_i, \theta_l)$ is the Gaussian probability density function.

3. EM Framework

The MRF model parameters $\theta = \{\theta_l; l \in L\}$ need to be estimated. Specifically for Gaussian MRF model for the observed image y , the mean and standard deviation of each Gaussian class parameters $\theta_l = (\mu_l, \sigma_l)$ need to be estimated. Since the class label and the model parameter are unknown and interdependent, the data set is said to be incomplete and hence the problem is incomplete data problem. EM algorithm [7] is one of the extensively used techniques to solve this problem.

In EM algorithm, the following is adopted; (i) The missing part \hat{x} is estimated with the current θ estimate and then \hat{x} used to form the complete data set $\{\hat{x}, y\}$, (ii) θ is estimated by maximizing the expectation of complete data log likelihood $E[\log P(X, Y | \theta)]$. In this case, in E step, the MAP estimates of the class labels are obtained to form the complete data set. In the M step, the ML estimate of the parameter is computed using the class labels in E step.

At the time step “t”

E - step

MAP estimate of X is obtained

$$\hat{X}^{(t)} = \arg \max_{x \in X} P(X | Y, \theta^{(t)}) \quad (5)$$

M - step

ML estimate of the parameters is obtained

$$\hat{\theta}^{(t+1)} = \arg \max_{\theta} P(Y | \theta, X^{(t)}) \quad (6)$$

4. MAP Estimate of Labels

In the labeling problem, let x^* denote the true but unknown labeling configuration and \hat{x} denote the estimate for x^* . x^* and \hat{x} are realization of random field X, which is modeled as MRF. The observed image y is a realization of GHMRF. The problem is to recover x^* from the observed image y . The following optimality criterion is considered.

$$\hat{x} = \arg \max_x P(X | Y, \theta) \quad (7)$$

where θ is the model parameter.

Using Baye's rule,

$$\hat{x} = \arg \max_x \frac{P(Y | X, \theta)P(X)}{P(Y)} \quad (8)$$

Since Y is known, the denominator of (8) is a constant

$$\hat{x} = \arg \max_x P(Y | X, \theta)P(X) \quad (9)$$

Since, X is a MRF, the prior probability distribution is given as

$$P(X) = \frac{1}{Z} e^{-U(x)}$$

It is also assumed that the pixel intensity y_i follows a Gaussian distribution with parameters $\theta_i = \{\mu_i, \sigma_i\}$, given the class label $x_i = l$

$$P(y_i | x_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(y_i - \mu_i)^2}{2\sigma_i^2}\right) \quad (10)$$

Using the assumption of conditional independence

$$\begin{aligned} P(Y | X) &= \prod_{i \in S} P(y_i | x_i) \\ &= \prod_{i \in S} \left[\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} - \log(\sigma_{x_i})\right) \right] \end{aligned} \quad (11)$$

(11) can be expressed as

$$P(Y | X) = \frac{1}{Z'} \exp(-U(Y | X))$$

Where

$$U(Y | X) = \sum_{i \in S} u(y_i | x_i) = \sum_{i \in S} \left[\frac{(y_i - \mu_{x_i})^2}{2\sigma_{x_i}^2} + \log(\sigma_{x_i}) \right]$$

and $z' = (2\pi)^{N/2}$

Using the above, (9) can be expressed as

$$\begin{aligned} \hat{x} &= \arg \max_x \left[\frac{1}{z} \exp(-U(x)) \frac{1}{z'} \exp(-U(Y | X)) \right] \\ &= \arg \max_x \exp[-\{U(Y | X) + U(X)\}] \end{aligned} \quad (12)$$

(12) is equivalent to minimizing the following

$$\hat{x} = \arg \min_x [U(Y | X) + U(X)] \quad (13)$$

The MAP estimate in (13) is obtained by employing the Tabu search algorithm.

5. Tabu Search

Conventional local search algorithm suffers from the problem of local minima trapping. Global search methods like random optimization, simulated Annealing algorithm and Genetic algorithm yield almost always global optimal solution at the cost computational burden. Such algorithms are time consuming because the algorithm revisits the point already visited in the search space. Tabu search algorithms ameliorates such problems by keeping track of points already visited, as the Tabu moves [9,10,11]. Tabu search can be viewed

as a strategy to solve combinational optimization problems and is an adaptive procedure which overcomes the limitations of local optimality. Tabu search has been successfully applied to traveling sales man problem, scheduling, integrated circuit design etc. Recently, Tabu search has also been applied for cell image segmentation [12]. The strategy of Tabu search has also been applied to determine the optimal coefficient of a digital filter [13].

5.1 Tabu Search for MAP Estimation

In this article we have proposed the Tabu Search algorithm which mostly makes the Tabu array with recent moves of minimum energy but also with moves with higher energy has been accepted with a probability. This strategy is our aspiration condition. The basic steps of the algorithm to obtain the MAP estimate is as follows.

5.2 Tabu Algorithm

1. Initialize the initial temperature T_{in} .
2. The initial image for the algorithm is the degraded image.
3. A Tabu list, i.e. Tabu image set is created to store the recent moves, i.e. the image estimates of the algorithm. The set is of fixed length.
4. From the current move or image, the next Tabu image is generated. i) Perturb $x_{ij}(t)$ with a zero mean Gaussian Distribution with a suitable variance. ii) Evaluate the energy $Up(x_{ij}(t+1))$ & $Up(x_{ij}(t))$. If $\Delta f = (Up(x_{ij}(t+1)) - Up(x_{ij}(t))) < 0$, assign the modified value as the new value. If $\Delta f > 0$, accept the $x_{ij}(t+1)$ with a probability (if $\exp(-\Delta f/T(x)) > \text{random}(0,1)$). iii) Repeat step (ii) for all the pixels of the image.
5. Compute the power of the updated image $x(t+1)$ as $Px(t+1)$ and compare it with the powers of the tabu list named as Tabu energy, if $Px(t+1) < P_{Tabu}$ then $x(t+1)$ is a Tabu image.
6. Aspiration condition: If $Px(t+1) > P_{Tabu}$, accept $x(t+1)$ as Tabu image with probability
7. Update the Tabu list.
8. Decrease the Temperature according to the logarithmic cooling schedule.
9. Repeat step 4 – 8 for a fixed number of iterations

6. Tabu Expectation Maximization Algorithm (TEM)

The maximum likelihood estimate of the GHMRF model parameters are obtained by maximizing the likelihood function $P(Y | \theta, X^{(t)})$. In the similar

approach of zhang [8], the update equation reduces to the following.

$$\mu_l^{(t+1)} = \frac{\sum_{i \in S} P^{(t)}(l | y_i) y_i}{\sum_{i \in S} P^{(t)}(l | y_i)} \quad (15)$$

$$(\sigma_l^{(t+1)})^2 = \frac{\sum_{i \in S} P^{(t)}(l | y_i) (y_i - \mu_l)^2}{\sum_{i \in S} P^{(t)}(l | y_i)} \quad (16)$$

6.1 Tabu-HMRF-EM Algorithm

1. Initialize the class label to random values and take an arbitrary parameter set.
2. Compute the likelihood distribution $P^{(t)}(y_i | x_i)$ and estimate the class labels by MAP estimation.

$$\hat{x}^{(t)} = \arg \max_x P(X | Y, \theta)$$

Or in other words

$$\hat{X}^{(t)} = \arg \min_x [U(Y | X) + U(X)]$$

This is obtained by the proposed Tabu Search algorithm.

3. Compute the posterior distribution.

$$P^{(t)}(l | y_i) = \frac{g^{(t)}(y_i | \theta_l) P^{(t)}(l | x_{\eta_i})}{P(y_i)}$$

4. Update the parameters

$$\mu_l^{(t+1)} = \frac{\sum_{i \in S} P^{(t)}(l | y_i) y_i}{\sum_{i \in S} P^{(t)}(l | y_i)}$$

$$(\sigma_l^{(t+1)})^2 = \frac{\sum_{i \in S} P^{(t)}(l | y_i) (y_i - \mu_l)^2}{\sum_{i \in S} P^{(t)}(l | y_i)}$$

5. Steps 2-5 are repeated until a fixed number of iterations.

7. Simulation

We have considered both 4 and 3 class synthetic images in our simulation. These images are synthesized using Gibb's sampler as shown in Fig 1(a) and 4(a). The noisy image of SNR 18dB corresponding to 4 class

image is shown in Fig 1(b). This noisy image is modeled as GHMRF model with the parameters μ , σ and the apriori model clique potential function is $V_c(x_{ij}) = -\delta$ if $x_i = x_j$ and δ if $x_i \neq x_j$. Our proposed TEM algorithm is applied to the noisy image of Fig 1(b). The number of classes is assumed to be 6 and the initial values of the parameters corresponding to these classes are presented in table 1(a) and 1(b). The initial parameters were selected not with any prior estimation rather arbitrarily. With these initial parameters the MAP estimate of the image labels is obtained from the noisy image. The length of the Tabu Image array is 10. The initial temperature is chosen to be $T_{in}=0.55$ and the cooling schedule followed an exponentially decaying function. The δ is chosen to be $\delta=0.92$. The MAP estimated image together with the given noisy image are used in the EM algorithm to estimate the model parameters.

Thus the model parameters and image labels are recursively estimated for all the classes and eventually the algorithm converges to stable labelization. Hence, the segmented image obtained is as shown in Fig 1(c). It is clear from Fig. 1(c) that there are few points in different classes. These points have been assumed as classes and hence the algorithm converged to a very low value of the model parameters as given in Table 1(a) and 1(b). The estimated parameters (μ , σ) of class 1 and class 5 are significantly low as observed from table 1(a) and 1(b). This implies that the algorithm could segment the image broadly into four classes i.e. class 2, 3, 4 and 6. The algorithm started with 6 classes and eventually the algorithm converged to 4 distinct classes. The arbitrary initial values of the parameters are given in Tabu 1(a) and 1(b). The algorithm converged in 15 iterations. The estimated value of μ and σ over these 15 iterations are shown in Fig 2. It is observed that the initial parameters are not close to estimated parameters. The estimates of parameters do not show any monotonically decreasing trend. This is intuitively appealing because of the use of stochastic algorithm. The algorithm is also tested with another noisy image of SNR 18 dB as shown in Fig. 3(a). The corresponding segmented image is shown in Fig. 3(b). The parameters used for this image are $\delta=0.98$, iteration $K=20$, $T_{in}=0.1$. It is observed from Fig. 4(b) that a spurious class is retained in the segmented image. This is because of the increased noise strength in the observed image.

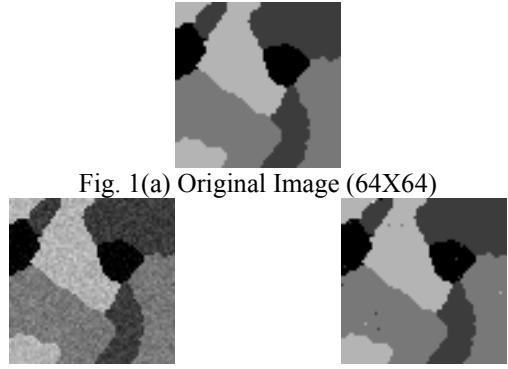


Fig. 1(a) Original Image (64X64)

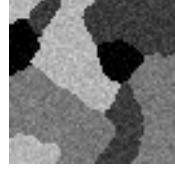


Fig. 1(b) Noisy Image



Fig. 1(c) Segmented Image

Fig. 1 Segmentation of 4 class synthetic image with SNR=20dB

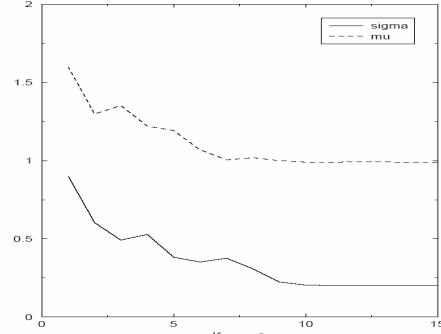


Fig. 2 Estimation of model parameters over iteration corresponding to the noisy image with SNR = 20dB



Fig. 3(a) Noisy image

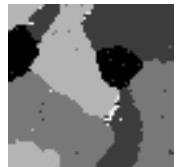


Fig. 3(b) Segmented image

Fig. 3 Segmentation of 4 class synthetic image with SNR= 18dB

Table 1(a)

	μ_1	μ_2	μ_3	μ_4	μ_5	μ_6
20dB	Initial	.2	1.6	2.3	2.2	1
	Final	.051	.991	2.00	2.989	.683
18dB	Initial	2.2	0.6	1.3	1.2	1
	Final	.005	.994	2.00	2.99	.989

Table 1(a) The initial and estimated parameters “ μ ” for different classes GHMRF model

	σ_1	σ_2	σ_3	σ_4	σ_5	σ_6
20dB	Initial	.5	.9	.3	.5	.2
	Final	.080	.200	.199	.201	.001
18dB	Initial	1.5	.2	.8	1.5	.5
	Final	.088	.253	.252	.250	.228

Table 1(b) The initial and estimated parameters “ μ ” for different classes GHMRF model

We have also tested our algorithm for a 3 class image of size (128X128) as shown in Fig. 4(a). The corresponding noisy and segmented images are shown in Fig. 4(b) and Fig. 4(c) respectively. As observed from Fig. 4(c) there are few white dots in one class and they are the misclassified labels. The phenomenon observed in this case is similar to that of four class image. The parameter of the algorithm are $T_{in}=0.2$, $\delta=1.2$, number of iterations $K=10$. The initial and final estimates of the model parameters are presented in table 2(a) and Table 2(b). The parameters estimates over the iterations are shown in Fig. 5, where it is seen that the parameter estimates follow a monotonically decreasing trend. The results of algorithm tested with SNR=18 dB are shown in Fig. 6(b). The few misclassified points are due to the high noise strength of the image. However, three distinct classes could be obtained from our algorithm. Hence the algorithm could be successfully tested for 3 and 4 class images.



Fig. 4(a) Original Image (128X128)

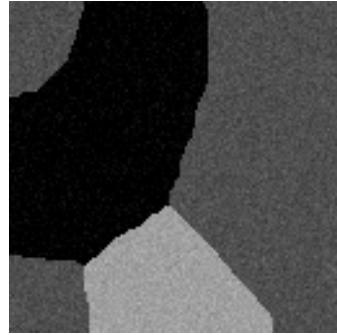


Fig. 4(b) Noisy Image (SNR=20dB)



Fig. 4(c) Segmented Image

Fig. 4 Noisy and segmented images for a three class

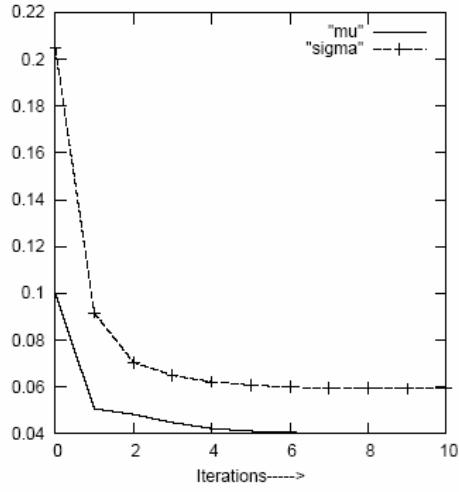


Fig. 5 estimation of μ and σ over iteration corresponding to 3 class image with SNR =20 dB



Fig. 6(a) Noisy Image (SNR=18 dB)



Fig. 6(b) Segmented Image
Fig. 6 Noisy and Segmented for a three class image with SNR=18dB

8. Conclusion

In this paper we have addressed the problems of image segmentation in an unsupervised framework. We have modeled the image the noisy image using Hidden Markov Random Field (HMRF) model. In the expectation step of the EM algorithm, the MAP estimate of the image labels are obtained by employing the proposed Tabu Search algorithm. Thus the proposed TEM algorithm does not depend on a good initial choice of the parameters and hence global convergent property of the algorithm is retained. The algorithm converged to stable labelization starting from an arbitrary set of parameters. This implies that the algorithm could converge to the correct estimates starting from a parameter set. The parameters are the ML estimates. The algorithm could be tested successfully with three as well as four class noisy images. The performance of the algorithm deteriorated with increase in the noise strength. Currently attempts are made to estimate the model parameters using the notion of Tabu search.

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