Estimating Ultimate Load Carrying Capacity of Shell Foundation: Neural Network model and Sensitivity Analysis

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Abstract. The use of computational intelligence techniques is becoming popular across several disciplines. One of the key criteria for examining an existing algorithm is to apply it on different data sets. This paper is an application oriented work that uses a computational intelligence technique to analyze an important civil engineering problem which evaluates the suitability of neural network in estimating the ultimate load of shell foundations. In addition, to understand the relative importance of input parameters, sensitivity analysis using various methods are presented. Neural interpretation diagrams are drawn to know the relation between inputs and the output. An empirical equation developed using the connection weight and biases of the trained ANN model with reasonable accuracy.

Keywords: Shell foundation; Artificial Neural Network; Empirical equation; Sensitivity analysis

1 Introduction

Shell foundations are complex yet important components of modern engineering practice. The complicated structural constitution makes the analysis and engineering design of these types of foundations difficult. In spite of the complexity in their structure, shell foundations have proven economical in many situations. The research on shell foundation on its structural behavior has proved that shells save considerably in material as compared to their flat counterparts. The geotechnical aspects of shell foundation have shown that they possess more bearing capacity and lesser settlement as compared to their flat counterparts. In the literature available on the shell foundations, were analyzed using mathematical formulations, finite element method, and finite difference method. And few reports that to simulate the soil structure interaction beneath the shell, linear Winkler and Pasternak soil models were utilized (Abdel-Rahman [1]; Bagherizadeh et al. [2]; Paliwal et al. [3]). Currently, the structural analysis of shell foundation is using membrane theory that assumes uniform contact pressure under the footing. But in actual cases, it is reported that the contact pressure under footing is non-uniform. Literature review reveals that, the amount of research on shell foundations is far less as compared to the research on their flat counterparts viz. strip and square foundations. Hence, there is need to explore suitable techniques which will be able to understand proper behavior of these foundations. In this study, one of the key statistical methods i.e. artificial neural networks is considered to study the response of the shell foundations.

Artificial Intelligence (AI) has overtaken most of the fields in engineering since past few decades. Among which neural network algorithm which was proposed in 1940's has been the hotspot among the researchers since several decades (Maozhun and Ji [4]). It has shown a certain degree of success to determine the structure and parameters of the geotechnical models in which the relationship between the physical processes is complex. The main advantage of Artificial Neural Network is that no assumptions has to be made during its modelling. The structure and operations of ANN's have been explained in detail by Hagen et al. [5]. Several researchers have studied the scope of application of ANN's in geotechnical engineering (Chang et al. [6], Adeli [7]; Pichler et al. [8]; Shahin et al. [9]- [13]). The application includes modelling the monotonic and hysteretic behaviour of geomaterials (Basheer [14]), modelling load capacities of pile foundations (Das and Basudhar [15]), liquefaction prediction (Hanna et al. [16]), estimating the compaction characteristics and permeability (Sinha and Wang [17]), prediction of ultimate bearing capacity of shallow foundations (Behera and Patra [18], estimation of cyclic load induced settlement of strip footing (Sasmal and Behera [19]).

The application of intelligent techniques in analyzing the behavior of shell foundations is not explained in the Literatures of last two decades which can be observed from the systematic literature review presented before. Hence, in this study an attempt has been made to apply the Levenberg-Marquardt technique to a new problem domain. Not only the accuracy of the technique is verified but also an empirical expression is also derived to ease the tasks of the practicing engineers. The details of the model are explained below.

2 Artificial Neural Network (ANN)

ANNs has been reported as an effective tool for analyzing civil engineering problems in recent years. Prior knowledge of the input and output is not required in this approach, which makes them different from other statistical analysis. In this study, the ultimate load of shell foundation is expressed as a function of its associated factors like the type of shell (t), embedment ratio (D/B) and angle of shearing resistance (ϕ).

2.1 Preprocessing of dataset

The database for the ANN analysis was generated from the experimental study of shell foundation in dry sand by Hanna and Abdel-Rahman [20]. Experiments on 9 prototypes in loose, medium and dense sands both as surface and embedded footing were available. The parameters available in the data are the type of footing (t), embedment ratio (D/B) and angle of shearing resistance (ϕ) and ultimate load (Q_u). Total of fifty-four number of results is used for the neural analysis. Out of which thirty-six numbers of results are used for training the network and the rest eighteen numbers of results are used for testing the network. The input parameters are the type of footing (t), embedment ratio (D/B) and angle of shearing resistance (ϕ) and the output is ultimate load (Q_u). The type of footing is defined as

t =1	Strip flat model
=2	Triangular (1) shell model (*a/b*=1/2)
=3	Triangular (2) shell model (*a/b*=1)
=4	Circular flat model
=5	Conical (1) shell model (*a/b*=1/2)
=6	Conical (2) shell model (*a/b*=1)
=7	Square flat model
=8	Pyramidal (1) shell model (*a/b*=1/2)
=9	Pyramidal (2) shell model (*a/b*=1)

Where *a/b* is rise-to-half width ratio.

Inputs are normalized in the range [-1, 1] using Eq. (1) according to Behera and Patra [18].

$$X_{normalized} = 2\left(\frac{X - X_{minimum}}{X_{maximum} - X_{minimum}}\right) \quad (1)$$

Where, $X_{normalized}$ = normalized input value, x = actual value of input, $x_{maximum}$ = maximum value of input , $x_{minimum}$ = minimum value of input.

3 Development of neural network model

The training is done using a Levenberg-Marquardt technique. Mean Squared Error (MSE) and correlation coefficient (R) are taken as performance evaluators.

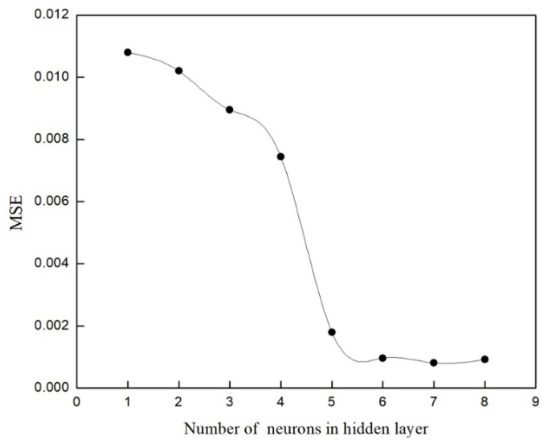

Fig. 1. Model Selection

By varying the number of neurons in the hidden layer the maximum R and minimum MSE value are obtained as 0.99852 and 0.000811 respectively when the number of hidden layer neuron = 7. But for keeping the ANN model simple the number of neurons in the hidden layer is chosen as 6 where the R-value of 0.99824 and MSE of 0.000968 is obtained as it gives comparable results. So 3-6-1 (number of inputs-number of hidden layer neurons-number of outputs) is chosen as the final neural architecture. The model performance with increase in the number of hidden layer neuron is given in Fig. 1.

Table 1. Details of weights and biases

Neuron	Weight				Bias	
	w_{ik}			w_k		
	Type	D/B	φ	Q_u	b_{hk}	b_o
Hidden Neuron k (= 1)	-0.53	1.73	-1.27	0.37	2.66	0.36
(= 2)	-0.46	-0.02	-0.75	-1.43	1.07	
(= 3)	0.65	2.24	0.73	0.17	-0.41	
(= 4)	3.94	0.03	0.08	-0.29	1.46	
(= 5)	-2.13	-0.02	-0.81	-0.29	-1.48	
(= 6)	2.36	-0.50	-1.81	-0.13	2.84	

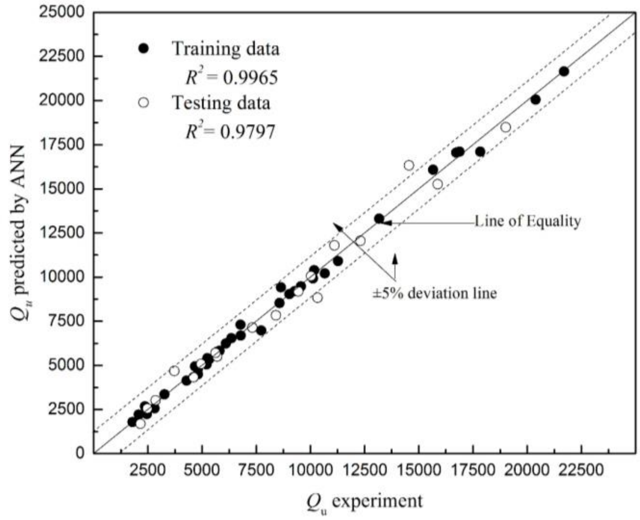

Fig. 2. Comparison between the reported results and results from the LMNN model

After the simulation of the neural network model using the optimal conditions, weights and biases obtained from the analysis are shown in Table 1. The parameters listed in Table 1 and there use in developing the empirical expression, is discussed in section 5. Comparison between predicted ultimate load (Q_u) with experimental Q_u is shown in

Figure 2. The R^2 value between the observed and predicted data for training data is 0.9965 and that for testing data is 0.9797 which indicates that the model is the best fit.

4 Sensitivity analysis (SA)

Sensitivity Analysis (SA) is a significantly important tool used during pre-processing of a model in neural network. It helps to understand the inner complex relationship in the system model. Sensitivity analysis investigates the influence degree of a certain parameter on the output of the model by changing that parameter of the system models within its reasonable range (Maozhun and Ji [4]). Based on the trained weights and biases (Table 1) of the neural network model, Pearson's correlation, Spearman's rank correlation, Variable perturbation method, Garson's algorithm, connection weight approach and Weight magnitude analysis are the methods used in this study.

Table 2 shows the Pearson's and Spearman's correlation. From the correlation indices it is evident that the ultimate load (Q_u) is highly correlated with the parameter φ, followed by D/B and type of footing (t).

Table 2. Correlation analysis

Variables	Pearson's correlation				Spearman's correlation			
	Type (t)	D/B	φ	Q_u	Type (t)	D/B	φ	Q_u
Type (t)	1.00	0.00	0.04	0.15	1.00	-0.10	0.28	0.14
D/B		1.00	0.00	0.57		1.00	-0.03	0.27
φ			1.00	0.74			1.00	0.67
Q_u				1.00				1.00

4.1 Variable Perturbation technique

Using this method, the most influencing input parameter is determined by varying the inputs by same percentage. The most influencing input will be that which makes highest deviation in output. The parameters are perturbed in the range of $\pm$20% @10% from their mean value. Then the effect of perturbation is evaluated for each parameter. The sensitivity of parameter is based on the change in a parameter i.e. Sensitivity Index (S_i). This parameter is calculated following the formula for sensitivity (S_i, %) according to Liong et al. [21] using Equation (2). More the deflection from the base value (sensitivity index = 0), the parameter is more sensitive. From Fig. 3, it can be concluded that φ is the most influencing parameter on Q_u.

$$S_i = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{\%\ change\ in\ output}{\%\ change\ in\ input}\right) \times 100 \quad (2)$$

Where, N = 54

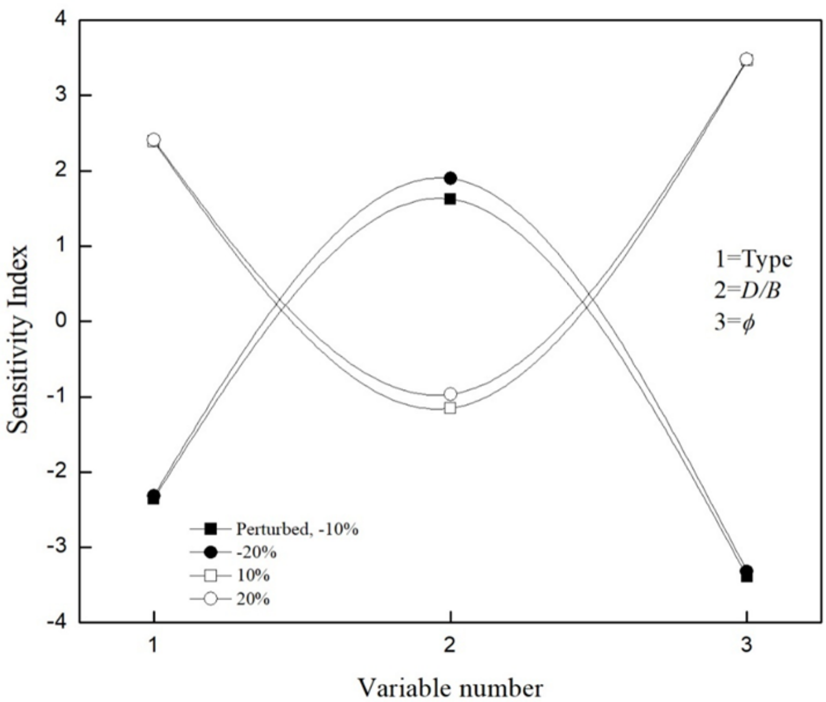

Fig. 3. Variable perturbation

4.2 Weight methods

The relative importance of the variables using the Garson's algorithm, Connection weight approach and the Weight magnitude analysis are presented in the Table 3. Analysis using first two methods are carried out by using the steps as described by (Olden et al. [22]). Weight magnitude analysis is done using the procedures mentioned in (Yao et al. [23]). The rankings of the input variables are tabulated as Table 4.

Table 3. Importance of input parameters

	Sensitivity index				Relative importance		
Parameters	-20%	-10%	10%	20%	Garson algorithm	Connection weight	Weight magnitude analysis
Type (*t*)	-2.32	-2.35	2.39	2.41	48.36	-0.24	2.56
D/B	1.90	1.62	-1.15	-0.97	20.70	1.11	1.15
φ	-3.32	-3.39	3.47	3.48	30.94	1.18	1.38

Table 4. Ranking of input parameters

	Method adopted			
Parameters	Variable perturbation	Garson's algorithm	Connection weight	Weight magnitude analysis
Type (*t*)	2	1	3	1
D/B	3	3	2	3
φ	1	2	1	2

4.3 Neural Interpretation Diagram

Neural interpretation diagram (NID) is a visualization tool to observe the connection weights. The line thickness demonstrates the relative magnitude of the weight. Solid and dotted lines represent positive and negative connection weights respectively. Grey circles show the inputs those affect the output adversely, while blank circle shows the input that affect the output directly.

With the weights obtained from the neural analysis as shown in Table 1, a NID is presented in Figure 4. From the connection weights shown in Table 3 and from Figure 4, it can be seen that type of footing is having indirect relation with Q_u. And the inputs *D/B* and φ is having direct relation with Q_u.

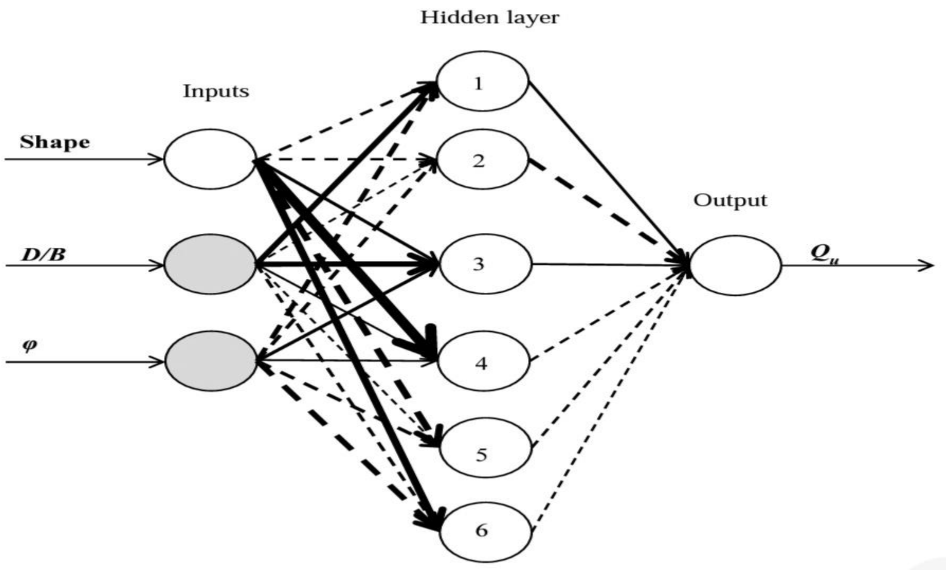

Fig. 4. Neural Interpretation Diagram

5 Development of ANN model equation

The weights and biases listed in Table 1 are used to develop a new empirical expression for calculating the ultimate load capacity of shell foundations using the above mentioned input parameters. The procedures given in Goh et al. [24] are adapted in this study to develop the equation. The steps are explained with the help of Eqs. (3) – (16).

$$Z_n = f\{b_o + \sum_{k=1}^{h}[w_k f(b_{hk} + \sum_{i=1}^{m} w_{ik}X_i)]\} \tag{3}$$

Where
Z_n= normalized value of output
f = transfer function
b_o = bias at the output layer
h = no. of neurons in the hidden layer
w_k = connection weight between k^{th} neuron of hidden layer and single output neuron

b_{hk} = bias at the k^{th} neuron of hidden layer
m = no. of input variables
w_{ik} = connection weight between i^{th} layer of input and k^{th} neuron of hidden layer
X_i = normalized value of inputs in the range [- 1, 1]

$$A_1 = -0.53t + 1.73\left(\frac{D}{B}\right) - 1.27\varphi + 2.66 \quad (4)$$

$$A_2 = -0.46t - 0.02\left(\frac{D}{B}\right) - 0.75\varphi + 1.07 \quad (5)$$

$$A_3 = 0.65t + 2.24\left(\frac{D}{B}\right) + 0.73\varphi - 0.41 \quad (6)$$

$$A_4 = 3.94t + 0.03\left(\frac{D}{B}\right) + 0.08\varphi + 1.46 \quad (7)$$

$$A_5 = -2.13t - 0.02\left(\frac{D}{B}\right) - 0.81\varphi - 1.48 \quad (8)$$

$$A_6 = 2.36t - 0.5\left(\frac{D}{B}\right) - 1.81\varphi + 2.84 \quad (9)$$

$$B_1 = 0.37\left(\frac{e^{A_1} - e^{A_1}}{e^{A_1} + e^{A_1}}\right) \quad (10)$$

$$B_2 = -1.43\left(\frac{e^{A_2} - e^{A_2}}{e^{A_2} + e^{A_2}}\right) \quad (11)$$

$$B_3 = 0.17\left(\frac{e^{A_3} - e^{A_3}}{e^{A_3} + e^{A_3}}\right) \quad (12)$$

$$B_4 = -0.29\left(\frac{e^{A_4} - e^{A_4}}{e^{A_4} + e^{A_4}}\right) \quad (13)$$

$$B_5 = -0.29\left(\frac{e^{A_5} - e^{A_5}}{e^{A_5} + e^{A_5}}\right) \quad (14)$$

$$B_6 = -0.13\left(\frac{e^{A_6} - e^{A_6}}{e^{A} + e^{A_6}}\right) \quad (15)$$

$$Q_n = B_1 + B_{2_1} + B_3 + B_4 + B_5 + B_6 + 0.36 \quad (16)$$

Equation 16 gives the normalized value of output *i.e.* the ultimate load (Q_u). It can be denormalized using the following expression:

$$Q_u = 0.5(Yn + 1)(Q_{max} - Q_{min}) + Q_{min} \quad (16)$$

Where Q_{max} = 21708 kN and Q_{min} = 1794 kN.

Eq. (16) can be used to find out the ultimate load of shell foundation in cohesionless soil with reasonable accuracy. The developed equation i.e. Eq. (16) has already been tested for generalization ability and its suitability is confirmed from R^2 value of 0.9797 as indicated in Fig. 2. Therefore, the developed equation can be used for fast predicting the shell foundation capacity by practicing engineers and researchers. Figure 5 shows a histogram of the residuals from which it is seen that majority of the residuals are close to zero.

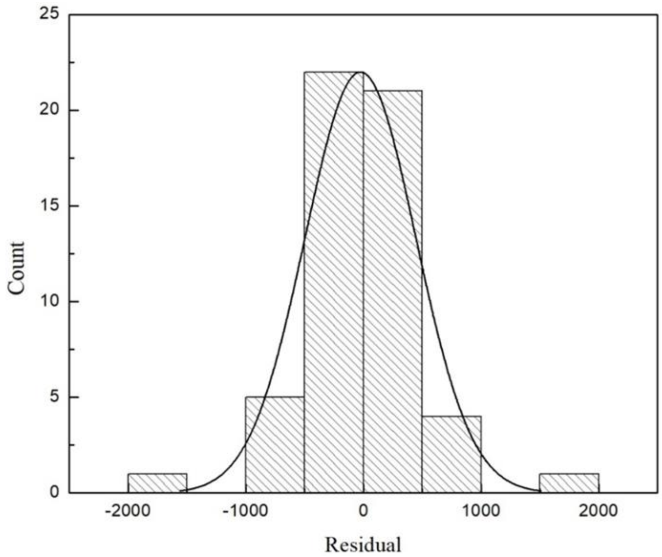

Fig. 5. Histogram showing the residual count along with the normal distribution curve for the dataset

6 Conclusions

The database for the ANN analysis was generated from the experimental investigation of shell foundation in dry sand by Hanna and Abdel-Rahman, 2011. Using fifty-four number of data set generated, a couple of empirical equations are developed to predict the ultimate load of shell foundation. It is expressed as a function of its associated factors like the type of shell (t), embedment ratio (D/B) and angle of shearing resistance (ϕ). The statistical computing based on Artificial Neural Network suggests the following major inferences.

- In the residual analysis using the histogram, the normal distribution curve drawn over the histogram shows that a maximum number of residuals are nearer to zero which indicates Levenberg Marquardt model can be successfully applied to find out the static ultimate load of shell foundations.

- From the Pearson correlation coefficient, it is found that ultimate load (Q_u) is highly correlated with the parameter φ, followed by *D/B* and type of footing *(t).*
- Equation (16) is derived to predict the ultimate load of shell foundation, with reasonable accuracy.
- The inputs *D/B* and φ are having a direct relation with Q_u, type of footing is having indirect relation with ultimate load (Q_u) as observed from the Neural Interpretation Diagram (NID).

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