VIBRATION ANALYSIS AND STABILITY STUDIES IN MILLING OF THIN-WALLED WORKPIECES

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ABSTRACT:-Milling is one of the most commonly used machining processes. Now-a-days, aeronautical and automobile industries require preparation of thin walled components. Machining of such components is a difficult task, because of high work flexibility, which in turn causes undesirable deformation that compromises the tolerances given on the workpiece. Focus of the present work is to design and analyze the milling process with flexible workpiece considerations via an interactive cutting force model. The tool and workpiece considered are having two degrees of freedom each in X and Y direction and further analysis has done in time domain by solving both the coupled delay-differential equations. The vibrational amplitudes and corresponding cutting force components from a four-fluted milling tool at different cutting speeds are observed. The Fast Fourier Transform (FFT) of the time domain signals show the dominant chatter frequencies, which is of main concern in determining the process stability. The FRF of the signals are also predicted from the FFTs of X and Y displacements and corresponding forces. It is found out that the chatter frequency varies with spindle speed. The effect of stiffness of workpiece on vibration response is also studied. The inter-dependent effect of the tool and workpiece model is studied with a varying stiffness model of the workpiece and the behavioral change of the system with respect to different parameters is observed.

Index terms: Thin walled workpiece, Chatter vibrations, Two degree of freedom, In process workpiece, Time domain analysis.

I. INTRODUCTION

Today, in several industrial applications, high-speed milling of the thin surfaces is required. Components like turbine blades, propellers and rotors needs precise dimensional accuracy. Manufacturing of such complex devices is an important task for industries like automotive, aviation, food industry and in machine engineering. Many times the milling process considered as a finishing process because of high machining cost and complexities of the geometry. Dynamic behavior of the tool workpiece system is important because of the slenderness of the tool and workpiece. The slenderness of the tool and workpiece affects the stability of the system and can generate the regenerative chatter vibrations. Additionally, the vibrations generated can compromise the surface quality of the machined surface and further can increase tool wear. In some cases, it may lead to the destruction of a tool or a workpiece.

The chatter vibrations mainly generated in the milling process are of two kinds; named as primary and secondary chatter. The primary chatter is of frictional types, which generates by rubbing action of the clearance face. This causes the excitation of the vibration along the direction of the cutting force. It is limited in the thrust force direction. The secondary chatter is of dominant nature, which is the commonly generated chatter vibrations also called as regenerative chatter. As the name suggest, the regenerative chatter creates itself repeatedly. It can occur often because many metal cutting operations includes overlapping cuts, which leads to vibration amplification. While machining process, the vibrations leaves a wavy surface behind. A new wave is generated when the next cutting tooth is under cutting operation. Because of this, the chip thickness of the material varies. This causes variation in the cutting forces on the milling surface. It happens due to the phase difference between the wave left by the previous teeth and the wave left by the current one. This is the main reason for minimizing the chatter vibrations produced in the milling process.

Junjin et al. [1] proposed a method for vibration suppression in milling of thin walled workpiece by using damping properties of magnetorheological fluid for flexible fixture. The workpiece is considered equivalent to the cantilever beam and is solved according to the Euler–Bernoulli beam assumptions. Kalinski et al. [2] used modern approach of control law, which is applied for active force generation by the piezo-ceramic plate actuator. Wan et al. [3] created the stable process condition by attaching additional masses for milling process. Zhang et al. [4] used
method for minimizing the chatter vibrations by submerging the milling system in viscous fluid. The milling force coefficients reduce significantly because of the lubrication provided by viscous fluid condition and damping of milling system increases due to viscous fluid condition. Diez et al. [5] used piezoelectric system for compensating workpiece deformations in flexible peripheral milling. Qu et al. [6] have done the three dimensional stability prediction with chatter analysis in thin walled plate milling. The three dimensional stability lobe diagram is used for choosing proper cutting parameters to avoid the vibrations. The workpiece-holder system is modelled considering it as 2-DOF system with assumption that the tool system have higher rigidity than the thin-walled plate. In some cases, the tool orientation is also important for reducing the vibration. Huang et al. [7] reduced the deformation produced by properly choosing the orientation of the tool for end milling of the impeller blades. In this method, the direction of the cutting force is adjusted in a particular plane such that the stiffness of the blade is relatively higher in that plane. This causes cutting force components to be small in the high flexibility direction. Ren et al. [8] have created the multi-pocket structure by combining the Kirchhoff plates. Subdomain decomposition method is used for finding the analytical solution for the thin walled structure vibration. The semi discretization method is used to find stability of the thin walled milling process. There is another method for stabilizing the milling process by creating the counteracting vibrations. It is done by using complex mechatronic devices, which can actuate the workpiece during machining process. The active fixture is designed based on the black-box control-logic. Weremczuk et al. [9] presented non-linear model by considering it as a two degrees of freedom system. This model is very useful because it considers the susceptibility of the tool and workpiece system. Since it is a dynamic problem so it is described by using the delay discontinuous differential equations. It causes the instability for different system parameters. Altintas et al. [10] used stability laws in time as well as frequency domain for milling process. The stability lobe diagram is generated directly and analytically depending upon the time varying factors. Wang et al. [11] done the experiment on the Titanium alloy for high-speed vertical milling. It determines the critical boundary where the chatter occurs mostly. The finishing machining process is done by using the C++ software for calculating the chatter stability. Yuwen Sun et al. [12] studied force induced deformation effect in milling process. Liping Wang et al. [13] predicted the surface form errors in manufacturing of aerospace parts in five-axis flank milling process. Lorenzo Sallese et al. [14] designed an intelligent fixture, which will generate the counteracting vibrations for stabilizing the process.

The objective of this work is to analyze the behavior of the tool with flexible workpiece system in milling operation. A four fluted end mill is modeled and the dynamic analysis is conducted to obtain the time and frequency domain responses. Effect of workpiece stiffness properties on the tool frequency response functions are studied. The time-domain studies for understanding the chatter instabilities in the system are based on two degrees of freedom model of tool and workpiece. The remaining part of the paper has four sections: mathematical modeling, solution methodology, results, discussion, and conclusions.

II. MATHEMATICAL MODELING

The cutting tool and in process workpiece is considered as thin and cantilevered. It is fixed at the one end and the other end is free to move in both X and Y directions. The vibrations produced in Y direction are of major concern because it is present in the direction perpendicular to the machining surface. The stiffness of the thin walled workpiece is considered less as compared to the cutting tool and the holder system. Considering the tool to be having 2-DOF in X and Y direction. The equation of motion for cutting tool is given as-

\[
M_x\ddot{x}(t) + C_x\dot{x}(t) + K_xx(t) = F_x, \quad (1)
\]

\[
M_y\ddot{y}(t) + C_y\dot{y}(t) + K_yy(t) = F_y, \quad (2)
\]

Where \(M_x=\mu M, \ C_x, K_x, \ C_y, K_y\) are the mass, damping and stiffness of the tool in X and Y direction respectively. \(F_x\) and \(F_y\) are the cutting forces acting on the tool in X and Y directions. The cutting force is a function of axial depth of cut \(dz\), feed per tooth \(f\). Phase angle between the two adjacent teeth \(\delta\), spindle speed \(\omega\) and time \(t\).

\[
\begin{align*}
\text{Fig.1 Tool- workpiece model}
\end{align*}
\]
Eq. (1) and (2) represent equations of motion of the tool. Such equations can be written for workpiece as well with respective parameters. The combined equations for tool and workpiece model as shown in Fig.1, can be written in coupled form as:

\[ M_t \dddot{U}(t) + C_t \ddot{U}(t) + K_t U(t) = F_t \]
\[ M_w \dddot{V}(t) + C_w \ddot{V}(t) + K_w V(t) = F_w \]

Where \( M \), \( C \), \( K \) are mass, damping, stiffness matrices for tool (\( T \)) and workpiece (\( W \)). \( U(t) \) and \( V(t) \) are deformations produced for tool and workpiece respectively. The cutting forces \( F_t \) and \( F_w \) are dependent on each other as \( F_w = - F_t \).

In the cutting process, the forces known are mainly tangential, radial and axial forces. These forces are the function of the cutting parameters like axial depth of cut, chip thickness, and expressed as follows:

\[ F_t = K_{rc} * h(t) * dz \]
\[ F_r = K_{rc} * h(t) * dz \]
\[ F_a = K_{ac} * h(t) * dz \]

Where \( K_{rc}, K_{rc}, K_{ac} \) are known as cutting force coefficients which depends upon material properties, \( h(t) \) is dynamic chip thickness. These cutting forces can be converted into the global cutting forces in X, Y and Z directions.

\[ F_{global} = g(\theta) * [T(\theta) * F] \]

\[ F_x = \begin{pmatrix} \cos \theta & -\sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} F_t \\ F_r \\ F_a \end{pmatrix} \]

Where \( \theta \) is instantaneous tooth tip angle. The \( g(\theta) \) is a step function used to show whether the tool engaged in cutting action or not.

\[ g(\theta) = \begin{cases} 1 & 0 < \theta < \theta_{exit} \\ 0 & Otherwise \end{cases} \]

The instantaneous chip thickness \( h(t) \) can be given by the relation

\[ h(t) = f * \sin(\theta) + [X(t) - X(t-T)] * \sin(\theta) + [Y(t) - Y(t-T)] * \cos(\theta) \]

Where \( X(t) \) and \( Y(t) \) are instantaneous displacements in X and Y direction respectively. The displacements \( X(t-T) \) and \( Y(t-T) \) are produced by the previous tooth which has cut the material in the process. Thus, \( T \) is the tooth passing period which is given as \( T = 60/Z \), where \( Z \) is the total number of cutting tooth and \( N \) is the spindle speed. The passive component of the chip thickness is dependent of the feed rate \( f \), which is given by the component \( f * \sin(\theta) \).

During the milling process, mass of the workpiece changes continuously with time. Since the dimensions of the thin walled workpiece also changing with time, it affects the stiffness of the workpiece. Hence, the stiffness of the workpiece is not constant but varies from time to time. The mass and stiffness is considered to be varying exponentially. The material removal rate can be calculated from the cutting parameters and while the stiffness can be approximated to be reducing exponentially from a start value to final value defined according to eq. (11):

\[ K_w(t) = K_{w0} * \exp(C_1*t) \]

Where \( K_{w0}, K_w(t) \) are initial and instantaneous stiffness of the workpiece respectively. The constant \( C_1 \) can be calculated by the boundary condition. It is to be noted that the model is considered as multiple point contact model instead of the single point contact model, which is commonly used in other machining process. This is because of the fact that in milling process both the tool and the in process workpiece are flexible in cutting region. Since the deformation cannot be avoided in this region, the system undergoes very steep vibration along the tool axis. This is the reason for not using the single point contact model because of lack of accuracy in predicting the dynamic behavior of the tool and workpiece.

### III. SOLUTION METHODOLOGY

In order to solve the coupled delay differential equations in time domain, computer programs are implemented in Matlab15.0a on a core-i7 processor. The equations of motion are solved using Runge-Kutta fourth order method with two starting time step data. The time varying cutting forces are obtained from the dynamic chip thickness. FFT of the time responses and corresponding cutting forces are obtained to predict the vibration amplitudes. The detailed flow diagram for the proposed approach is illustrated in Figure 2. In order to study the stability of the process in time domain, the state transition matrix obtained from state space transformations is to be employed. Alternatively, frequency response function is used to draw the stability lobe diagram.
IV. RESULTS AND DISCUSSION

The parameters used in the analysis are taken from literature as in Table 1.

Table 1. Modal parameters of tool [9]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of tool (m₁)</td>
<td>0.1824(Kg)</td>
</tr>
<tr>
<td>Damping coefficient of tool (c₁)</td>
<td>12.8177(Ns/m)</td>
</tr>
<tr>
<td>Stiffness of tool (k₁)</td>
<td>1.366e5(N/m)</td>
</tr>
<tr>
<td>Mass of workpiece (m₂)</td>
<td>3.02(Kg)</td>
</tr>
<tr>
<td>Damping coefficient of workpiece (c₂)</td>
<td>76.2829 (Ns/m)</td>
</tr>
<tr>
<td>Stiffness of workpiece (k₂)</td>
<td>3.0718e5(N/m)</td>
</tr>
<tr>
<td>K₉₅</td>
<td>450 (MPa)</td>
</tr>
<tr>
<td>K₉₆</td>
<td>160 (MPa)</td>
</tr>
<tr>
<td>No. of cutting tooth (Z)</td>
<td>4</td>
</tr>
<tr>
<td>Spindle Speed(N)</td>
<td>1500rpm/3000 rpm</td>
</tr>
<tr>
<td>Feed per tooth(f)</td>
<td>0.01(mm)</td>
</tr>
<tr>
<td>Depth of cut (dₐ)</td>
<td>1 (mm)</td>
</tr>
</tbody>
</table>

The displacements in the X and Y directions of the tool due to vibrations are given in the Figures 3 and 4. The vibrations produced in the milling process gives the velocity in X and Y direction out of which the velocity of workpiece in Y direction is of main concern because of regenerative vibrations occurs in Y-direction. It clearly shows that the displacements produced due to vibrations are major in Y direction as compared to that of in X-direction. They show the displacements and cutting force to have a better understanding of the system behavior. The analysis is done mainly for two cutting speeds, 1500- rpm and 3000-rpm.

The signals become like periodic waves for the 1500-rpm speed after 2 seconds whereas for the 3000-rpm speed, the signal becomes almost constant afterwards. This means there is no significant effect of chatter on the cutting process. However, the cutting force amplitudes have reduced considerably as the speed increases.

Fig.5 and Fig.6 show the variation of the cutting forces in X and Y directions. The cutting forces vary in between the two limits for both the speeds. The initial disturbance in the forces is more at lower speed especially in the Y direction. The cutting forces for lower speed has more wavy nature as compared to the higher speed. It shows the cutting forces stabilizes easily at higher speeds. Hence, the cutting at the higher speed is more stable since constant forces are acting on the workpiece.
Fig. 5 Cutting forces of the tool at 1500 rpm.

Fig. 6 Cutting forces of the tool at 3000 rpm.

The time domain analysis is sometimes difficult for understanding the behaviour of the system. This is the reason to convert the time domain analysis into frequency domain to have better understanding of the system. Fig. 7 and Fig. 8 represent the Fast Fourier Transform (FFT) of the relative displacements of the tool with respect to workpiece. The displacements at 1500-rpm having a peak at 50Hz chatter frequency. The initial peaks formed shows the two modes of the tool. The same peak formed at 100 Hz frequency for 3000-rpm spindle speed. FFT of the displacements is used to calculate the FRF of the system, which helps in better understanding of the milling process.

Figures 9 and 10 represent the direct FRF of the tool as a ratio of FFT of displacement components and force components. The FRF plot in fact contains real and imaginary components, which indicate the frequency and the damping of the system. The FRF of the system gives the frequency of the system as well as the chatter frequency. In present work, the FRF corresponds to the tool tip, which can be employed in constructing the stability lobe diagram. This shows that the amplitude of the FRF function depend upon the cutting parameters but the frequency of the FRF is changing with respect to the spindle speed. The stability lobe diagrams are also of very much importance in finding the correct machining parameters.
This shows that the chatter frequency depends upon the cutting tool parameters mainly spindle speed. Hence, the proper selection of the cutting parameters is an important task in machining process. Whenever the operating (cutting) speed exceeds the chatter frequency, it results in unstable cutting leading to chatter.

Fig. 11 shows the effect of workpiece flexibility on the tool vibrations at 1500 and 3000 rpm in the form of bar chart. It shows that the displacement in Y direction mainly varies with spindle speed and stiffness of the workpiece. These results are formulated for three stiffness values as $K_1=3.0718 \times 10^4$ N/m, $K_2=3.0718 \times 10^5$ N/m, $K_3=3.0718 \times 10^6$ N/m.

While milling, the dimensions of the workpiece decrease with time. Hence, the stiffness of the workpiece also decreases due to the decrease in thickness of the workpiece. Fig. 12 shows the displacements of the workpiece for constant stiffness value at 1500 rpm. Fig. 13 shows the displacements of the workpiece for exponentially decreasing value of stiffness from $K_2$ to $K_1$. Because of decrease in the stiffness of the workpiece, the magnitude of the vibrational displacements of workpiece increases as compared to the constant stiffness values. Therefore, the decrease in the thickness of the workpiece causes instability in the milling as the machining time proceeds.
Stability issues are further considered as next part.

V. CONCLUSION

The results show that the behavioral changes in system happens because of the different cutting and design parameters. The study shows that the spindle speed is one of the most important parameters deciding the chatter vibrations produced in the milling of thin walled workpiece. It also shows that the chatter frequency can be changed depending upon the machining parameters. The significant changes in the vibrational amplitude of the workpiece show that the stiffness of the workpiece cannot be taken as constant. The frequency domain analysis gives better idea about the tool and workpiece milling system. It is also to be noted that the stiffness and damping of the material to be used for milling plays an important role in the whole process. Hence, rearranging the workpiece in proper direction to have better stiffness and damping is also important for reducing the vibrations produced while milling process. Using the present FRF data, the stability region has to be plotted in the form of stability lobe diagram of the system. The experimental work has to be conducted to further verify the results obtained.

REFERENCES