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Buckling of Curved Panels Subjected to Non-uniform Loading Using Finite Element Method

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Abstract: The static stability of the curved panels subjected to varieties of non-uniform loading including partial and concentrated in-plane compressive edge loading is studied using finite element method, considering the effects of transverse shear deformation and rotary inertia. An eight nodded quadratic isoparametric element is employed in the present analysis. The study reveals that the stability behaviour of the flat and curved panels is greatly influenced by the geometry, boundary conditions, the type and position of loads.

Key Words: Buckling, Curved panel, Finite Element Method, Non-uniform Load

INTRODUCTION

Plates and shells are extensively used as structural parts in civil, aerospace, automotive and marine engineering structures. This wide range of practical applications demands a fundamental understanding of their vibration and static stability characteristics. Experience showed that such structures may fail in many cases not on account of high stresses, surpassing the strength of material but owing to insufficient stability of slender members. Due to its significance in structural mechanics, large number of references in the published literature deal with vibration and static stability behaviour of plates subjected to uniform in-plane stresses. There is a renewed interest with development of aviation and aerospace programs during the 1960's which is still expanding to off-shore and nuclear engineering. Timoshenko and Gere (1961) studied analytically the buckling of flat plates subjected to various uniform loads and a pair of concentrated load at mid breadth. The buckling of plate subjected to localized edge loading was investigated for few cases by Khan and Walker (1972). Cases of practical interests arise when the in-plane stresses are caused by concentrated forces acting along the boundaries. The free vibration and buckling analysis of a rectangular plate were studied for a pair of oppositely directed in-plane concentrated forces by Leissa and Ayoub (1988) using Ritz and finite element method (FEM). The buckling of plates with different end conditions was analyzed using the finite strip method by Bradford and Azhari (1995). Recently, Liew and Chen (2004) studied the buckling of flat plates subjected to partial in-plane edge loads using the radial point interpolation method. Deolasi and Datta (1993) investigated the parametric instability of flat rectangular plates subjected to localized edge loading. Kang and Leissa (2005) presented solutions for the buckling of flat rectangular plates subjected to linearly varying in-plane loading on two opposite simply supported edges. The buckling of cylindrical shells subjected to uniform loads was studied by Mandal and Calladine (2000) experimentally. Matsunaga (1999) investigated analytically the stability of curved panels subjected to uniform in-plane loads.

Objective and Scope of Present Study

Plethora of studies have been made by many researchers on buckling of plates and shells subjected to varieties of in-plane edge loads. But studies on static stability of shells with different types of non-uniform in-plane loads are scarce. The study of static stability of curved panels with non uniform in-plane load is new. This further necessitates an extensive study on buckling of curved panels subjected to non-uniform in-plane loads for different boundary conditions, various aspect ratio and b/h ratio.

FINITE ELEMENT FORMULATION

The governing equation for static stability or buckling of structures in matrix form is:

$[[K]-\lambda[K_g]]\{q\} =\{0\}$ where

Element elastic stiffness matrix $$[K]_e = \int_{-1}^{1}\int_{-1}^{1}[B]^T[D][B]d\xi d\eta$$

Element Geometric Stiffness matrix $$[K_g]_e = \int_{-1}^{1}\int_{-1}^{1}[G]^T[S][G]d\xi d\eta$$

An eight nodded isoparametric element is used in the present analysis. A computer program has been developed to perform all the necessary computations. The element stiffness and mass matrices are derived using a standard procedure. The geometric stiffness matrix is essentially a function of the in-plane stress distribution in the element due to applied edge loading. Since the stress field is non-uniform, plane stress analysis is carried out using the finite element techniques to determine the stresses and these stresses are used to formulate the geometric stiffness matrix. Reduced integration technique is adopted for the element matrices in order to avoid possible shear locking. The overall matrices are obtained by assembling the corresponding element matrices, using skyline technique. Subspace iteration method is adopted throughout to solve the eigenvalue problems. The boundary conditions are imposed restraining the generalized displacements in different nodes of the discretized structure.

RESULTS AND DISCUSSIONS

Convergence study

The convergence study is carried out for non-dimesional buckling load parameter for flat panel subjected to uniform in-plane load as shown in the Table 1. A mesh of 10x10 shows good convergence and all further results have been computed with this mesh.

Table 1: Convergence study for Non dimensional buckling load (λ)
For a/b=1, b/h=100, ν=0.3 , E=2.0X10^{11} N/m^2

Mesh	Non dimensional buckling load			
	a/h=10	20	40	100
4X4	36.8679	38.8349	39.4261	40.0312
8X8	36.8297	38.7770	39.3025	39.4542
10x10	36.8284	38.7755	39.3007	39.4511

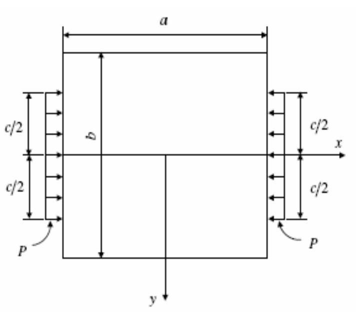
Comparison with previous Results

The Non-dimensional buckling load for a flat panel subjected to concentrated loading at the two opposite edges for different boundary conditions are computed and compared with the results of Liew and Chen (2004) by radial point interpolation method and with Kitipornchai *et al.* (1993). The present finite element formulation is further validated for buckling of flat panel subjected to partial in-plane edge loading in line with Liew and Chen (2004) and compared with the results obtained by radial interpolation method, wherever available.

Table 2 Comparison of Non dimensional buckling load parameter (N_xb^2/D) for a square Mindlin plate that is subjected to axial in-plane uniform edge loadings.
a/b=1, ν=0.3, E=2x10^{11}N/m^2

Thikness Ratio (h/b)	Boundaries	Liew and Chen (2004)	Kitipornchai *et al.* (1993)	Present
0.1	SSSS	37.36	37.38	36.828
	CCCC	81.85	81.84	79.225
	CFCF	34.65	34.62	33.727
0.05	SSSS	38.95	38.93	38.775
	CCCC	94.52	94.34	93.4228
	CFCF	37.69	37.51	37.2421

Table 3 Comparison of Buckling load parameters for a square Mindlin plate subjected to axial partial in-plane edge loading for different boundary condition
A = 0.5m . b = 0.5m, h/b = 0.01, ν = 0.3, E = $2x10^{11}$ N/m^2



c/a	Boundaries		
	SSSS	CCCC	CFCF
0	25.647(present) 25.814 (Leissa &Ayoub) 25.659 (RPIM)	66.40 (Present) 66.057 (Leissa &Ayub) 67.712 (RPIM)	35.982 (present) 36.290 (RPIM)
0.25	26.767 (Present) 26.925 (RPIM)	69.553 (Present) 70.925 (RPIM)	36.549(Present) 36.887 (RPIM)
0.5	29.956 (present) 30.024 (RPIM)	79.200 (Present) 81.024 (RPIM)	37.775(Present) 38.656 (RPIM)
1.0	39.451 (present) 39.183 (RPIM) 39.478 (T &Gere)	99.396 (Present) 100.178 (RPIM) 99.3887(T& James)	38.636(Present) 38.894 (RPIM)

Numerical Results

After convergence study and comparison with previous studies, numerical results on buckling of curved panels subjected to varieties of in-plane edge loads are presented.

Table-4 Non dimensional buckling load parameter for doubly curved panel subjected to concentrated load at the two opposite ends for different aspect ratio(a/b) and boundary conditions. $\Lambda = N_x b^2/D$,ν = 0.3,E=$2x10^{11}$ N/m^2, b/h=100

a/b	Hyperbolic Paraboloid			Cylindrical			Spherical		
	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF
0.5	55.233	128.552	123.951	71.765	114.655	107.814	78.970	136.809	136.751
1.0	25.617	116.123	96.873	74.899	118.876	102.062	82.949	127.366	120.401
2.0	30.143	138.309	68.050	80.198	140.690	45.497	93.791	154.190	50.3287
3.0	32.381	149.532	32.392	90.861	151.120	27.650	111.842	171.780	21.357

Table-5 Non dimensional buckling parameters for doubly curved panel subjected to uniform load at two opposite ends for different aspect ratio(a/b and boundary conditions.
$\Lambda = N_x b^2/D$,ν = 0.3,E=2x10^{11} N/m^2 ,b/h=100

a/b	Hyperbolic Paraboloid			Cylindrical			Spherical		
	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF
0.5	123.355	330.927	266.625	171.987	293.474	189.694	220.244	381.072	272.937
1.0	39.431	118.2565	54.616	46.356	113.048	41.803	67.092	121.218	55.650
2.0	38.968	186.587	59.804	166.137	196.552	32.824	187.695	213.817	44.336
3.0	39.080	195.9317	33.8098	170.659	205.894	24.208	191.808	214.744	20.4469

Table-6 Non dimensional buckling load parameters for partial edge loading at one end for different aspect ratio(a/b), a = 0.5m . b = 0.5m, ν = 0.3, E = 2x10^{11} N/m^2, b/h=100

c/a	Hyperbolic Paraboloid			Cylindrical			Spherical		
	A/b=0.5	a/b=1	a/b=2	a/b=0.5	a/b=1	a/b=2	a/b=0.5	a/b=1	a/b=2
0.2	88.647	52.492	42.678	105.264	95.641	95.184	119.947	115.093	117.741
0.4	73.644	41.761	39.646	95.051	100.425	103.860	123.329	117.252	125.116
0.6	83.502	35.534	36.835	112.975	117.023	118.991	146.684	133.953	141.456
0.8	101.894	35.698	37.121	140.883	142.670	142.395	180.946	157.129	164.481
1.0	123.356	38.958	38.969	171.987	171.527	166.137	220.245	185.875	187.695

Table-7 Non dimensional Buckling load parameters for a square Mindlin curved panel subjected to axial in-plane edge loading for different boundary condition a=0.5m, b=0.5m, h/b=0.01, ν=0.3, E=2x10^{11}N/m^2

c/a	Hyperbolic Paraboloid			Cylindrical			Spherical		
	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF	SSSS	CCCC	CFCF
0	25.617	116.123	96.873	74.899	118.876	102.062	82.949	127.366	120.401
0.25	26.730	109.219	88.557	73.368	111.031	85.248	83.315	121.185	104.939
0.5	29.835	125.327	98.926	88.299	127.475	89.386	103.082	141.435	112.741
1.0	38.958	213.923	123.036	171.527	218.995	70.307	185.875	234.168	111.725

As shown in Table 4 to Table 7, the buckling characteristics of curved panels subjected to non-uniform loads are different from flat panels.

CONCLUSION

Investigation of buckling behaviour of curved panels subjected to in-plane edge loading and concentrated loading at the two opposite edges for different aspect ratio(a/b), thickness ratio (h/b) and boundary conditions have been carried out using the finite element method. The conclusions are summarized as given below.

1) The non dimensional buckling load parameter for a curved panel subjected to uniform in-plane edge loading and concentrated loading are large for rectangular panels than square panels.
2) Non dimensional buckling load parameter for a curved panel is subjected to axial in-plane edge loadings for different boundary conditions increases as c/a increases.
3) Non dimensional buckling load parameter for a square Mindlin panel that is subjected to axial in-plane uniform edge loading for different boundary conditions increases with introduction of curvature.
4) Non dimensional buckling load parameter for a simply supported curved panel that is subjected to partial edge loading at one end is higher at c/a=0 i.e. at the edge x=0, as c/a increases, the buckling load decrease, however as c/a approaches unity, the buckling load again increases due to the restraining effect of the other edge, x=a.

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