Speed Dependent Dynamic Parameters Estimation of a Rotor–Bearing–Coupling System

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Abstract. In the modelling and analysis of turbine generator systems, bearing and coupling dynamic parameters are considered as the major unknowns. In the past, practitioners of rotor dynamics have modelled coupling as having speed independent stiffness and damping parameters that lead to modelling error, due to the fact that the amount of misalignment depends upon different modes of excitation. In this article, an identification algorithm has been developed for simultaneous estimation of the speed dependent bearing and coupling dynamic parameters along with residual unbalances. Lagrange's equation is used to derive the equations of motion of the system in generalized coordinates and least squares technique is used to develop identification algorithm. The novelty of the present identification algorithm is the estimation of speed dependent coupling dynamic parameters along with speed dependent bearing dynamic parameters. Numerical experiments have been performed for a simple rotor train model to illustrate the developed algorithm. To check the robustness of the identification algorithm, measurement noise has been added in numerically simulated response. Well agreement in the estimated parameters is observed for a different level of measurement noise.

Keywords: Speed Dependent, Bearing and Coupling Dynamic Parameters, Lagrange's Equation, Identification Algorithm, Residual Unbalances.

1 Introduction

Unbalance is one of the major causes of failure of turbo–machineries because a small amount of unbalance can be the reason for excessive vibrations at high speed that may cause shattering of the rotor system [1]. It also diminishes the effectiveness of the entire rotor system. After unbalance the second most common fault is the misalignment of shafts that causes unwanted moment and force at the critical locations in the rotor system i.e., coupling and bearings. Though misalignment of shafts at coupling and bearing is an eminent problem but still its identification is tough due to complexity in its modelling. Many researchers have developed algorithms for speed–independent bearing and coupling parameters [8-12] as well as speed–dependent bearing dynamic parameters [2, 13] but till now no work has been reported on the estimation of speed–dependent coupling parameters. The amount of misalignment may reflect in terms of stiffness and damping forces that depends upon the different modes of excitation and during motion the amount of misalignment would change with the operating speed. Hence, the coupling must be modelled as having speed dependent stiffness and damping parameters.

[3] established an identification algorithm to evaluate the bearing dynamic parameters and residual unbalance simultaneously for a multi–degrees–of–freedom system. [4] used impulse response measurements to develop an algorithm for estimation of residual unbalances and speed dependent bearing dynamic parameters simultaneously and acclaimed that the proposed methodology has flexibility to incorporate any number of bearings. [5] discussed various techniques to detect and quantify the faults in rotating machines. [6] developed a model–based fault diagnosis technique to identify the faults in a rotor–bearing–coupling system subjected to misalignment and unbalance. Residual generation method is incorporated to estimate residual forces. [7] performed an experimental investigation on four models of coupling and concluded that out of four models Nelson and Crandall's second model that includes rotational damping and stiffness along with mass and inertia is found to be best to describe the dynamics of the coupling. [13] developed an identification algorithm to estimate the speed–dependent parameters like displacement stiffness and current stiffness and residual unbalances in a rotor system suspended on AMBs. Least square fit method in the frequency domain is used to estimate the parameters. SIMULINK is used to obtain current and displacement response in time domain. Fast Fourier Transform (FFT) is used to convert time domain signal into frequency domain.

In the present paper, an identification algorithm is established for simultaneous estimation residual unbalance and speed–dependent bearing and coupling parameters in the rotor bearing coupling system. Two sets of forced responses are generated alternatively by considering and without considering trial mass and responses are incorporated in the development of an algorithm to find the estimates for different speeds. The effect of noisy response on the estimated parameters is discussed and the algorithm is found to be excellent.

2 System Configuration

A simple model of turbo–generator system comprises of two rigid rotors each having a rigid disc at mid–span of mass, m_i^d(where i = 1, 2) and diametral mass moment of inertia, I_i^d (where i = 1, 2) along with two residual unbalances (F^{res}=$m_i^d e$, magnitude of unbalance in each rotor, where e is the eccentricity of the rotor) is taken as shown in Fig. 1. Flexible coupling used to connect the rotors is modelled as having K_{cL}, the linear stiffness, and K_{cT}, the rotational stiffness. Each shaft is mounted on bearings having damping coefficient, C, and stiffness coefficient, K, as the properties. The diagram of the rotor–bearing–coupling model in deflected position in (x–z) plane is shown in Fig. 2 (a), in which X and Φy represents the linear and angular displacements respectively, and the schematic diagram of the coupling is shown in Fig. 2 (b).

2.1 Equation of Motion

For simplicity, a rotor model representing single–plane motion is taken under consideration and four generalized coordinates are used to define the motion of the turbo–generator system. Relationships among the displacements at various axial locations of the rotor in (*z*–*x*) plane in terms of generalized coordinates are

$$X_p = X_{c_1} + 1.25l\,\Phi_{y_1},\ X_q = X_{c_1} + 0.25l\,\Phi_{y_1},\ X_r = X_{c_2} + 0.25l\,\Phi_{y_2},\ X_s = X_{c_2} + 1.25l\,\Phi_{y_2},$$
$$X_1 = X_{c_1} + 0.75l\,\Phi_{y_1},\ X_2 = X_{c_2} + 0.75l\,\Phi_{y_2} \quad (1)$$

where 1.25l is the length of each shaft. Kinetic energy (*KE*) of the system and virtual work (δW) due to non-conservative forces can be obtained as,

$$KE = \frac{1}{2}m_1\dot{X}_1^2 + \frac{1}{2}I_{d_1}\dot{\Phi}_{y_1}^2 + \frac{1}{2}m_2\dot{X}_2^2 + \frac{1}{2}I_{d_2}\dot{\Phi}_{y_2}^2 \quad (2)$$

$$\begin{aligned}\delta W &= K_1X_p\delta X_p + K_2X_q\delta X_q + K_3X_r\delta X_r + K_4X_s\delta X_s + K_{cL}(X_{c_1} - X_{c_2})\delta(X_{c_1} - X_{c_2}) \\ &\quad + K_{cT}(\Phi_{y_1} - \Phi_{y_2})\delta(\Phi_{y_1} - \Phi_{y_2}) - (C_1\dot{X}_p\delta X_p + C_2\dot{X}_q\delta X_q + C_3\dot{X}_r\delta X_r + C_4\dot{X}_s\delta X_s) \\ &\quad + f_1(t)\delta(X_{c_1} + .75l\Phi_{y_1}) + f_2(t)\delta(X_{c_2} + .75l\Phi_{y_2})\end{aligned} \quad (3)$$

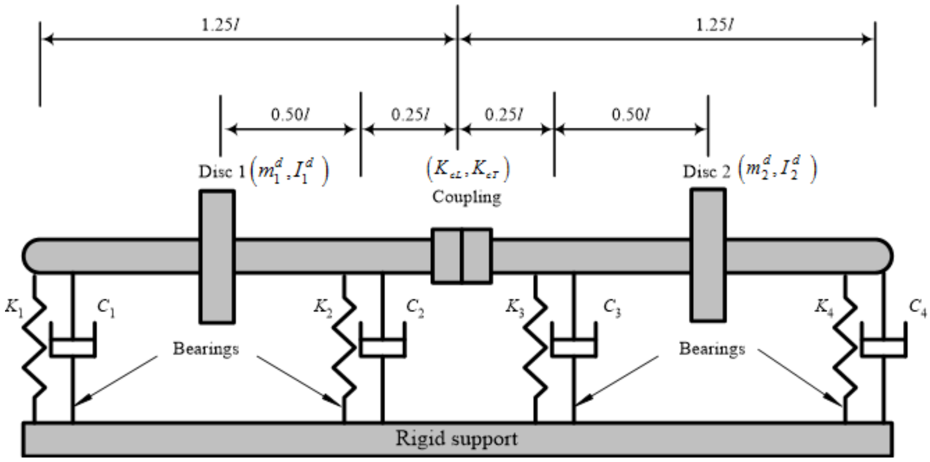

Fig. 1. Rotor–bearing–coupling system

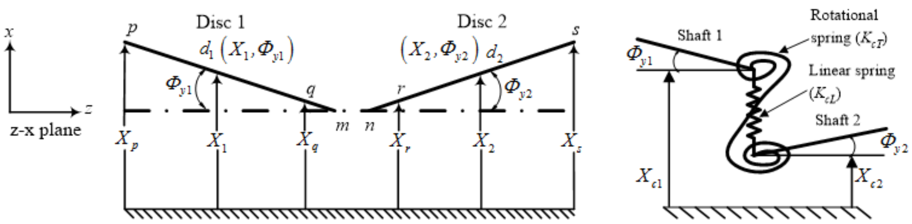

Fig. 2. (a) Diagram of deflected rotor in z–x plane (b) Coupling model

Lagrange's equation can be expressed as

$$\frac{d}{dt}\left(\frac{\delta(KE)}{\delta \dot{Q}_j}\right) = \frac{\delta W}{\delta Q_j} \tag{4}$$

where Q_j represents the generalized coordinates (X_{c_1} , X_{c_2} , Φ_{y_1} and Φ_{y_2}).

The equations of motion for the turbo–generator system can be written as

$$[M]\{\ddot{\eta}\}+[C]\{\dot{\eta}\}+[K]\{\eta\}=\{F(t)\} \tag{5}$$

Where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively. The force and displacement expressions used in the equation of motion are having forms $\{F(t)\}=\{\bar{f}\}e^{j\omega t}$ and $\{\eta(t)\}=\{\bar{\eta}(t)\}e^{j\omega t}$ respectively. Where vectors $\{\bar{f}\}$ and $\{\bar{\eta}\}$ comprises of magnitude and phase information. Substituting the above expressions of force and displacements in Eq. (5), it can be written as

$$\left([K]+j\omega[C]-\omega^2[M]\right)\{\bar{\eta}\}=\{\bar{f}\} \tag{6}$$

with

$$\{\bar{f}\}=\begin{Bmatrix} F_1^{res}\omega^2 e^{j\phi_1} \\ .75lF_1^{res}\omega^2 e^{j\phi_1} \\ F_2^{res}\omega^2 e^{j\phi_2} \\ .75lF_2^{res}\omega^2 e^{j\phi_2} \end{Bmatrix} e^{j\omega t}$$

Responses in complex forms can be appraised from Eq. (6) with the help of the assumed values of dynamic parameters of the bearing-coupling system and the unbalances.

2.2 Development of Identification Algorithm

Eq. (6) can be written in the form of regression equation in which all the unknown quantities i.e. the coupling and bearing dynamic parameters and residual unbalances are kept in a vector placed on the left–hand side and all the known quantities are kept in right–hand side vector. The regression equation for a spin speed can be written as

$$\left[A(\omega)\right]_{8\times 14}\left\{X(\omega)\right\}_{14\times 1}=\left\{B(\omega)\right\}_{8\times 1} \tag{7}$$

with

$$\left\{X(\omega)\right\}_{14\times 1}=\left\{K_1,K_2,K_3,K_4,K_{cL},K_{cT},C_1,C_2,C_3,C_4,F_{1r}^{res},F_{1i}^{res},F_{2r}^{res},F_{2i}^{res}\right\}^T$$

where subscript represents matrix or vector size. From Eq. (7) it can be seen that the number of equations is less than that of the number of unknown variables. So this is a case of undetermined system of linear simultaneous equations. In order to estimate the parameters, another set of response is required that can be generated with the help of trial mass. For trial unbalance, the equations of motion and force vector can be represented as

$$\left([K]+j\omega[C]-\omega^2[M]\right)\{\eta_t\}=\left\{\bar{f}_t\right\} \tag{8}$$

with

$$\left\{\bar{f}_t\right\}=\begin{Bmatrix} F_1^{res}\omega^2e^{j\phi_1}+U_1\omega^2e^{j\theta_1} \\ .75lF_1^{res}\omega^2e^{j\phi_1}+.75lU_1\omega^2e^{j\theta_1} \\ F_2^{res}\omega^2e^{j\phi_2} \\ .75lF_2^{res}\omega^2e^{j\phi_2} \end{Bmatrix}e^{j\omega t}$$

where U_1 represents the trial unbalance ($U_1= m_te_t$ is the trial unbalance with trial mass m_t and eccentricity e_t) attached to the first disc in the turbo–generator system. Both the sets of responses from Eq. (6) and Eq. (8) will be used in the formation of the determined regression equation for one spin speed, and this procedure will be implemented for several spin speeds in order to find the unbalances along with speed dependent dynamic parameters.

The determined system of regression equation by using least square fit technique for one spin speed can be presented as

$$\begin{bmatrix} A_1(\omega) \\ A_2(\omega) \end{bmatrix}_{16\times 14}\left\{X(\omega)\right\}_{14\times 1}=\begin{Bmatrix} B_1(\omega) \\ B_2(\omega) \end{Bmatrix}_{16\times 1} \tag{9}$$

From Eq. (9) it can be seen that the total number of equations (i.e., 16) is greater than the number of unknown parameters (i.e., 14), this represents the system of equations is determinate. Now, the regression equation is extended to estimate speed dependent dynamic parameters. In this article, for brevity, the algorithm is developed to accommodate dynamic parameters at three different speeds and could be expressed as

$$\begin{bmatrix} A_1(\omega_1) \\ A_2(\omega_1) \\ A_1(\omega_2) \\ A_2(\omega_2) \\ A_1(\omega_3) \\ A_2(\omega_3) \end{bmatrix}_{48\times 34} \{XX(\omega)\}_{34\times 1} = \begin{bmatrix} B_1(\omega_1) \\ B_2(\omega_1) \\ B_1(\omega_2) \\ B_2(\omega_2) \\ B_1(\omega_3) \\ B_2(\omega_3) \end{bmatrix}_{48\times 1} \tag{10}$$

Eq. (10), can be written as

$$\left[AA(\omega)\right]_{48\times 34}\{XX(\omega)\}_{34\times 1} = \{BB(\omega)\}_{48\times 1}$$

with

$$AA(\omega) = \left[Q(\omega)\ R(\omega)\right] \tag{11}$$

$$Q(\omega) = \begin{bmatrix} q(\omega_1) & 0 & 0 \\ 0 & q(\omega_2) & 0 \\ 0 & 0 & q(\omega_3) \end{bmatrix}; \quad R(\omega) = \begin{bmatrix} r(\omega_1) \\ r(\omega_2) \\ r(\omega_3) \end{bmatrix}; \quad XX(\omega) = \begin{bmatrix} X(\omega_1) \\ X(\omega_2) \\ X(\omega_3) \end{bmatrix};$$

In Eq. (11), the matrix $Q(\omega)$ contains displacement contributions from the speed dependent bearing and coupling dynamic parameters. Vector $R(\omega)$ having the coefficient contributions due to residual unbalances. Vector $B(\omega)$ contains known information such as masses (disc mass and trial mass). ω_1, ω_2 and ω_3 are three different spin speeds of rotor. With mathematical rearrangements Eq. (10) can be converted into the least squares form to estimate unknown parameters as

$$\{XX(\omega)\}_{34\times 1} = ([AA(\omega)]^T_{34\times 48}\left[AA(\omega)\right]_{48\times 34})^{-1}[AA(\omega)]^T_{34\times 48}\{BB(\omega)_{48\times 1}\} \tag{12}$$

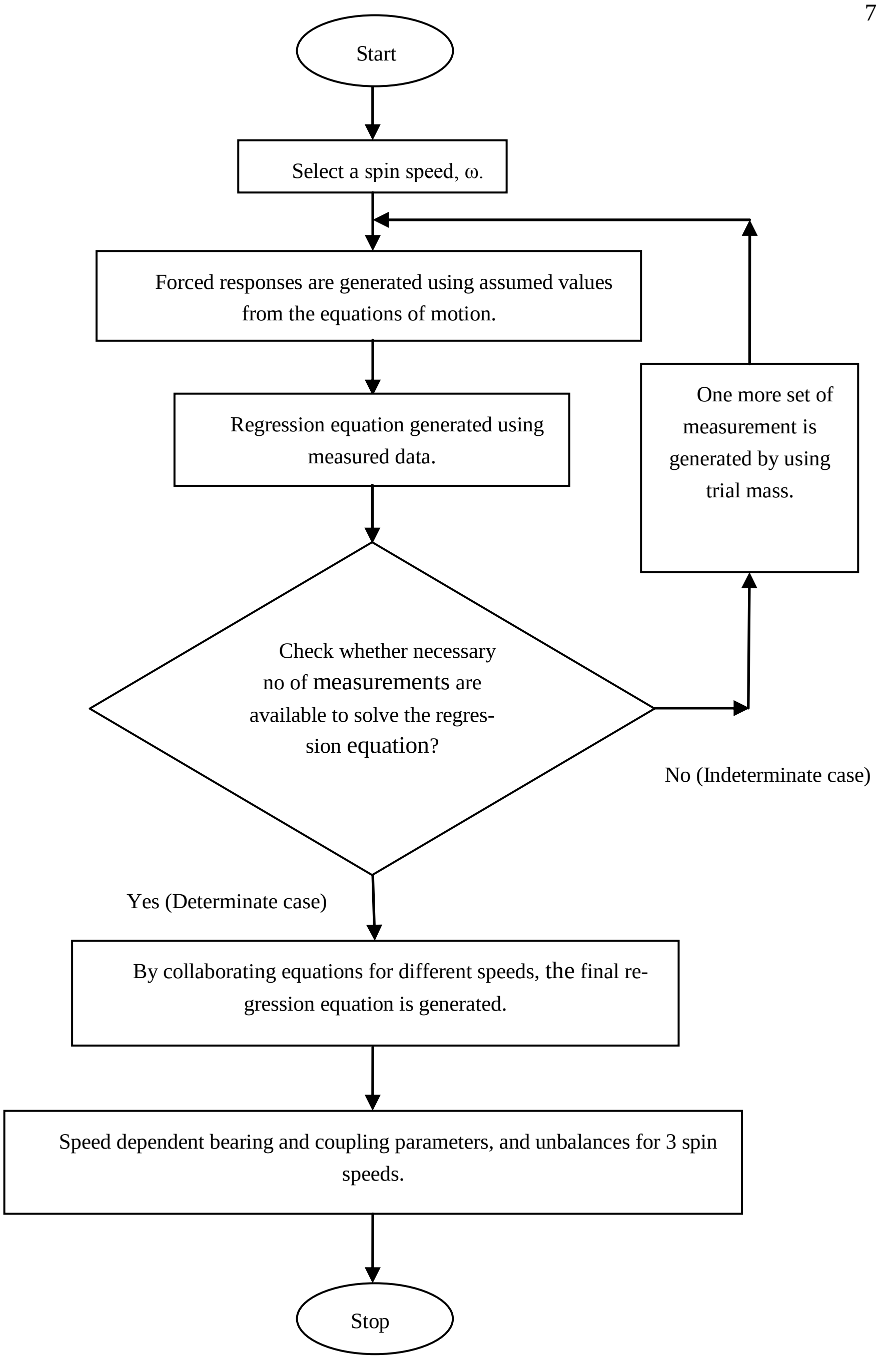

Fig. 3. Flow chart for identification algorithm

3 Results and Discussion

Numerical experiment is performed for the turbo–generator system shown in Fig. 1. The flow chart of the algorithm is presented in **Error! Reference source not found.**. The model parameters used to generate response of the system are: rigid rotors each of length 1.25 m, mass of 0.612 kg and diametral mass moment of inertia of 0.051 kgm^2: two rigid discs of masses 2 kg and 5 kg, and diametral mass moment of inertia, 0.005 kgm^2 and 0.014 kgm^2, respectively. The assumed and estimated coupling and bearing dynamic parameters for three different spin speeds (i.e., 20Hz, 30Hz and 40Hz) are summarized in Table 1, Table 2 and Table 3, respectively. The effectiveness of the identification algorithm is checked for various levels of measurement noise (up to 5%) fed to the simulated response. The speed dependent estimated parameters for three different speeds are shown in Fig. 4, Fig. 5 and Fig. 6. From Fig. 4–Fig. 6, it can be concluded that most of the parameters are in well agreement with assumed values except some of the damping parameters. The maximum percentage deviation occurred for 20 Hz, 30 Hz and 40 Hz are around 24%, 22% and 22%, respectively for 5% measurement noise condition. It can be noticed that for all the three spin speeds maximum percentage deviation of the estimated parameter is found for same bearing damping parameter.

Table 1. Comparison of assumed and estimated parameters for 20 Hz

Parameters	Assumed value	Estimated Parameters (% deviation)		
		Without noise	With 1% noise	With 5% noise
K_1 (kN/m)	25	24.99 (0.04)	24.94 (0.24)	24.75 (1.00)
K_2 (kN/m)	20	20.00 (0.04)	20.13 (0.65)	20.62 (3.10)
K_3 (kN/m)	15	14.99 (0.07)	14.44 (3.73)	12.41 (17.27)
K_4 (kN/m)	25	24.98 (0.08)	24.23 (3.08)	21.51 (13.96)
K_{cL} (kN/m)	10	9.99 (0.1)	9.93 (0.70)	9.65 (3.50)
K_{cT} (kNm/rad)	15	15.00 (0.04)	15.24 (1.60)	15.92 (6.13)
C_1 (Ns/m)	200	199.9 6(0.02)	200.16 (0.08)	200.77 (0.39)
C_2 (Ns/m)	150	150.09 (0.06)	149.78 (0.15)	148.91 (0.73)
C_3 (Ns/m)	140	139.92 (0.06)	132.37 (5.45)	106.69 (23.79)
C_4 (Ns/m)	250	249.96 (0.02)	239.85 (4.06)	204.50 (18.20)
F_1^{res} (kg×mm)	5.86	5.85 (0.17)	5.86 (0.00)	5.86 (0.03)
F_2^{res} (kg×mm)	7.47	7.46 (0.13)	7.32 (2.01)	6.78 (9.24)

Table 2. Comparisonof assumed and estimated parameters for 30 Hz

Parameters	Assumed value	Estimated Parameters (% deviation)		
		Without noise	With 1% noise	With 5% noise
K_1 (kN/m)	25.50	25.49 (0.04)	25.45 (0.19)	25.30 (0.78)
K_2 (kN/m)	20.60	20.60 (0.05)	20.74 (0.68)	21.26 (3.20)
K_3 (kN/m)	15.55	15.55 (0.01)	15.01 (3.47)	13.05 (16.08)
K_4 (kN/m)	25.65	25.64 (0.04)	24.90 (2.92)	22.23 (13.33)
K_{cL} (kN/m)	10.53	10.53 (0.05)	10.43 (0.95)	10.06 (4.46)
K_{cT} (kNm/rad)	15.62	15.64 (0.13)	15.75 (0.83)	16.10 (3.07)
C_1 (Ns/m)	230	229.92 (0.04)	230.11 (0.05)	230.43 (0.19)
C_2 (Ns/m)	185	185.13 (0.07)	184.86 (0.07)	184.42 (0.31)
C_3 (Ns/m)	175	175.85 (0.49)	166.14 (5.06)	137.12 (21.65)
C_4 (Ns/m)	285	286.04 (0.36)	273.36 (4.08)	234.26 (17.80)
F_1^{res} (kg×mm)	5.86	5.85 (0.17)	5.86 (0.00)	5.86 (0.03)
F_2^{res} (kg×mm)	7.47	7.46 (0.13)	7.32 (2.01)	6.78 (9.24)

Table 3. Comparisonof assumed and estimated parameters for 40 Hz

Parameters	Assumed value	Estimated Parameters (% deviation)		
		Without noise	With 1% noise	With 5% noise
K_1 (kN/m)	25.90	25.89 (0.04)	25.85 (0.19)	25.74 (0.62)
K_2 (kN/m)	20.80	20.81 (0.05)	20.95 (0.72)	21.52 (3.46)
K_3 (kN/m)	15.85	15.85 (0.04)	15.38 (2.96)	13.63 (14.01)
K_4 (kN/m)	25.95	25.95 (0.02)	25.27 (2.62)	22.80 (12.14)
K_{cL} (kN/m)	10.83	10.83 (0.01)	10.71 (1.02)	10.24 (5.45)
K_{cT} (kNm/rad)	15.92	15.92 (0.03)	15.97 (0.31)	15.91 (0.06)
C_1 (Ns/m)	260	259.96 (0.02)	260.03 (0.01)	260.14 (0.05)
C_2 (Ns/m)	205	205.07 (0.03)	204.97 (0.02)	204.88 (0.06)
C_3 (Ns/m)	195	195.25 (0.13)	186.03 (4.60)	152.59 (21.75)
C_4 (Ns/m)	300	300.26 (0.09)	288.20 (3.93)	244.12 (18.63)
F_1^{res} (kg×mm)	5.86	5.85 (0.17)	5.86 (0.00)	5.86 (0.03)
F_2^{res} (kg×mm)	7.47	7.46 (0.13)	7.32 (2.01)	6.78 (9.24)

Table 4. Evaluatedresidual unbalances for different disc masses for 5% measurement noise

S.N.	Assumed mass of disc(kg)	Estimated residual unbalance (kg-m @ deg)	
		Assumed unbalance (disc 1) 0.00586 @ 36.0	Assumed unbalance (disc 2) 0.00747 @ 144.0
Set-1	m1=2.0, m2=5.0	0.00558 @ 30.00	0.00735 @ 150.00
Set-2	m1=1.5, m2=3.0	0.00585 @ 30.02	0.00730 @ 150.27
Set-3	m1=2.0, m2=1.5	0.00585 @ 29.92	0.00731 @ 150.25

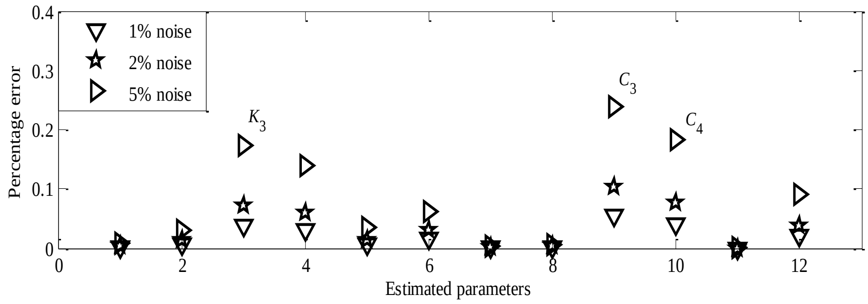

Fig. 4. Error plot for 20 Hz

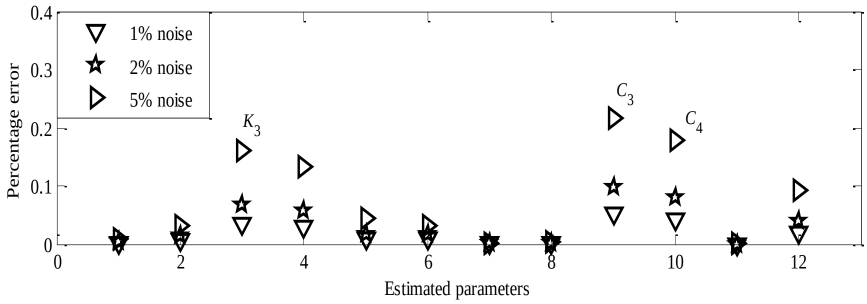

Fig. 5. Error plot for 30 Hz

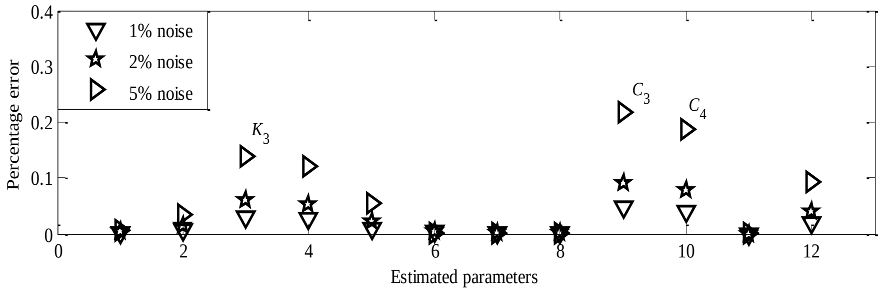

Fig. 6. Error plot for 40 Hz

The effect of modelling error (percentage deviation in the estimated parameters due to variation in disc masses) in the estimation of unbalance parameters (magnitude and phase) for 5% measurement noise case is given in Table 4. From Table 4, it can be seen that the magnitude of residual unbalance is representing well agreement with assumed values for all the three sets of disc masses. Also, slight variation in the estimated phase can be observed. But fair repeatability of the estimated parameters (magnitude and phase) can be observed for all the three sets. Hence it concluded that the estimated unbalance parameters show well agreement with assumed values in terms of magnitude whereas small deviation could be observed in phase estimation.

4 Conclusions

An identification algorithm is developed to estimate speed–dependent bearing and coupling parameters and the unbalances, simultaneously. The speed dependency of the parameters is checked for three different spin speeds and corresponding parameters are estimated that shows well agreement with each other. The effectiveness of identification algorithm is verified against measurement errors and modelling errors and found to be excellent. In future it will be interesting to see how these speed–dependent parameters behave while performing two-plane analysis with more numbers of unknown parameters.

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