Corrected John's Test Based Blind Spectrum Sensing Technique for Cognitive Radio Networks

Sreekanth Peddinti*,Shrishail M Hiremath†
* † Electronics and Communication Engineering Department
National Institute of Technology, Rourkela, Odisha, India-769008
Email: sreekanth1305@gmail.com, hiremaths@nitrkl.ac.in

Prof. Sarat Kumar Patra‡
‡Indian Institute of Information Technology, Vadodara
Email: skpatra@iiitvadodara.ac.in

***Abstract*—Spectrum sensing is one of the fundamental objective of cognitive radio network (CRN). In last one-decade the eigenvalue-based blind spectrum sensing methods have been extensively studied for CR applications. Eigenvalue-based techniques require large data sample for accurate detection of the signal under the low Signal-to-Noise ratio. This results in delayed sensing under large data sample. Thus, the recent trend is to explore the eigenvalue-detection methods under the low sample environment.In this paper we propose a Corrected John's Test (CJT) based eigenvalue technique for spectrum sensing. Asymptomatic test statistic and Probability of false alarm are obtained for the same. Performance analysis of CJT detector is compared with the previously proposed techniques like, Scaled Largest Eigenvalue (SLE), Maximum-Minimum Eigenvalue (MME), and Arithmetic to Geometric Mean (AGM) for relatively less number of samples. The simulation results show supremacy of CJT under Nakagami fading environment, for very less number of samples.**

***Index terms*—** Cognitive Radio, Spectrum Sensing, eigenvalue, low sample environment, Nakagami fading

I. Introduction

The rapid increment in the wireless devices has expanded the interest for high information rate and better multi-media applications. So, this has led to the spectrum scarcity problem. The Cognitive Radio network is proposed to make use of the spectrum efficiently by exploiting the under-used frequency bands by dynamic spectrum access[1, 2]. It consists of four main functions, Sensing, Sharing, Mobility and Management [2–4]. Out of them, the most fundamental and key function is the Spectrum Sensing. Spectrum Sensing is the process of detecting the presence or absence of Primary User (PU). The cognitive radio system will give access to the Secondary User (SU) in the case of PU's absence and quickly vacate as the PU reappears. There are several spectrum sensing methods like, Energy detection [4, 5], matched filter detection, cyclostationary detection [6] and recently Eigenvalue [7–9] based detection to detect the presence of PU signal [10].

The conventional energy detector [4, 5, 11] is simple and computationally less complex; however, the threshold depends on the noise variance and performance is poor under low Signal-to-Noise ratio (SNR) [4]. In practical case, obtaining accurate noise variance is not possible, hence fully-blind spectrum sensing schemes like eigenvalue detector are more robust. Eigenvalue detectors do not relay upon noise power and perform superior than energy detector under low SNR conditions [12]. Several eigenvalue methods like Maximum-Minimum Eigenvalue (MME) [9], Energy-Minimum Eigenvalue [9] (EME), Arithmetic to Geometric mean [13] (AGM), Eigenvalue-Moment-Ratio [13] (EMR) and Scaled Largest Eigenvalue [8] (SLE) uses different test statistics depending on the eigenvalues of the covariance matrix. The thresholds of SLE and MME detectors are obtained in the regime where $p, n \to \infty$ and hence they perform better under more number of samples (n) and sensors (p) scenario [9, 14]. The asymptotic threshold of EMR is also derived under $p, n \to \infty$ regime, but detection performance is better than SLE & MME for less number of samples. For more samples case, it has been known that the SLE surpasses all the detectors, because of it's agile increase in the probability of detection. This agitated behavior of these detectors, based on a number of samples, is due to the regime under which their thresholds are obtained. In other words, there will be a small error in performance for the above mentioned tests, under the finite number of samples and sensors [13]. So this paper proposes the Corrected John's Test (CJT) based spectrum sensing technique for low sample scenario under the classical regime, p fixed and $n \to \infty$.

The potential of a detector is known when it's ability to detect under low sample scenario is great. Also, due to some hardware preferences, the CRN ought not rely upon additional number of sensors to give better performance. These two reasons state that the detector implemented at CRN should give better performance under less and fixed sensors and low samples environment. Consequently, the classical regime is highly preferred for CRN applications. Corrected John's Test (CJT) is such a detector which outperforms SLE, MME, and AGM under the classical regime and for very low sample environment. The asymptotic expressions for probability of false alarm (P_{fa}) and threshold are derived for CJT. And furthermore, the numerical and theoretical results are validated using MATLAB simulations. Comparative analysis of CJT with previous proposed eigenvalue based sensing techniques is done in terms of probability of detection (P_d) $vs\ SNR$, for no fading and severe Nakagami-fading channel.

The rest of the paper is as per the following: section II describes the system model, which describes the binary hypothesis detection of the PU and some conventional test statistics. Section III presents the proposed analytical expressions of probability of false alarm and asymptotic threshold. The simulation results are presented in section IV which shows the performance of CJT compared to SLE, MME and AGM under Nakagami fading environment.Finally, the section V discusses the conclusion and further research that could be carried on.

II. System Model

A. Hypothesis Test

The signal detection can be done by testing the signal's particular characteristic i.e., energy or eigenvalue etc., with some threshold level. Two hypothesis, H_0 and H_1 are considered to characterize the presence or absence of PU. i.e., when the signal is not present it is considered as H_0 hypothesis and when the signal is existing it is H_1 hypothesis [10].

$$H_0 \ : x[i] = w[i] \quad (1)$$
$$H_1 \ : x[i] = Hs[i] + w[i] \quad (2)$$

Where $i = 1,..,n$ is the particular sample at i^{th} moment. As shown in Fig.1, a single PU transmits the signal and p number of sensors are used at the secondary receiver system to receive it by sensing the particular frequency band for a specific amount of time, collecting n samples [8].

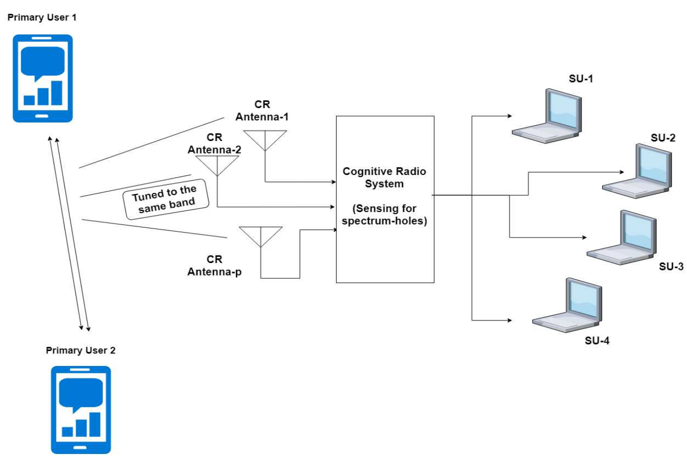

Fig. 1. System Model of CR

Let $x_{p\times n}[i] = [x_1(i)...x_p(i)]^T$, is a matrix comprising of all the n samples gathered by each p sensor at i^{th} instant. $s_{1\times n}[i] = [s(1)...s(n)]$ is the transmitted signal represented as data matrix, $h_{p\times 1}$ is the envelope of fading channel, which uniformly influences all the samples at an individual sensor independently. $w_{p\times n}[i] = [w_1(i)...w_p(i)]^T$ is the received complex Gaussian noise, with zero mean and variance σ^2, additively affecting each sample. Let $X = [x(0), x(1), ..x(i), ..x(n)]$ be the matrix which denotes the full data received through out the sensing time, then the sample covariance matrix of the received signal is defined as $R_{x_{p\times p}} \triangleq \frac{1}{N}XX^H$[15, 16]. Let T be a test statistic and t be the decision threshold with which it is compared. Then probability of false alarm is defined as,

$$P_{fa} = Pr(T > t|H_0) \quad (3)$$

And the probability of detection is defined as

$$P_d = Pr(T > t|H_1) \quad (4)$$

The decision threshold should not depend on the noise variance, since the noise level is not certain [4]. The optimum threshold level should be chosen with the end goal of low false alarm rate ($\leq$ 0.1 tolerable level, as per IEEE 802.22 standards for WRAN) and high detection rate. So to obtain a constant false alarm rate (CFAR) the threshold level should be fixed to a level by assuming P_{fa} is 0.1.

B. Corrected John's Test (CJT) detection Method

Consider the received signal, $X_{p\times n}$, from a p-dimensional multivariate distribution with population covariance matrix Σ_p. A significant task in multivariate analysis is to check sphericity of the test [17]. Under H_0 hypothesis, where only i.i.d normal distribution samples with zero mean and Σ_p will occur, the eigenvalues of $p \times p$ sample covariance matrix can be represented as $\{l_i\}_{1\leq i\leq p}$.

i. **John's Test:** The acclaimed John's Test is defined as[18],

$$T_2 = (\frac{np}{2})\frac{p^{-1}\sum_{i=1}^{p}(l_i - \bar{l})^2}{(\bar{l})^2} \quad (5)$$

where $\bar{l}$ being equal to,

$$\bar{l} = \frac{1}{n}\sum_{i=1}^{p}(l_i)$$

It can be observed that the John's Test T_2 proportional to the ratio of square of variation from sample eigenvalues to the square of mean of sample eigenvalues. Under H_0 hypothesis, the T_2 test statistic follows a Chi-Squared distribution, χ_f^2 with $f = \frac{1}{2}p(p+1) - 1$ degrees of freedom.

The distribution function of the statistic T_2 was obtained using the Box-Bartlett correction [17],

$$P(T_2 \leq t_{jt}) = P_f(t_{jt}) + \frac{1}{n}\{a_p P_{f+6}(t_{jt}) + b_p P_{f+4}(t_{jt}) + \quad (6)$$

$$c_p P_{f+2}(t_{jt}) + d_p P_f(t_{jt})\} + O(n^{-2}) \quad (7)$$

where, t_{jt} is threshold for John's test and $O(n^{-2})$ is the convergence factor and

$$P_k(t_{jt}) = P(\chi_k^2 \leq t_{jt})$$

$$a_p = \frac{1}{12}(p^3 + 3p^2 - 12 - 200p^{-1})$$

$$b_p = \frac{1}{8}(-2p^3 - 5p^2 + 7p - 12 - 420p^{-1})$$

$$c_p = \frac{1}{4}(p^3 + 2p^2 - p - 2 - 216p^{-1})$$

$$d_p = \frac{1}{24}(-2p^3 - 3p^2 + p + 436p^{-1})$$

ii. **Corrected John's Test (CJT):**
Corrected John's Test (CJT) is the modified version of John's Test, which has been proposed by Q. Wang et al. in [17].

$$U = 2(np)^{-1}T_2 \tag{8}$$

under classical scenario, p fixed and $n \to \infty$, U follows Chi-Squared distribution, which is represented in the form $nU - p$.

$$nU - p \Rightarrow \frac{2}{p}\chi_f^2 - p \tag{9}$$

C. Some other Conventional Detectors

i. **Scaled Largest Eigenvalue (SLE):** The test statistic of SLE is defined as [8],

$$T_{SLE} = \frac{l_1}{\frac{1}{p}\sum_{i=1}^{P} l_i} \tag{10}$$

The T_{SLE} has a Tracy-Widom distribution under H_0 hypothesis, when scaled and centralized by μ_{np} and σ_{np}[19]. The threshold is formulated as a function of Tracy-Widom distribution, and is represented as [13, 14]

$$t_{SLE} = (1 + \sqrt{\frac{p+1/2}{n+1/2}})^2 + \frac{(\sqrt{p+1/2} + \sqrt{n+1/2})^{4/3}}{n[(p+1/2)(n+1/2)]^{1/6}} F_2^{-1}(1-\epsilon) \tag{11}$$

Where F_2^{-1} is the Tracy-Widom distribution of order β =2

ii. **Arithmetic to Geometric Mean (AGM):** The test statistic of AGM is defined as [13],

$$T_{AGM} = 2(n-1)log\left(\frac{\frac{1}{p}\sum_{i=1}^{p} l_i}{(\prod_{i=1}^{p} l_i)^{1/p}}\right)^P \tag{12}$$

The threshold expression for AGM [13] is represented as,

$$t_{AGM} = \frac{2}{c_1}\bar{\Gamma}^{-1}(1-\epsilon, p^2-1) \tag{13}$$

iii. **Maximum-Minimum Eigenvalue (MME):** The test statistic of MME is defined as [9],

$$T_{MME} = \frac{l_1}{l_m} \tag{14}$$

According to the theorem by I. M. Johnstone, the covariance matrix of received signal under H_0 hypothesis has to be centered and scaled to make it follow the Tracy-Widom distribution[9]. So the R_X is to be formed in the form of $\frac{l_{max}(A_{MME})-\mu}{\nu}$, where the matrix $A_{MME} = \frac{N}{\sigma_n^2} * R_X$

And the threshold is given by [9],

$$t_{MME} = \frac{(\sqrt{n+1/2} + \sqrt{p+1/2})^2}{(\sqrt{n+1/2} - \sqrt{p+1/2})^2} (1 + \frac{(\sqrt{n+1/2} + \sqrt{p+1/2})^{-2/3}}{[(p+1/2)(n+1/2)]^{1/6}} F_2^{-1}(1-\epsilon)) \tag{15}$$

III. Probability of False Alarm and Asymptotic Threshold for CJT

A. Probability of False Alarm

Making use of the distribution function of John's test [17], the P_{fa} expression for CJT can be obtained as,

$$P_{fa} = P(nU - p > t_{CJT}|H_0) \tag{16}$$

$$= 1 - P(nU - p \leq t_{CJT}|H_0)$$

$$= 1 - [P(\frac{2}{p}\chi_f^2 - p \leq t_{CJT}) + \frac{1}{n}\{a_p P(\frac{2}{p}\chi_{f+6}^2 - p \leq t_{CJT}) + b_p P(\frac{2}{p}\chi_{f+4}^2 - p \leq t_{CJT}) + c_p P(\frac{2}{p}\chi_{f+2}^2 - p \leq t_{CJT}) + d_p P(\frac{2}{p}\chi_f^2 - p \leq t_{CJT})\}] \tag{17}$$

$$= 1 - [P(\chi_f^2 \leq \frac{(t_{CJT}+p)p}{2}) + \frac{1}{n}\{a_p P(\chi_{f+6}^2 \leq \frac{(t_{CJT}+p)p}{2}) + b_p P(\chi_{f+4}^2 \leq \frac{(t_{CJT}+p)p}{2}) + c_p P(\chi_{f+2}^2 \leq \frac{(t_{CJT}+p)p}{2}) + d_p P(\chi_f^2 \leq \frac{(t_{CJT}+p)p}{2})\}] \tag{18}$$

$$= 1 - [\frac{\gamma(\frac{f}{2}, \frac{(t_{CJT}+p)p}{4})}{\Gamma(\frac{f}{2})} + \frac{1}{n}\{a_p \frac{\gamma(\frac{f+6}{2}, \frac{(t_{CJT}+p)p}{4})}{\Gamma(\frac{f+6}{2})} + b_p \frac{\gamma(\frac{f+4}{2}, \frac{(t_{CJT}+p)p}{4})}{\Gamma(\frac{f+4}{2})} + c_p \frac{\gamma(\frac{f+2}{2}, \frac{(t_{CJT}+p)p}{4})}{\Gamma(\frac{f+2}{2})} + d_p \frac{\gamma(\frac{f}{2}, \frac{(t_{CJT}+p)p}{4})}{\Gamma(\frac{f}{2})}\}] \tag{19}$$

$$= 1 - [F_{\frac{f}{2}}(\frac{(t_{CJT}+p)p}{4}) + \frac{1}{n}\{a_p F_{\frac{f+6}{2}}(\frac{(t_{CJT}+p)p}{4}) + b_p F_{\frac{f+4}{2}}(\frac{(t_{CJT}+p)p}{4}) + c_p F_{\frac{f+2}{2}}(\frac{(t_{CJT}+p)p}{4}) + d_p F_{\frac{f}{2}}(\frac{(t_{CJT}+p)p}{4})\}] \tag{20}$$

where, $\gamma(\frac{f}{2}, \frac{(t_{CJT}+p)p}{4})$ is the lower incomplete gamma function, $\Gamma(\frac{f}{2})$ is the gamma function and $F_{\frac{f}{2}}(\frac{(t_{CJT}+p)p}{4})$ is the CDF of Chi-Squared distribution, considering the left tail region, with f degrees of freedom and t_{CJT} being the threshold of CJT test statistic.

B. Asymptotic Theoretical Threshold

From the Eq. 20, it can be seen that there are five chi-squared distribution functions with different degrees of freedom which makes it difficult to obtain the threshold value. It can be observed that the a_p, b_p, c_p, d_p terms can be neglected when multiplied by it's CDF. Following, an asymptotic threshold expression has been proposed. The approximated probability of false alarm expression can be obtained as,

$$P_{fa} = F_{\frac{f}{2}}\left(\frac{(t_{CJT}+p)p}{4}\right) \tag{21}$$

$$F_{\frac{f}{2}}^{-1}(P_{fa}) = \frac{(t_{CJT}+p)p}{4}$$

Taking inverse, the asymptotic threshold is represented as,

$$t_{CJT} \approx \frac{4}{p} F_{\frac{f}{2}}^{-1}(P_{fa}) - p \tag{22}$$

where, t_{CJT} is the asymptotic threshold for statistic $nU - p$ and $F_{\frac{f}{2}}^{-1}(P_{fa})$ denotes the inverse of chi-square CDF, considering the right-tail region, with f degrees of freedom.

IV. Simulation Results and Analysis

The detection performance of CJT can be observed by implementing a MATLAB based simulation. It has been also compared with SLE, AGM and MME to inspect it's dominance under low sample and fading environment. The distribution of CJT under H_1 hypothesis has not been proposed in [17]. Henceforth to avoid any disruption in performance, the signal is considered to be normal distributed. The detection of CJT has been analyzed under severe Nakagami fading environment (m=0.5)[20, 21], and also considering very less number of samples and sensors (n=12, p=3). Severe Nakagami fading has been taken to analyze the worst case performance of CJT detector. The monte carlo simulation has been implemented for 10^6 number of iterations.

The accuracy of proposed theoretical threshold expression can be observed by comparing it with the simulated threshold. Fig. 2 indicates two different combinations of sensors and samples scene. For low sample case, $n = 12$ and $p = 3$, it depicts a very minute difference between the proposed and simulated thresholds under target false alarm value $P_{fa} = 0.1$. However, as the value of P_{fa} increases, both are exactly the same. For relatively more number of samples and sensors case, i.e., $n = 24$ and $p = 8$, it has been seen that the difference between simulated and theoretical curves remain almost negligible. The main conclusion is that, the proposed asymptotic threshold respects the classical regime, i.e., it has been observed by the authors that, even by keeping the number of sensors fixed, the accuracy of proposed threshold expression increases as $n \to \infty$.

The receiver operating characteristics of CJT under no fading and severe Nakagami fading channel is demonstrated in Fig. 3 . There is almost 5-10 % difference in the probability detection with respect to no fading. Likewise, Fig. 4 shows the plot of P_d vs SNR for CJT in comparison to the traditional eigenvalue methods, under sample starving environment. It can been observed that, there is almost ten percentage of improvement in the probability of detection for zero to five dB SNR.

The receiver operating characteristics (ROC) of CJT in Fig. 5 shows that under target false alarm $P_{fa} = 0.1$, the detection performance of CJT dominates over the SLE, MME and AGM even under very less number of samples and also in the presence of Nakagami fading effect. Thus, our simulation results shows the superiority of CJT over other existing techniques in terms of probability detection.

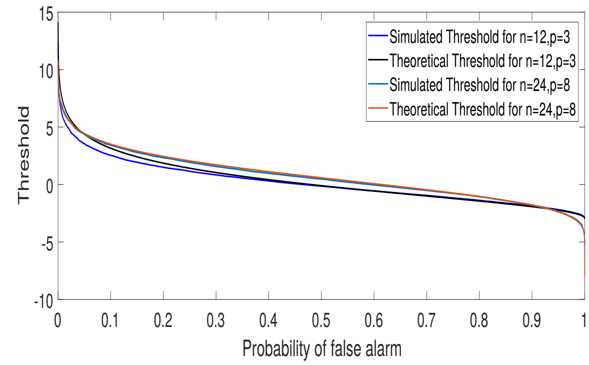

Fig. 2. Accuracy of proposed threshold expression

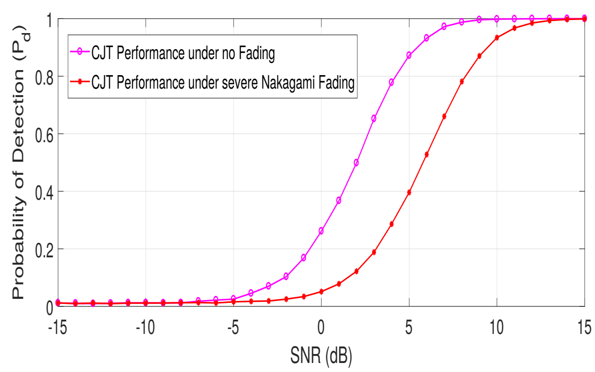

Fig. 3. Comparison of CJT performance under no fading channel and Nakagami channel

V. Conclusion

Corrected John's Test (CJT) is a detector which can perform better under low samples environment. The CJT's performance is analyzed under the presence of severe Nakagami fading case, and it can be seen that CJT dominates all traditional eigenvalue detectors. However, as the number of samples increases, SLE surpasses CJT. The EMR test's performance dominates the CJT's even for less samples. In any case, EMR's regime of convergence is different from that of CJT's. Furthermore, the proposed threshold expression has been approximated by neglecting the smaller terms. Therefore, further statistical analysis is required so that Eq. 20 can be solved

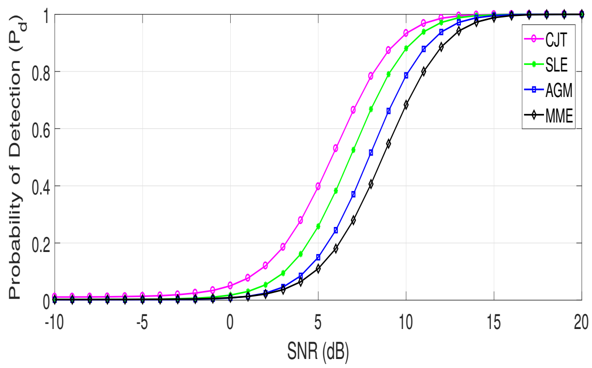

Fig. 4. SNR vs P_d of CJT compared with SLE, MME and AGM

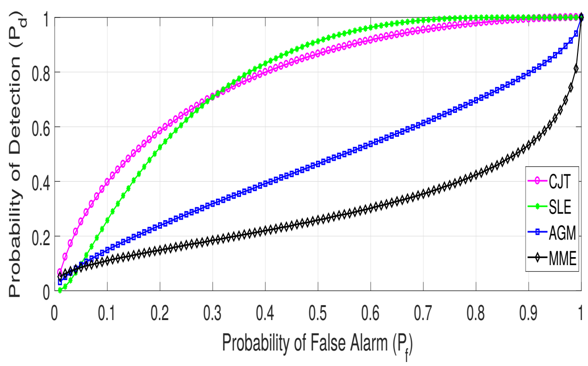

Fig. 5. ROC of CJT compared to SLE, MME, AGM for SNR=5dB.

for better threshold value and in turn the accuracy may also improve. In future, CJT would be also tested for multiple primary users transmission.

REFERENCES

[1] A. Khattab, D. Perkins, and M. Bayoumi, *Cognitive radio networks: from theory to practice*. Springer Science & Business Media, 2012.

[2] O. Holland, H. Bogucka, and A. Medeisis, *Opportunistic spectrum sharing and white space access: The practical reality*. John Wiley & Sons, 2015.

[3] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE journal on selected areas in communications*, vol. 23, no. 2, pp. 201–220, 2005.

[4] R. Umar, A. U. Sheikh, and M. Deriche, "Unveiling the hidden assumptions of energy detector based spectrum sensing for cognitive radios," *IEEE Communications Surveys & Tutorials*, vol. 16, no. 2, pp. 713–728, 2014.

[5] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proceedings of the IEEE*, vol. 55, no. 4, pp. 523–531, 1967.

[6] T. Yucek and H. Arslan, "A survey of spectrum sensing algorithms for cognitive radio applications," *IEEE communications surveys & tutorials*, vol. 11, no. 1, pp. 116–130, 2009.

[7] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE transactions on communications*, vol. 57, no. 6, 2009.

[8] B. Nadler, F. Penna, and R. Garello, "Performance of eigenvalue-based signal detectors with known and unknown noise level," in *Communications (ICC), 2011 IEEE International Conference on*. IEEE, 2011, pp. 1–5.

[9] Y. Zeng and Y.-C. Liang, "Maximum-minimum eigenvalue detection for cognitive radio," in *Personal, Indoor and Mobile Radio Communications, 2007. PIMRC 2007. IEEE 18th International Symposium on*. IEEE, 2007, pp. 1–5.

[10] S. M. Kay, "Fundamentals of statistical signal processing, vol. ii: Detection theory," *Signal Processing. Upper Saddle River, NJ: Prentice Hall*, 1998.

[11] M. Hamid, N. Björsell, and S. B. Slimane, "Energy and eigenvalue based combined fully blind self adapted spectrum sensing algorithm," *IEEE Transactions on Vehicular Technology*, vol. 65, no. 2, pp. 630–642, 2016.

[12] Y. Zeng and Y.-C. Liang, "Eigenvalue-based spectrum sensing algorithms for cognitive radio," *IEEE transactions on communications*, vol. 57, no. 6, 2009.

[13] L. Huang, J. Fang, K. Liu, H. C. So, and H. Li, "An eigenvalue-moment-ratio approach to blind spectrum sensing for cognitive radio under sample-starving environment," *IEEE Transactions on Vehicular Technology*, vol. 64, no. 8, pp. 3465–3480, 2015.

[14] A. Taherpour, M. Nasiri-Kenari, and S. Gazor, "Multiple antenna spectrum sensing in cognitive radios," *IEEE transactions on wireless communications*, vol. 9, no. 2, pp. 814–823, 2010.

[15] A. Edelman, B. D. Sutton, and Y. Wang, "Random matrix theory, numerical computation and applications," *Modern Aspects of Random Matrix Theory*, vol. 72, p. 53, 2014.

[16] I. M. Johnstone, "On the distribution of the largest eigenvalue in principal components analysis," *Annals of statistics*, pp. 295–327, 2001.

[17] Q. Wang, J. Yao *et al.*, "On the sphericity test with large-dimensional observations," *Electronic Journal of Statistics*, vol. 7, pp. 2164–2192, 2013.

[18] S. John, "The distribution of a statistic used for testing sphericity of normal distributions," *Biometrika*, vol. 59, no. 1, pp. 169–173, 1972.

[19] B. Nadler, "On the distribution of the ratio of the largest eigenvalue to the trace of a wishart matrix," *Journal of Multivariate Analysis*, vol. 102, no. 2, pp. 363–371, 2011.

[20] N. Pillay and H. Xu, "Eigenvalue-based spectrum holedetection for nakagami-m fading channels with gaussian and impulse noise," *IET communications*, vol. 6, no. 13, pp. 2054–2064, 2012.

[21] M. K. Simon and M.-S. Alouini, *Digital communication over fading channels*. John Wiley & Sons, 2005, vol. 95.