Outage Analysis for Underlay Cognitive TWRN with Multi-antenna Sources in Nakagami-m fading

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***Abstract*—In this paper, we investigate the outage performance of a cooperative and cognitive radio system where two secondary users (SUs) communicate through a relay via the two-way relaying. The coexistence of SUs with the primary users (PUs) in the same spectrum is possible by using underlay protocol, where the interference at the PU receiver is under a predefined threshold. Focusing on the cooperation process among the SU nodes and making use of the underlay cognitive approach, we derive the exact user outage probability(OP) in closed-form. Furthermore, the tight closed-form expression for the end to end OP is also derived, which is declared when any of a SU is in the outage. Our analysis employs transmit/receive beamforming at the SU nodes and analog network coding at relay node where opportunistic selection algorithm is deployed for selecting best relay in underlay scenario. Furthermore, we have derived the asymptotic performance and thus focus on the performance gain achieved by multi-relay and multiantenna diversity under Nakagami-m fading. Besides, we also evaluate the optimized locations for the relay under practical scenarios to minimize the OP. Monte Carlo simulation results are presented to corroborate the proposed analysis.**

***Index Terms*—Cognitive radio, primary user, secondary user, two-way relay, beamforming, AF relaying, outage probability.**

I. INTRODUCTION

Increasing data traffic and the limited spectrum resources are two major concerns for the future wireless technologies [1]. Thus, there is a high demand for a technology that can effectively and efficiently reuse the wireless spectrum. Among the various technologies, cognitive radio networks (CRNs) is a promising candidate to fulfill these requirements by allowing the coexistence of licensed users (a.k.a. primary users - PUs) and unlicensed users (a.k.a. secondary users - SUs) [2]. In this technique, underlay, overlay, and interweave are the most popular spectrum sharing protocols [3]. In the underlay approach, the transmission power of SUs is constrained so that the interference to the PUs is minimum or under predefined threshold [4].

Recently, cooperative relaying communication (CRC) has emerged as a powerful technology to overcome the fading severity and limited coverage area [5]. The idea behind CRC is to imitate an antenna array by allowing multiple users to relay the signal to each other. Hence, the target destination can receive the same signal through multiple paths, thus being able to exploit diversity. There is two basic relaying protocol that has been extensively studied in literature are amplify-and-forward (AF) and decode-and-forward (DF). On the other hand, CRC allows the user to transmit signal with low power, which could help to improve the throughput of underlay secondary network [6].

Owing to the remarkable perspective for throughput enhancement achieved with the integration of cooperative relaying and cognitive radio has attracted significant attention [6]–[9]. In [6], authors investigate the outage performance of underlay one-way relaying network (OWRN) in Nakagami-m fading channel. In [7] authors examined the outage performance of multiuser underlay spectrum sharing network, where the user with the highest instantaneous signal-to-noise ratio (SNR) is scheduled for transmission. In [8], yeoh *et al.* have analyzed the performance of a one-way UOWRN by assuming the multiple antennas at all terminals in the presence of multiple PUs, where authors include the transmit antenna selection (TAS) among the multiantenna terminals to improve the received signal-to-interference-noise ratio (SINR). Finally, multiantenna multihop UOWRN was investigated in [9], where authors compare maximal ratio combining (MRC) and selection combining schemes in the presence of multiple interferences. Due to limited throughput of OWRN protocol, two-way relaying (TWR) protocol is proposed [10].

Prior related research on underlay TWRN: In [10], the authors addressed the DF underlay cognitive TWRN over Nakagami-m fading channel, where analysis is limited to the bounds on the system performance. In [11], bidirectional AF underlay CRN has been considered, opportunistic relay selection (ORS) is adopted and bounds on the performance are studied in Rayleigh fading. Authors in [12] studied the outage performance of underlay two-way multi-relay network, apart from that the performance of the secondary system was analyzed by terminals location. Multiantenna zero-forcing (ZF) beamforming in underlay TWRN is considered in [13], where authors investigated the outage performance for fixed gain Rayleigh fading.In [14], multi-antenna multi-relay (MAMR) distributed beamforming is used instead of network coding to evaluate the system performance. Recently, study of optimal power allocation in massive MIMO multi-relay based underlay TWRN is done in [15].

Motivation and contribution: In [10], [11] and [16], authors considered only single antenna source terminal. In [13] authors considered only multiantenna beamforming in UTWRN while in [14], [15], authors considered both multiantenna and multi-relay. The distributed beamforming instead of network coding is used in [14], on the other hand, [15] obtain the limited analysis of the considered model by considering optimal power allocation in massive MIMO scenario. To the authors best knowledge no prior work has been done in underlay cognitive TWRN using both MRC/MRT(Maximal ratio transmission)

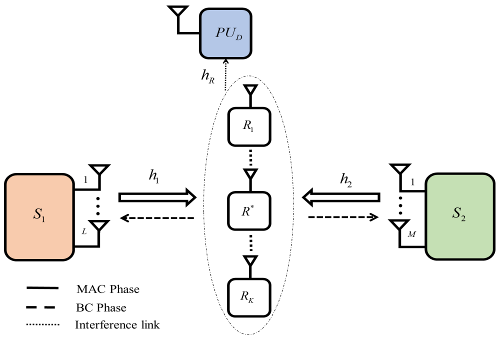

Fig. 1. Multi-user two-way AF relaying

and ORS. Note that such investigations are crucial for the design of an ad-hoc TWRN in practical applications. The major contributions of this paper can be encapsulated as follows:

- We obtain the two exact end-to-end (e2e) instantaneous SNRs for the secondary network after applying partial self-interference cancellation.
- Based on the user SNR, we derive a closed-form expression for the OP. To gain further insights, we also develop high SNR asymptotic OP expressions.
- We further derive a tight approximate expression of the e2e outage probability (OP) and its asymptotic expression.
- To minimize the secondary network system end to end OP, we present the relay location optimization problem.

Above all, numerical and simulation analyses are conducted to illustrate the secondary system performance under various practical scenarios and different values of primary outage threshold.

II. System and Channel Model

Here we investigate the performance of MAMR-underlay TWRN, where the secondary network consists of a source S_1 and S_2 communicate with each other via one intermediate relay R_k selected from K number of relays in the presence of a PU receiver (as shown in Fig. 1). The nodes S_1 and S_2 (i.e., end nodes) are MIMO enabled with L and M antennas respectively, and the opportunistic selection is adopted at the relay terminal with single antenna[1]. MRT and MRC is performed by S_1 and S_2 while k-th relay R_k perform amplify-and-forward (AF) transmission, where $k \in \{1, 2, .., K\}$. Furthermore, self-interference cancellation is performed by the end sources. It is assumed that the S_1 and S_2 is having no direct links, which may be due to high shadowing or path loss. All nodes are further assumed to operate in a half-duplex manner. Primary network consists of primary receiver node having the single antenna and primary transmitter is assumed to be far away from the secondary network.

The secondary transmission process is completed in two phases, namely multiple access (MAC) phase and broadcast (BC) phase. In MAC phase (which is represented as solid line in Fig. 1), the secondary source S_1 and S_2 transmits the collaborative beamforming to the selected relay, without incurring the interference to the primary user (PU) with transmit powers P_1 and P_2. In the second phase, the selected relay amplifies the received signal from the source S_1 and S_2 and then forwards it to the destination with the power P_R. The transmit power at P_1, P_2 and P_R are constrained so that the interference impinged on the PU receiver remains below the maximum tolerable interference power Q [6]. It is further assumed that the R_k is having complete knowledge of instantaneous channel state information (CSI) of $R_k \rightarrow PU_D$ link while S_1 and S_2 is having average CSI of $S_1 \rightarrow PU_D$ and $S_2 \rightarrow PU_D$ respectively.

Source S_1 transmits symbol x_1 to R_k using L antennas and S_2 transmits x_2 to R_k using M antennas, both the symbol x_1 and x_2 are assumed to be having unit energy. In MAC phase (which is represented by solid lines in Fig. 1), the signal received at k-th relay R_k is given as[2]

$$y_{R_k} = \sqrt{P_1}\mathbf{h}_{1_k}^T \mathbf{w}_{1_k} x_1 + \sqrt{P_2}\mathbf{h}_{2_k}^T \mathbf{w}_{2_k} x_2 + n_{R_k} \tag{1}$$

where x_i and P_i denote the respective transmit signal (with unit energy) and transmit power from the source $\mathtt{S_i}$, for $i \in \{1, 2\}$, and n_{R_k} represents the additive white Gaussian noise (AWGN) at R_k. $\mathbf{h}_{1_k} = [h^1_{1_k}, h^1_{2_k}, ...h^1_{L_k}]$ and $\mathbf{h_{2_k}} = [h^2_{1_k}, h^2_{2_k}, ...h^2_{M_k}]$ defines the channel fading coefficients between $S_1 \rightarrow R_k$ and $S_2 \rightarrow R_k$ respectively, Therefore,the transmit power at S_1, S_2 and R_k can be mathematically written as $P_1\Omega_{S_1P} + P_2\Omega_{S_2P} \leq Q$ and $P_R \leq \frac{Q}{|h_{R_k}|^2}$, where Ω_{S_1P} and Ω_{S_2P} denote the average channel gain between $S_1 \rightarrow PU_D$ and $S_2 \rightarrow PU_D$ respectively. $\mathbf{w}_{1_k} = \left(\mathbf{h}^\dagger_{1_k}/\|\mathbf{h}_{1_k}\|\right)^T$ and $\mathbf{w}_{2_k} = \left(\mathbf{h}^\dagger_{2_k}/\|\mathbf{h}_{2_k}\|\right)^T$ defines the transmit weight vector at S_1 and S_2 respectively. The transmit power from S_1 and S_2 are given as $\mathbb{E}[\|\sqrt{P_1}\mathbf{w}_{1_k}\|^2] = P_1$ and $\mathbb{E}[\|\sqrt{P_2}\mathbf{w}_{2_k}\|^2] = P_2$, respectively. In BC phase (which is represented by dashed lines in Fig. 1), the received signal at S_1 and S_2 are given by

$$y_{S_1} = \mathbf{w}_1^\dagger \left(\mathcal{G}\mathbf{h}_{1_k} y_{R_k} + \mathbf{n}_1\right) \tag{2}$$

$$y_{S_2} = \mathbf{w}_{2_k}^\dagger \left(\mathcal{G}\mathbf{h}_{2_k} y_{R_k} + \mathbf{n}_2\right) \tag{3}$$

where, $\mathbf{n}_1$ and $\mathbf{n}_2$ represents AWGN at S_1 and S_2 respectively and $\mathcal{G}^2 = P_R/\left(P_1 \parallel \mathbf{h}_{1_k}\|^2 + P_2 \parallel \mathbf{h}_{2_k}\|^2 + N_0\right)$ defines the variable gain[2]. After canceling the self interference term at S_1 and S_2 leads to

$$\tilde{y}_{S_1} = \mathcal{G}\sqrt{P_2}\mathbf{w}_{1_k}^\dagger \mathbf{h}_{1_k}\mathbf{h}_{2_k}^\dagger \mathbf{w}_{2_k} x_2 + \mathcal{G}\mathbf{w}_{1_k}^\dagger \mathbf{h}_{1_k} n_{R_k} + \mathbf{w}_{1_k}^\dagger \mathbf{n}_1 \tag{4}$$

[1]In rela life applications, the SU terminals could be two wireless access points in two micro-cell base stations or two separate homes connected temporarily by the relay, which provides a wireless backhaul service.

[2]**Notations**: In this paper scalar and vectors are represented as an italic symbol and lower case boldface symbols respectively. For any scalar a, absolute value is denoted by $|a|$. For a given complex vector $\mathbf{a}$, $(\mathbf{a})^T$ represents the transpose, $(\mathbf{a})^\dagger$ represent the conjugate transpose and $\|\mathbf{a}\|$ denotes the Euclidean norm. $\mathcal{CN}(\mu, \sigma^2)$ denotes a complex circular Gaussian random variable with mean μ and variance σ^2. where $\Upsilon(\cdot, \cdot)$ indicates the lower incomplete gamma function [17, Eq. (8.350.1)]. $\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ denote the gamma function [17, Eq. (8.310.1)] and the upper incomplete gamma function [17, Eq. (8.350.2)],respectively. $\mathbb{E}[\cdot]$ shows the expectation.

[2]Without loss of generality we have assumed that the noise at all the terminal (n_{R_K}, n_1, n_2) follows $\mathcal{CN}(0, N_0)$.

$$\tilde{y}_{S_2} = \mathcal{G}\sqrt{P_1}\mathbf{w}_{2_k}^{\dagger}\mathbf{h}_{2_k}\mathbf{h}_{1_k}^{\dagger}\mathbf{w}_{1_k}x_1 + \mathcal{G}\mathbf{w}_{2_k}^{\dagger}\mathbf{h}_{2_k}n_{R_k} + \mathbf{w}_{2_k}^{\dagger}\mathbf{n}_2 \quad (5)$$

Thus the received end-to-end instantaneous SNR at S_1 can now be written as

$$\gamma_{S_2\to R_k\to S_1} = \frac{\mathcal{G}^2 P_2 \, \|\mathbf{h}_{2_k}\|^2\|\mathbf{h}_{1_k}\|^2}{\mathcal{G}^2 \, \|\mathbf{h}_{1_k}\|^2 \, N_0 + N_0} \quad (6)$$

Now substituting $\mathcal{G}$ in above equation and after some mathematical manipulations we obtain

$$\gamma_{S_2\to R_k\to S_1} = \frac{\lambda_1 \left[\frac{P_1\|\mathbf{h}_{1_k}\|^2}{N_0}\right]\left[\frac{P_2\|\mathbf{h}_{2_k}\|^2}{N_0(\lambda_1+1)}\right]}{\left[\frac{P_1\|\mathbf{h}_{1_k}\|^2}{N_0}\right]+\left[\frac{P_2\|\mathbf{h}_{2_k}\|^2}{N_0(\lambda_1+1)}\right]} \quad (7)$$

where $\lambda_1 = \frac{P_R}{P_1}$. The above SNR expression can be upper bounded as

$$\gamma_{S_2\to R_k\to S_1} \le \gamma^{up}_{S_2\to R_k\to S_1} = \lambda_1 \min\left(\frac{P_1 \, \|\mathbf{h}_{1_k}\|^2}{N_0}, \frac{P_2 \, \|\mathbf{h}_{2_k}\|^2}{N_0\,(\lambda_1+1)}\right) \quad (8)$$

Similarly

$$\gamma_{S_1\to R_k\to S_2} \le \gamma^{up}_{S_1\to R_k\to S_2} = \lambda_2 \min\left(\frac{P_1 \, \|\mathbf{h}_{1_k}\|^2}{N_0\,(\lambda_2+1)}, \frac{P_2 \, \|\mathbf{h}_{2_k}\|^2}{N_0}\right) \quad (9)$$

where $\lambda_2 = \frac{P_R}{P_2}$. Each relay is trained independently so that the end secondary users on both sides of a relay can calculate equivalent e2e SNRs. By doing so, the best relay R_{k^*}, is selected opportunistically as given by

$$k^* = \arg\max_{k\in\{1,\dots,K\}} \gamma^{up}_J \quad (10)$$

where $J \in \{S_2\to R_k\to S_1, S_1\to R_k\to S_2\}$.

Herein, we assume that all channel coefficients undergo Nakagami-m fading. As a result, $\|\mathbf{h}_{1_k}\|^2$, $\|\mathbf{h}_{2_k}\|^2$ and $|h_{R_k}|^2$ are gamma distributed with fading severity parameters m_1, m_2 and m_{R_k} and channel powers Ω_1, Ω_2 and Ω_{R_k} respectively. Thus,the probability density function and cumulative distribution function (CDF) of $\mathcal{W}$, for $\mathcal{W} \in \left\{\|\mathbf{h}_{1_k}\|^2, \|\mathbf{h}_{2_k}\|^2, |h_{R_k}|^2\right\}$,can be formulated in compact form as,

$$f_{\mathcal{W}}(w) = \frac{\alpha^m w^{m-1}}{\Gamma(m)}e^{-\alpha w}, \quad \mathcal{F}_{\mathcal{W}}(w) = 1 - \frac{\Gamma(m,\alpha w)}{\Gamma(m)} \quad (11)$$

where $\alpha \in \left\{\alpha_1 = \frac{m_1}{\Omega_1}, \alpha_2 = \frac{m_2}{\Omega_2}, \alpha_{R_k} = \frac{m_{R_k}}{\Omega_{R_k}}\right\}$ and $m \in \{Lm_1, Mm_2, m_{R_k}\}$;

III. Performance Analysis

In this section, outage probability (OP) and asymptotic analysis for considered systems under spectrum sharing constraint are derived.

A. Tight Lower Bound Expression

The user OP is defined as the probability that the instantaneous SNR at S_1 is below a certain threshold γ_{th}, i.e.,

$$\mathcal{P}_{out} = \Pr\left(\gamma_{S_2\to R_{k^*}\to S_1} \le \gamma_{th}\right) \quad (12)$$

Proposition 1: *The OP of instantaneous SNR $\gamma^{up}_{S_2\to R_{k^*}\to S_1}$ for multiantenna multirelay TWR spectrum sharing network can be defined as*

$$\mathcal{P}_{out} = \prod_{k=1}^{K}\left(1 - e^{-\frac{\alpha_2\gamma_{th}N_0\Omega_4}{(1-\xi)Q}} \sum_{i=0}^{m_1L-1}\sum_{j=0}^{m_2M-1}\sum_{t=0}^{j} \binom{j}{t} \frac{\left(\frac{\alpha_1\gamma_{th}N_0}{Q}\right)^i}{i!} \frac{\left(\frac{\alpha_2\gamma_{th}N_0\Omega_4}{(1-\xi)Q}\right)^j}{j!} \left(\frac{\xi}{\Omega_3}\right)^t \frac{\Gamma\left(m_{R_{k^*}}+i+t\right)}{\Gamma\left(m_{R_{k^*}}\right)} \times \alpha_{R_{k^*}}^{m_{R_{k^*}}} \left(\alpha_{R_{k^*}} + \frac{\gamma_{th}N_0}{Q}\left(\alpha_1 + \alpha_2\frac{\xi\Omega_4}{(1-\xi)\,\Omega_3}\right)\right)^{-m_{R_{k^*}}-i-t}\right) \quad (13)$$

Proof: See Appendix-A.
Similarly the OP at S_2 can be defined.

B. Asymptotic Analysis for OP

For better insight, we present high SNR asymptotic expression of (12) by applying the first order Taylor series expansion $e^{-x} \approx (1-x)$ and using [17, Eq. (8.354.1)] for asymptotic representation of incomplete gamma function i.e, $\Upsilon(m,\alpha x) \overset{x\to 0}{=} \frac{(\alpha x)^m}{m}$ into (12) and by neglecting the higher order terms we get

$$\mathcal{P}^{\infty}_{out} \underset{\lambda_Q\to\infty}{\simeq} \mathcal{G}_c\left(\frac{\gamma_{th}}{\lambda_Q}\right)^{\mathcal{G}_d}, \quad (14)$$

where diversity gain and coding gain are represented as $\mathcal{G}_d$ and $\mathcal{G}_c$ respectively which are defined as

$$\mathcal{G}_d = K \times \min(m_1L, m_2M) \quad (15)$$

$$\mathcal{G}_c = \begin{cases} \mathcal{G}_{c_1}, & m_1L < m_2M \\ \mathcal{G}_{c_2}, & m_1L > m_2M \\ \mathcal{G}_{c_1} + \mathcal{G}_{c_2}, & m_1L = m_2M, \end{cases} \quad (16)$$

where $\mathcal{G}_{c_1}$ and $\mathcal{G}_{c_2}$ are represented as

$$\mathcal{G}_{c_1} = \prod_{k=1}^{K}\frac{\alpha_1^{m_1L}}{\Gamma(m_1L+1)} \times \frac{\alpha_{R_{k^*}}^{-m_1L}}{\Gamma\left(m_{R_{k^*}}\right)} \times \Gamma\left(m_{R_{k^*}} + m_1L\right) \quad (17)$$

$$\mathcal{G}_{c_2} = \prod_{k=1}^{K}\frac{\left(\frac{\alpha_2\Omega_4}{(1-\xi)}\right)^{m_2M}}{\Gamma(m_2M+1)} \sum_{q=0}^{m_2M}\binom{m_2M}{q}\left(\frac{\xi}{\Omega_3}\right)^q \times \frac{\alpha_{R_{k^*}}^{-q}\,\Gamma\left(m_{R_{k^*}}+q\right)}{\Gamma\left(m_{R_{k^*}}\right)} \quad (18)$$

IV. End To End Outage Probability

End to end OP of a system can be defined as the probability with which the instantaneous SNR using k-th relay $\gamma_{S_1\to R_k\to S_2}$ or $\gamma_{S_2\to R_k\to S_1}$, drops below a predefined threshold γ_{th}

$$\mathcal{P}_{out_{e2e},k}(\gamma_{th}) = \Pr(\min\left(\gamma^{up}_{S_1\to R_k\to S_2}, \gamma^{up}_{S_2\to R_k\to S_1}\right) \le \gamma_{th}) \quad (19)$$

$$= 1 - \Pr(\gamma^{up}_{S_1\to R_k\to S_2} > \gamma_{th},\, \gamma^{up}_{S_2\to R_k\to S_1} > \gamma_{th})$$

After some manuplations in $\gamma^{up}_{S_1\rightarrow R_k\rightarrow S_2}$ and $\gamma^{up}_{S_2\rightarrow R_k\rightarrow S_1}$, the above equation can be written as ...

$$\mathcal{P}_{out_{e2e},k} \approx 1 - \Pr\left(\min\left(\eta_1\left(\gamma_{th}\right)\|\mathbf{h}_{1_k}\|^2, \eta_2\left(\gamma_{th}\right)\|\mathbf{h}_{2_k}\|^2\right) \geq 1\right) \tag{20}$$

where $\eta_1\left(\gamma_{th}\right) \approx \frac{\lambda_2}{\gamma_{th}}\frac{P_1}{N_0(\lambda_2+1)}$ and $\eta_2\left(\gamma_{th}\right) \approx \frac{\lambda_1}{\gamma_{th}}\frac{P_2}{N_0(\lambda_1+1)}$. Now

$$\begin{aligned}\mathcal{P}_{out_{e2e},k} &\approx \Pr\left(\min\left(\eta_1\left(\gamma_{th}\right)\|\mathbf{h}_{1_k}\|^2, \eta_2\left(\gamma_{th}\right)\|\mathbf{h}_{2_k}\|^2\right) \leq 1\right)\\ &\approx \Pr\left(\|\mathbf{h}_{1_k}\|^2 \leq \frac{1}{\eta_1\left(\gamma_{th}\right)}, \|\mathbf{h}_{2_k}\|^2 \leq \frac{1}{\eta_2\left(\gamma_{th}\right)}\right)\end{aligned} \tag{21}$$

(21) can be further expressed as

$$\begin{aligned}\mathcal{P}_{out_{e2e},k} \approx 1 - &\left(1 - \mathcal{F}_{\|\mathbf{h}_{1_k}\|^2|Z}\left(\frac{1}{\eta_1\left(\gamma_{th}\right)}\right)\right)\\ &\times\left(1 - \mathcal{F}_{\|\mathbf{h}_{2_k}\|^2|Z}\left(\frac{1}{\eta_2\left(\gamma_{th}\right)}\right)\right)\end{aligned} \tag{22}$$

Now using the (10), the CDF of $\|\mathbf{h}_{1_k}\|^2$ and $\|\mathbf{h}_{1_k}\|^2$ as

$$\mathcal{F}_{\|\mathbf{h}_{1_k}\|^2|Z}\left(\frac{1}{\eta_1\left(\gamma_{th}\right)}\right) = 1 - \frac{\Gamma\left(m_1L, \frac{\alpha_1 N_0\gamma_{th}\Omega_3(\Omega_4+Z(1-\xi))}{\Omega_4\xi Q}\right)}{\Gamma\left(m_1L\right)} \tag{23}$$

and

$$\mathcal{F}_{\|\mathbf{h}_{2_k}\|^2|Z}\left(\frac{1}{\eta_2\left(\gamma_{th}\right)}\right) = 1 - \frac{\Gamma\left(m_2M, \frac{\alpha_2 N_0\gamma_{th}\Omega_4(\Omega_3+Z\xi)}{\Omega_3(1-\xi)Q}\right)}{\Gamma\left(m_2M\right)} \tag{24}$$

respectively. Now using the expansion of gamma function as $\Gamma\left(1+\mu, y\right) = \mu! e^{-y}\sum_{m=0}^{\mu}\frac{y^m}{m!}$ [17, Eq. (8.352.2)] and substituting the CDF of $\|\mathbf{h}_{1_k}\|^2$ & $\|\mathbf{h}_{1_k}\|^2$ in (22) and finally averaging w.r.t. Z, we obtain

$$\begin{aligned}\mathcal{P}_{out_{e2e},k} = 1 - &\int_{z=0}^{\infty} e^{-\frac{\alpha_1 N_0\gamma_{th}\Omega_3(\Omega_4+z(1-\xi))}{\Omega_4\xi Q}-\frac{\alpha_2 N_0\gamma_{th}\Omega_4(\Omega_3+z\xi)}{\Omega_3(1-\xi)Q}}\\ &\times\sum_{g=0}^{m_1L-1}\sum_{v=0}^{m_2M-1}\frac{1}{g!}\left(\frac{\alpha_1 N_0\gamma_{th}\Omega_3\left(\Omega_4+z\left(1-\xi\right)\right)}{\Omega_4\xi Q}\right)^g\\ &\times\frac{1}{v!}\left(\frac{\alpha_2 N_0\gamma_{th}\Omega_4\left(\Omega_3+z\xi\right)}{\Omega_3\left(1-\xi\right)Q}\right)^v f_z(z)dz\end{aligned} \tag{25}$$

Utilizing $f_Z(z) = \frac{\alpha_{R_k}^{m_{R_k}}}{\Gamma(m_{R_k})}z^{m_{R_k}-1}e^{-\alpha_{R_k}z}$ in (25), evaluating the integrals using [17, Eq. (3.351.3)] and after some mathematical manipulation we will get

$$\begin{aligned}\mathcal{P}_{out_{e2e},k} = &\Bigg(1 - e^{-N_0\gamma_{th}\left(\frac{\alpha_1\Omega_3}{\xi Q}+\frac{\alpha_2\Omega_4}{(1-\xi Q)}\right)}\frac{\alpha_{R_{k^*}}^{m_{R_{k^*}}}}{\Gamma\left(m_{R_{k^*}}\right)}\\ &\times\sum_{g=0}^{m_1L-1}\sum_{v=0}^{m_2M-1}\sum_{c=0}^{g}\sum_{l=0}^{v}\binom{g}{c}\binom{v}{l}\left(\frac{\xi}{\Omega_3}\right)^c\left(\frac{1-\xi}{\Omega_4}\right)^l\\ &\times\Gamma\left(m_{R_{k^*}}+c+l\right)\frac{\left(\frac{\alpha_1 N_0\gamma\Omega_3}{\xi Q}\right)^g}{g!}\frac{\left(\frac{\alpha_2 N_0\gamma_{th}\Omega_4}{(1-\xi)Q}\right)^v}{v!}\\ &\left(\alpha_{R_{k^*}}+\left(\frac{\alpha_1\left(1-\xi\right)\Omega_3}{\xi\Omega_4}+\frac{\alpha_2\xi\Omega_4}{\left(1-\xi\right)\Omega_3}\right)\frac{N_0\gamma_{th}}{Q}\right)^{-m_{R_{k^*}}-c-l}\Bigg)\end{aligned} \tag{26}$$

Now, the e2e OP after using selected relay as

$$\mathcal{P}_{out_{e2e}} = \prod_{k=1}^{K}\mathcal{P}_{out_{e2e},k} \tag{27}$$

A. Asymptotic Analysis

To draw better insight on diversity order and divesity gain of considered system model, here we present the asymptotic analysis of

$$\mathcal{P}^{\infty}_{out_{e2e}} \underset{\lambda_Q\rightarrow\infty}{\simeq} \bar{\mathcal{G}}_c\left(\frac{\gamma_{th}}{\lambda_Q}\right)^{\bar{\mathcal{G}}_d}, \tag{28}$$

where diversity gain and coding gain are represented as $\bar{\mathcal{G}}_d$ and $\bar{\mathcal{G}}_c$ respectively, which are defined as

$$\bar{\mathcal{G}}_d = K\times\min(m_1L, m_2M) \tag{29}$$

$$\bar{\mathcal{G}}_c = \begin{cases}\bar{\mathcal{G}}_{c_1}, & m_1L < m_2M\\ \bar{\mathcal{G}}_{c_2}, & m_1L > m_2M\\ \bar{\mathcal{G}}_{c_1}+\bar{\mathcal{G}}_{c_2}, & m_1L = m_2M,\end{cases} \tag{30}$$

where $\bar{\mathcal{G}}_{c_1}$ and $\bar{\mathcal{G}}_{c_2}$ are represented as

$$\begin{aligned}\bar{\mathcal{G}}_{c_1} = \prod_{k=1}^{K}\frac{\left(\frac{\alpha_1\Omega_3}{\xi}\right)^{m_1L}}{\Gamma\left(m_{R_{k^*}}\right)\Gamma\left(m_1L+1\right)}\sum_{u=0}^{m_1L}\binom{m_1L}{u}\\ \times\Gamma\left(m_{R_{k^*}}+u\right)\left(\frac{1-\xi}{\alpha_{R_{k^*}}\Omega_4}\right)^u\end{aligned} \tag{31}$$

and

$$\begin{aligned}\bar{\mathcal{G}}_{c_2} = \prod_{k=1}^{K}\frac{\left(\frac{\alpha_2 N_0\gamma_{th}\Omega_4}{(1-\xi)Q}\right)^{m_2M}}{\Gamma\left(m_{R_{k^*}}\right)\Gamma\left(m_2M+1\right)}\sum_{r=0}^{m_2M}\binom{m_2M}{r}\\ \times\Gamma\left(m_{R_{k^*}}+r\right)\left(\frac{\xi}{\alpha_{R_{k^*}}\Omega_3}\right)^r\end{aligned} \tag{32}$$

V. Relay Location Optimization

In this section, we address the optimization problem of relay location to minimize the end to end OP of our considered system under power constraint. The problem of relay location optimization can be formulated for total distance between S_1 and S_2 is normalized to unity,while d_1 and d_2 denote the distance between $S_1\rightarrow R_{k^*}$ and $S_2\rightarrow R_{k^*}$ respectively, so that $d_1+d_2=1$. The optimal relay position can be formulated as

$$\begin{aligned}&d_1^* = \arg\min_{d_1}\Im\left(\gamma_{th}\right)\\ &\text{subject to } 0 < d_1 < 1.\end{aligned} \tag{33}$$

where $\Im\left(\gamma_{th}\right)$ is the objective function, and $d_2 = 1-d_1$ in (31)[3].We assume that the average fading power Ω_1 and Ω_2 follow exponential distribution with distance such that $\Omega_1 = d_1^{-\varepsilon}$ and $\Omega_2 = d_2^{-\varepsilon}$. Note that ε is the path-loss exponent, which lies in the range of 2 to 6. Now, substituting the path loss definition and neglecting higher order terms, (31) can be expressed as

$$\Im\left(\gamma_{th}\right) = d_1^{\varepsilon m_1L}A + \left(1-d_1\right)^{\varepsilon m_2M}B \tag{34}$$

[3]Root finding algorithms such as Newton-Raphson method or bisection method can be used in solving optimal power or distance for higher values of m_h, m_g, L and M.

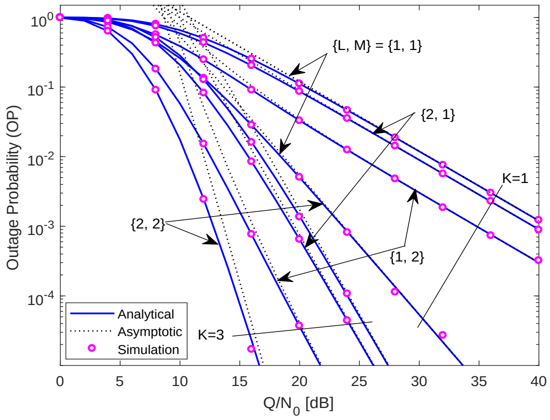

Fig. 2. OP at S_1 vs SNR of MAMR-UTWR system under different practical scenarios

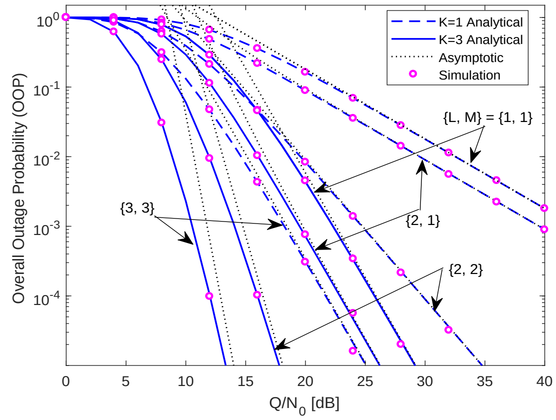

Fig. 3. OOP vs SNR of MAMR-UTWR system under different practical scenario

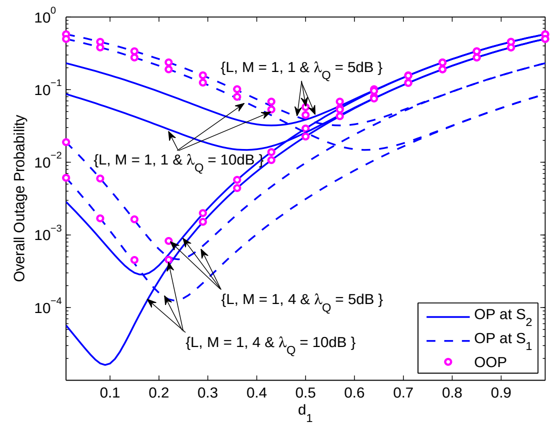

Fig. 4. OOP vs distance of MAMR-UTWR system under different practical scenarios

where

$$A = \frac{\left(\frac{m_1 N_0 \gamma_{th} \Omega_3}{\xi Q}\right)^{m_1 L} \sum\limits_{u=0}^{m_1 L} \binom{m_1 L}{u} \left(\frac{1-\xi}{\alpha_{R_{k^*}} \Omega_4}\right)^u \Gamma\left(m_{R_{k^*}} + u\right)}{\Gamma\left(m_{R_{k^*}}\right) \Gamma\left(m_1 L + 1\right)} \tag{35}$$

$$B = \frac{\left(\frac{m_2 N_0 \gamma_{th} \Omega_4}{(1-\xi) Q}\right)^{m_2 M} \sum\limits_{r=0}^{m_2 M} \binom{m_2 M}{u} \left(\frac{\xi}{\alpha_{R_{k^*}} \Omega_3}\right)^r \Gamma\left(m_{R_{k^*}} + r\right)}{\Gamma\left(m_{R_{k^*}}\right) \Gamma\left(m_2 M + 1\right)} \tag{36}$$

The second derivative of (32) w.r.t. d_1 is equal to

$$\begin{aligned} \frac{\partial^2 \Im\left(\gamma_{th}\right)}{\partial d_1^2} =& \varepsilon m_1 L \left(\varepsilon m_1 L - 1\right) d_1^{\varepsilon m_1 L - 2} A \\ &+ \varepsilon m_2 M \left(\varepsilon m_2 M - 1\right) \left(1 - d_1\right)^{\varepsilon m_2 M - 2} B \end{aligned} \tag{37}$$

By observing above equation we can say it is strictly convex with respect to $d_1 \in (0, 1)$. Thereby, the optimal relay location can be obtained by equating the first derivative to 0 as

$$\frac{\partial \Im\left(\gamma_{th}\right)}{\partial d_1} = \varepsilon m_1 L d_1^{\varepsilon m_1 L - 1} A - \varepsilon m_2 M \left(1 - d_1\right)^{\varepsilon m_2 M - 1} B \tag{38}$$

(36) can be solved by assuming $L = M = N$ and $m_1 = m_2 = m$ to obtain

$$d_1 = \frac{1}{1 + \left(\left(\frac{(1-\xi)\Omega_3}{\xi \Omega_4}\right)^{mN} \frac{\sum\limits_{u=0}^{mN} \binom{mN}{u} \left(\frac{(1-\xi)}{\alpha_{R_{k^*}} \Omega_4}\right)^u \Gamma\left(m_{R_{k^*}} + u\right)}{\sum\limits_{r=0}^{mN} \binom{mN}{u} \left(\frac{\xi}{\alpha_{R_{k^*}} \Omega_3}\right)^r \Gamma\left(m_{R_{k^*}} + r\right)}\right)^{\frac{1}{\varepsilon m N - 1}}} \tag{39}$$

As special case, $m_1 = m_2 = 1$ and $L = M = 1$ optimum relay location can be given as[3]

$$d_1 = \left(1 + \left(\frac{\frac{A_1 + (1-\xi)\Omega_3 B_1}{\xi \Omega_4}}{\frac{A_1 + \xi \Omega_4 B_1}{(1-\xi)\Omega_3}}\right)\right)^{\frac{-1}{\varepsilon - 1}} \tag{40}$$

where, $A_1 = \Gamma\left(m_{R_{k^*}}\right)$ and $B_1 = \frac{\Gamma\left(m_{R_{k^*}} + 1\right)}{\alpha_{R_{k^*}} \Omega_3 \Omega_4}$

VI. Numerical and Simulation Results

In this section, numerical results are presented to validate our analysis through Monte-Carlo simulations. Without losing generality, we have assumed γ_{th}=3dB and all the average channel gains and noise variances to be unity. Simulations are averaged over 1 million iterations.

Fig. 2 and 3 illustrates the user OP and end to end OP expressions (given in (13) and (26)) versus (vs) $\lambda_Q = Q/N_0$, respectively. It is clear that the simulation result having a good match with the analytical curve shows the validity of our analysis. We have shown two sets of simulation results in the figure, one with a different number of relay and another with varying the number of antennas on each source terminal. We observed that system OP decreases with increasing L and M. On the other hand, increasing number of relay (K) also helps the system to achieve the performance gain.

In Fig. 4, the user and end to end OP given in (13) and (26) respectively, is plotted as a function of d_1. We observe that there is a perfect agreement between simulation result and analytical curves. The optimal relay location w.r.t. S_1 and S_2 is determined for two cases: (i) symmetric e.g., $(L, M) = (1, 1)$ (ii) asymmetric e.g., $(L, M) = (1, 4)$. As expected, the worst case of user OP at S_1 and S_2 coincides with the e2e OP. For case (i), the optimal relay location is midway $(d_1 = 0.5)$ and

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for case (ii), the best performance can be achieved by placing the relay closer to the terminal with weaker hop. For example, when $M > L$ (asymmetric case), the performance is limited by the weaker hop ($S_1 \rightarrow R_k$), therefore, a more shorter ($S_1 \rightarrow R_k$) link ($d_1 = 0.22$) will improve the performance. For case (ii), from increasing λ_Q it is visible that the optimal SU relay location is further shifted towards the weaker SU link. A reasonable justification for that is, by increasing the value of λ_Q, the opportunistic selection of relay and transmit/receive beamforming process can be realized in a more efficient way.

VII. Conclusions

Outage analysis has been conducted for multiantenna underlay AF-TWRN by employing opportunistic relaying in the presence of primary receivers. For Nakagami-m fading, we derived OP and its asymptotic expressions and highlighted the impact of system/network parameters on the performance gains due to antenna and relay diversity. Moreover, the finding of this paper suggests that the optimum SU relay placement not only depends on the number of antennas of the SU but it also depends on the maximum allowed transmitted power of SU. Finally, Monte Carlo simulation results are presented to validate the proposed analysis.

Appendix

According to (12), the OP using k-th relay as

$$\begin{aligned}
\mathcal{P}_{out,k} &= \Pr\left(\gamma^{up}_{S_2\rightarrow R_k\rightarrow S_1} \le \gamma_{th}\right) \\
&= 1 - \Pr\left(\frac{\lambda_1 P_1 \parallel \mathbf{h}_{1_k}\|^2}{N_0} > \gamma_{th}\right) \Pr\left(\frac{\lambda_1 P_1 \parallel \mathbf{h}_{2_k}\|^2}{(\lambda_1+1)\,N_0} > \gamma_{th}\right) \\
&= 1 - \left(1 - \Pr\left(\parallel \mathbf{h}_{1_k}\|^2 \le \frac{\gamma_{th} N_0}{\lambda_1 P_1}\right)\right) \\
&\quad \times \left(1 - \Pr\left(\parallel \mathbf{h}_{2_k}\|^2 \le \frac{\gamma_{th}\,(\lambda_1+1)\,N_0}{\lambda_1 P_1}\right)\right)
\end{aligned} \tag{41}$$

Now, substituting $P_1 = \xi Q/\Omega_3$, $P_2 = (1-\xi)Q/\Omega_4$, $\lambda_1 = P_R/P_1$ and $P_R = Q/Z$, where $Z = |h_{R_{k^*}}|^2$.

$$\begin{aligned}
&= 1 - \frac{1}{\Gamma(m_1 L)\Gamma(m_2 M)} \int_{z=0}^{\infty} \Gamma\left(m_1 L, \frac{\alpha_1 \gamma_{th} N_0 z}{Q}\right) \\
&\quad \times \Gamma\left(m_2 M, \frac{\alpha_2 \gamma_{th} N_0 \Omega_4}{(1-\xi)Q}\left(1 + \frac{\xi z}{\Omega_3}\right)\right) f_Z(z) dz
\end{aligned} \tag{42}$$

We obtain the probability density function of Z under Nakagami-m fading with m_{R_k} and α_{R_k} are shape and scale parameter of interference link as $f_Z(z) = \frac{\alpha_{R_k}^{m_{R_k}}}{\Gamma(m_{R_k})} z^{m_{R_k}-1} e^{-\alpha_{R_k} z}$. Now, utilizing $f_Z(z)$ and evaluating the integral (42) using [17, Eq. (3.351.3)] and after some mathematical manipulations we will get (13).

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