Proceedings of ICTACEM 2017
International Conference on Theoretical, Applied, Computational and Experimental Mechanics
December 28-30, 2017, IIT Kharagpur, India
ICTACEM-2017/XXXX (paper No.)

Free vibration analysis of fiber metal laminated plates

Prasad E. V.[a*] and Sahu, S. K.[b]

[a,b] Department of Civil Engineering, N.I.T., Rourkela, Odisha, India

ABSTRACT

The free vibration analysis of a new aircraft material fiber metal laminated (FML) plates is investigated in the present study. A finite element based formulation is developed using first-order Reissner/Mindlin plate theory. A four-node isoparametric quadratic element is employed in the present study with five degrees of freedom per node. The modal testing of FML plates is performed with the help of Bruel and Kjaer FFT analyzer and PULSE software. The present study deals with the effects of various parameters including aspect ratio and boundary conditions on the dynamic characteristics of the fiber metal laminates. The analysis is carried out both experimentally and numerically. There is a good agreement between numerical and experimental results. From both the results, it is concluded that with the increase of aspect ratio of FML plate the natural frequency is also increased. The boundary conditions are also considerably affecting the natural frequencies of FML plates because of restraining at edges.

Keywords: FML plate, vibration, aspect ratio, boundary conditions.

1. INTRODUCTION

Laminated composite materials are advanced class of materials formed by combining one or more materials in the solid state with different chemical and physical properties. Fiber–metal laminate (FML) composites are consists of alternate layers of fiber-reinforced polymer and metal layers. These kind of hybrid composite materials offer outstanding mechanical properties over the conventional polymer composite laminates and the high-strength aluminum layers. As a result, the resistance to foreign object impact and environmental attack can be enriched. The specific stiffness and strength in the fiber orientation of reinforced-polymeric composites are improved over the high-strength aluminum metals used in the FML composites and contribute considerably to weight savings in the design of tension-dominated stresses in structural elements. Glass Reinforced (GLARE) and Aramid Reinforced Aluminum Laminates (ARALL), and belong to this new kind of materials GLARE is the successful fiber–metal laminate which is currently used for the development of basic

[*] Further author information: (Send correspondence to Prasad, E. V.)
Prasad, E. V.: E-mail: ellurivenkataprasad@gmail.com, Telephone: +91-9583574684
Sahu, S. K.: E-mail: sksahu.nitrkl@gmail.com, Telephone: 0661 2462322, Address: Department of Civil Engineering, N.I.T, Rourkela, Odisha-769008, India

aerospace components such as the fuselage of the Airbus A380 airplane. It is also used in aircraft cargo floors of Boing 777, stiffeners with a wide variety of shapes, engine cowlings, bonded GLARE patch repair, seamless GLARE tubes and cargo containers.

Roebroeks (1994) studied the fatigue crack growth behavior, crack initiation behavior and fire resistance behavior of fiber metal laminates under realistic loading conditions. Hashagen et al. (1995) introduced solid-like shell element and its applications for modelling the fiber metal laminates and also compared these numerical results with experimental results. Kawai et al. (1998) applied the classical laminated plate theory (CLT) for explaining the off-axis inelastic response of GLARE 2 laminated coupons. Harras et al. (2002) investigated the linear and nonlinear dynamic behavior of GLARE 3 using spectral method and experiments.

Viscoelastic properties such as elastic and viscous responses for glass epoxy, carbon epoxy and glass fiber metal and carbon fiber metal laminates were investigated by Botelho et al. (2006). The calculated young’s modulus values based on micromechanics approach and experimental values are in good agreement. Shooshtari and Razavi (2010) examined the linear and nonlinear dynamic behavior of composite and FML plates using FSDT. They have solved the nonlinear ordinary differential equations using Galerkin method.

Based on the literature review, it is observed that very less amount of work is available on fiber metal laminated plates. So the present study deals with numerical and experimental vibration analysis of fiber metal laminated plates. The effects of different factors such as aspect ratio and boundary conditions on the natural frequencies of woven fiber metal laminated plates are presented.

2. MATHEMATICAL FORMULATION

A fiber metal laminated plate of size a x b and thickness h consisting of n layers each of which oriented at angle θ as shown in **Figure 1** is considered in the present study.

2.1. Strain displacement relations

The displacement function in the shear deformation theory is assumed to be of the form

$$u(x,y,z) = u_0(x,y) + z\theta_x(x,y)$$
$$v(x,y,z) = v_0(x,y) + z\theta_y(x,y)$$
$$w(x,y,z) = w_0(x,y) \tag{1}$$

Where u_0, v_0, w_0 are the mid plane displacements.

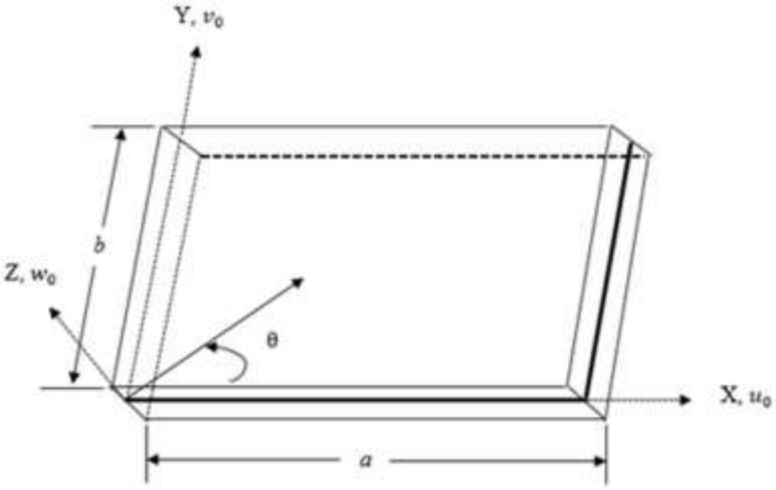

Figure 1. Fiber metal laminated plate.

The Green-Lagrange strain relations are:

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \\ \gamma_{xz} \\ \gamma_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} \end{Bmatrix} \quad (2)$$

2.2. Constitutive Relations

The constitutive equations for the plate are (Rath and Sahu (2012)):

$$\{F\} = [D]\{\varepsilon\} \quad (3)$$

Where

$$[D] = \begin{bmatrix} A_{ij} & B_{ij} & 0 \\ B_{ij} & D_{ij} & 0 \\ 0 & 0 & S_{ij} \end{bmatrix} \quad (4)$$

Where A_{ij}, B_{ij}, D_{ij} and S_{ij} are the extensional, bending-stretching coupling, bending and transverse shear stiffness.

$$\left(\mathrm{A}_{ij}, \mathrm{B}_{ij}, \mathrm{D}_{ij}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{k} \left[\overline{Q}_{ij}\right]_k \left(1, z, z^2\right) dz \ \left(i, j = 1, 2, 6\right) \tag{5}$$

$$\left(S_{ij}\right) = \mathrm{K} \sum_{k=1}^{n} \int_{z_{k-1}}^{k} \left[\overline{Q}_{ij}\right]_k \left(1, z, z^2\right) dz \ \left(i, j = 4, 5\right)$$

Where K is the shear correction factor. In the present study a shear correction factor of 5/6 is considered.

$(\overline{Q}_{ij})$ in Eq. (5) is defined as

$$\left(\overline{Q}_{ij}\right)_k = [T_1]^T \left[\overline{Q}_{ij}\right]_k [T_1] \ \left(i, j = 1, 2, 6\right)$$

$$\left(\overline{Q}_{ij}\right)_k = [T_2]^T \left[\overline{Q}_{ij}\right]_k [T_2] \ \left(i, j = 4, 5\right) \tag{6}$$

$$[T_1] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \tag{7}$$

$$[T_2] = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\left[Q_{ij}\right]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \left(i, j = 1, 2, 6\right)$$

$$\left[Q_{ij}\right]_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \left(i, j = 4, 5\right) \tag{8}$$

in which

$$Q_{11} = E_{11}/(1-\nu_{12}\nu_{21}),\ Q_{12} = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21}),$$
$$Q_{22} = E_{22}/(1-\nu_{12}\nu_{21}),\ Q_{44} = G_{13}\ ,\ Q_{55} = G_{23} \tag{9}$$

E_{11}, E_{22} = Young's moduli of lamina along longitudinal and transverse direction of the fibers, respectively.

G_{12}, G_{13}, G_{23} = Shear moduli of lamina w.r.t 1, 2, 3 axes.

ν_{12}, ν_{21} = Poisson's ratios.

3. EXPERIMENTAL PROGRAMME

The FML specimens were fabricated by considering weight fraction of 70:30 using hand layup method. Woven roving glass fiber and aluminum sheets were cut into required size for preparation of test samples. Epoxy resin matrix was prepared by taking hardener and 10% of weight of epoxy. A mould releasing plastic sheet was placed on the flat plywood and polyvinyl alcohol was applied on the sheet. Lamination starts with the application of epoxy resin matrix on the plastic sheet and whose purpose was to get a uniform surface and to avoid the exposure of aluminum to the environment. After application of epoxy resin gel the aluminum sheet was placed and with the help of steel roller rolling was done to avoid the formation of voids in the specimens. Repeat the same procedure for remaining layers also after completion of all layers apply polyvinyl alcohol on the plastic sheet and placed it over the last layer. The FML plate specimens were then cured in servo hydraulic hot pressing machine for 20 minutes at a temperature of 60^0 C. After curing of specimens they were cut into 235 x 235 mm for modal testing and 250 x 25 mm coupons for tensile testing. The vibration measurement of FML plates is performed with the help of Bruel and Kjaer (B&K) FFT analyzer and PULSE software. Natural frequencies are observed from the FRF and coherence of each specimen.

4. RESULTS AND DISCUSSIONS

Convergence study is carried out and based on the convergence study a mesh of 16 x 16 is considered for the present study. For validation of present formulation, the five lowest natural frequencies of fiber metal laminated plates are considered and compared with experimental studies, engineering sciences data unit method (ESDU) and finite element method of Harras et al. (2002) as shown in **Table 1**. The present numerical results show good agreement with the previous studies in the literature.

Table 1. Comparison of the natural frequencies in (Hz) with previous results a = 0.45 m, b = 0.3 m, h = .0014 m, (Al/GFC/Al/GFC/Al), all sides clamped.

Mode number	Present FEM	Harras et al. (2002)		
		FEM	ESDU Method	Experimental
1	105.11	105.37	105.60	93.5
2	162.02	162.31	162.90	153
3	255.27	258.34	259.40	245
4	265.82	259.13	260.30	253
5	300.70	310.45	311.90	298

In the present study numerical and experimental studies are performed for fiber metal laminated plates. The dimensions of the FML plate are a = 0.235 m, b = 0.235 m and thickness h = 0.0035 m. The material properties considered in the present study are shown in

Table 2. In this article, the effect of various factors such as aspect ratio and boundary conditions on natural frequencies of FML plates are presented.

Table 2. Material properties of the aluminum layer and the Glass fiber reinforced composite.

Materials	*E* (GPa)	*G* (GPa)	ρ (kg/m^3)	ν
Aluminum layer	58.03	22.30	2742.85	0.3
Glass fiber reinforced composite	E_{11} = 16.63 E_{22} = 16.63	G_{12} = G_{13} =2.15 G_{23} = 2.15	1866.66	ν_{12} = ν_{13} = 0.21

4.1. Effect of aspect ratio on the natural frequency of FML plates

To study the effect of aspect ratio on natural frequencies of FML plate (Al/GFRP (0/90)/ Al/ GFRP (0/90)/Al), three types of aspect ratios i.e. *a*/*b* = 1.0, 1.5 and 2.0 are considered. For different aspect ratios, the plate dimension varied, by keeping the plate length (0.235 m) and thickness (h = 0.0035 m) constant. For the aspect ratio of 1.0, *a* = *b* = 0.235 m; for 1.5, *a* = 0.235 m and *b* = 0.157 m; for 2.0, *a* = 0.235 m and *b* = 0.117 m. The variation of natural frequency with aspect ratio for clamped boundary condition (C-C-C-C) is shown in **Figure 2**. The experimental results well agree with their numerical (FEM) results.

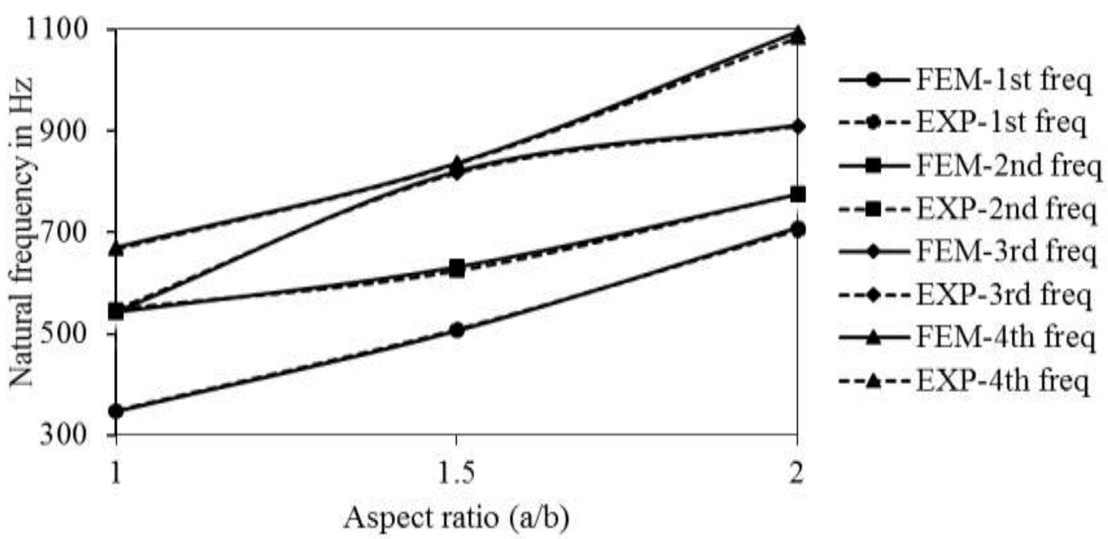

Figure 2. Variation of natural frequency with aspect ratio for clamped boundary condition.

The fundamental frequency of FML plate of aspect ratios 1.5 and 2.0 is seen to increase linearly with increase of aspect ratio. This represents that increase in the aspect ratio of a fiber metal laminated plate, the natural frequency is also increasing. The higher modes of frequencies vary differently to that of fundamental mode for increase in aspect ratio. The second and third lowest frequencies show same frequency for square plates but quite different for higher aspect ratio. **Figure 3** shows that effect of aspect ratio on natural frequency of FML plate for cantilever boundary condition (C-F-F-F). The experimental values measured from vibration test are matching with computational values. These values are low when compared to clamped FML plate results because of restraining at the edges. In this case it is observed that the fundamental frequency is decreasing with the increase of aspect ratio and higher modes of frequencies are increasing with the increase of aspect ratio of FML plate. The increment in the second natural frequency of FML plate is 15.34% and 23.33% for aspect ratios 1.5 and 2.0 respectively, as compared with the aspect ratio of 1.0.

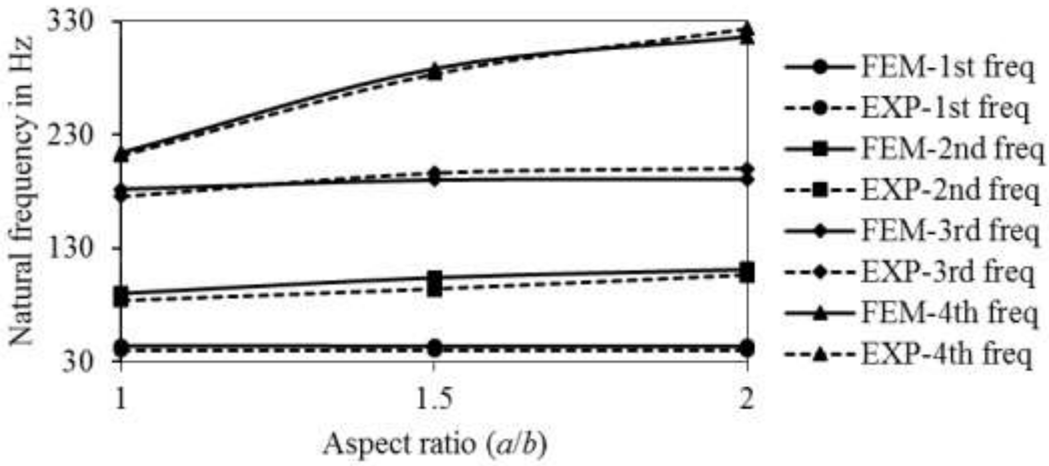

Figure 3. Variation of natural frequency with aspect ratio for cantilever boundary condition.

The effect of aspect ratio on natural frequency of FML plate is shown in **Figure 4** for simply supported condition (S-S-S-S). The natural frequencies of FML plate for this case are lies between cantilever and clamped FML plates.

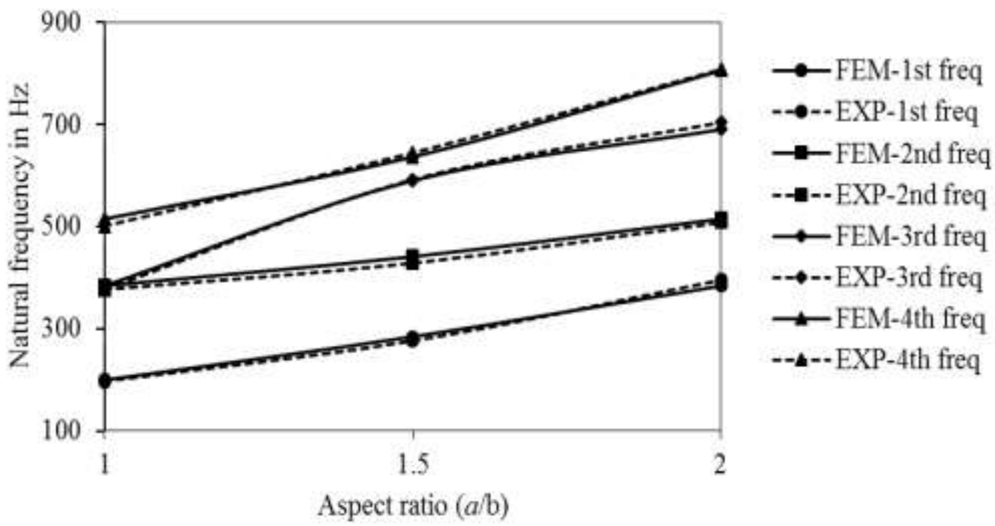

Figure 4. Variation of natural frequency with aspect ratio for simply supported boundary condition.

The fundamental frequency of FML plate is risen by 1.5 times and 2 times for aspect ratios of 1.5 and 2.0 respectively with respect to an aspect ratio of 1.0. The parabolic variation is observed for higher mode of frequencies of FML plate. The variation of natural frequency of FML plate with aspect ratio is shown in **Figure 5** for two side simply supported and two sides clamped boundary condition (S-C-S-C). The fundamental frequency is risen by 6.95% and 38.67% for an aspect ratio of 1.5 and 2.0 respectively, when compared to an aspect ratio of 1.0. The second frequency is significantly higher than fundamental frequency of FML plate. The frequency is varying linearly with the increase of aspect ratio.

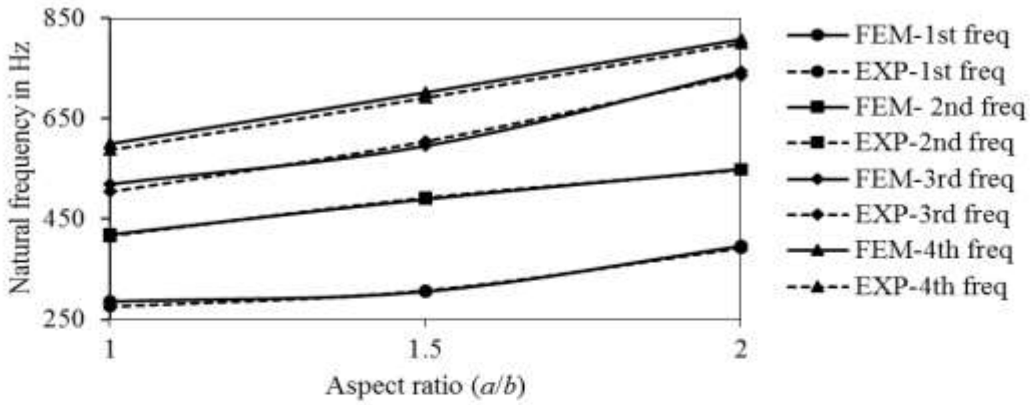

Figure 5. Variation of natural frequency with aspect ratio for S-C-S-C boundary condition.

In the case of three sides clamped and one side free (C-C-C-F) boundary condition as shown in **Figure 6**, the fundamental and second mode of frequency is varying linearly with the increase of aspect ratio of FML plate. The first frequency is risen by 1.7 times and 2.5 times for the aspect ratio of 1.5 and 2.0 respectively with respect to aspect ratio of 1.0.

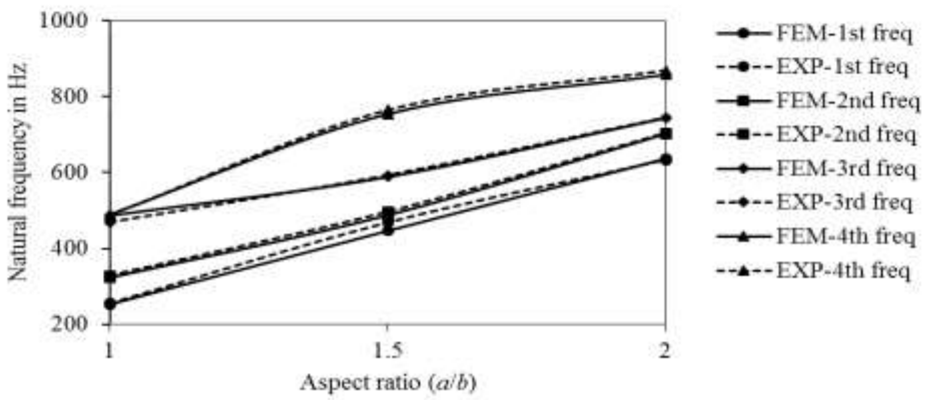

Figure 6. Variation of natural frequency with aspect ratio for C-C-C-F boundary condition.

4.2. Effect of boundary conditions on the natural frequency of FML plates

To examine the effect of boundary conditions on the natural frequencies of FML plates, three types of boundary conditions are taken and they are:

1. S-S-S-S (all edges simply supported)
2. C-C-C-C (all edges clamped)
3. C-F-F-F (cantilever), where $u = v = w = \theta_x = \theta_y = 0$ at $x = 0$.

The clamped and simply supported boundary condition details are shown in **Figure 7**, from the **Figures 2-4**, it is seen that the experimental results well agree with the numerical results for all the boundary conditions. From the experimental modal testing 1st, 2nd, 3rd and 4th natural frequencies are found to be the minimum (40, 84,176 and 212 Hz) for C-F-F-F boundary condition and the maximum (348, 548, 548 and 668 Hz) for C-C-C-C boundary condition. The fundamental frequency of FML plate with cantilever and simply supported boundary conditions decrease by 88.51% and 43.68% correspondingly, when compared to all sides clamped boundary condition. From the experimental and computational studies, it is noticed that the natural frequencies of FML plates are significantly dependent on the boundary conditions, i.e., the more firmly the plate is restrained, and the effect on the natural frequencies is also more for all the boundary conditions due to restraint at the edges.

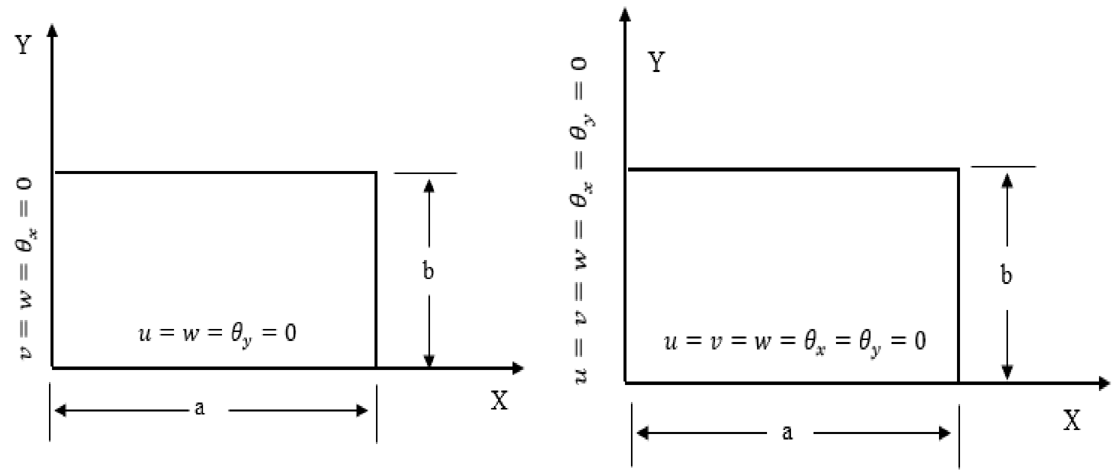

Figure 7. (Left) Simply supported and (right) clamped boundary conditions.

5. CONCLUSIONS

Based on the experimental and numerical studies of fiber metal laminated plates the following conclusions were made.

1. The numerical results shows good agreement with the experimental results.
2. With the increase of aspect ratio the natural frequency is also increasing.
3. In the case of cantilever FML the fundamental frequency is decreasing with the increase of aspect ratio and second frequency onwards it is increasing.
4. For clamped FML plate we found maximum natural frequencies where as in the case of cantilever FML plate they are minimum because of restraining at the edges.

REFERENCES

1. G. H. J. J. Roebroeks, "Fibre-metal laminates: recent developments and applications," *International journal of fatigue*.**16**, pp. 33-42, 1994.
2. F. Hashagen, J. C. J. Schellekens, R. De Borst, and H. Parisch, "Finite element procedure for modelling fibre metal laminatesl," *Composite Structures.* **32**, pp. 255-264, 1995.
3. M. Kawai, M. Morishita, S. Tomura, and K. Takumida, "Inelastic behavior and strength of fiber-metal hybrid composite: GLARE," *International Journal of Mechanical Sciences.* **40**, pp. 183-198, 1998.
4. B. Harras, R. Benamar, and R. G. White, "Experimental and theoretical investigation of the linear and non-linear dynamic behavior of a glare 3 hybrid composite panel," *Journal of Sound and Vibration.* **252**, pp. 281-315, 2002.
5. E. C. Botelho, A. N. Campos, E. De Barros, L. C. Pardini, and M. C. Rezende, "Damping behavior of continuous fiber/metal composite materials by the free vibration method," *composites part B: Engineering.* **37**, pp. 255-263, 2006.
6. A. Shooshtari, and S. Razavi, "A closed form solution for linear and nonlinear free vibrations of composite and fiber metal laminated rectangular plates," *Composite Structures.* **92**, pp. 2663-2675, 2010.
7. M. K. Rath, and S. K. Sahu, "Vibration of woven fiber laminated composite plates in hygrothermal environment," *Journal of vibration and control.* **18**, pp. 1957-1970, 2012.