Determination of Parameters for Boundary Layer Development in Rough Plates
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ABSTRACT

Research on the development of boundary layer and its parameters such as nominal boundary layer thickness, the displacement thickness, the momentum thickness and the energy thickness has been carried out for various objects. In this paper, the velocity distribution and the boundary layer development has been studied for flat plates with four different roughness (125μm, 290μm, 345μm, 375μm) by using emery paper. Experimental observations undertaken in a low speed wind tunnel at National Institute of Technology, Rourkela for such surfaces are taken into consideration. The effect of roughness, on these plates has been studied here. The plates have been inclined at four different angles (0°, 2°, 4°, 6°) with respect to the leading edge in order to create a negative pressure and the variation in boundary layer is observed.

Due to no-slip condition, the velocity of fluid near the wall is assumed to be zero. The speed of fluid increases from zero at the boundary of the plate to the free stream velocity of the fluid in the direction normal to the boundary. The locale of fluid within which the gradient of velocity occurs is called the boundary layer. Boundary layer is illustrated by four fundamental parameters to describe its size and state, namely boundary layer thickness, displacement thickness, momentum thickness and the energy thickness.

In this paper, the variation of these four different parameters of boundary layer has been studied. The parameters are observed to depend upon the main stream velocity (V), the distance from the leading edge (χ), the roughness coefficient (γ) and the angle of inclination of the flat plates (Φ).

Keywords: boundary layer, flat plate, displacement thickness, momentum thickness, energy thickness.

1. INTRODUCTION

Motions are always relative. When wind blows over an object due to difference in air density a phenomenon of relative motion occurs. Resistance due to the surface of the object occurs which generates a velocity gradient. After some height above the surface, where the velocity gradient does not exist, Bernoulli equation is applied.

Blom and Wartena (1969) described the development of a turbulent boundary layer in a neutral atmosphere due to an abrupt change of surface roughness. They emphasised on the correct evaluation of the adapted layer height for the determination of surface shear stress and roughness height from measured velocity profiles. They observed pressure drag of obstacles in the atmospheric boundary layer computed with a mesoscale numerical model of the troposphere. The different parts of drag force was studied and separated from the numerical results; total pressure drag was determined from the surface pressure distribution, hydrostatic drag from the temperature distribution in the atmosphere, and form drag as a residual. The dependence of the different parts of drag on the main influencing parameters, such as geometric parameters, dynamical and thermal parameters, and the surface roughness, was illustrated. Antonia and Krogstad (2000) studied the classical treatment of rough wall turbulent boundary layer. They studied the effect roughness on the mean velocity profile in terms of a roughness function.

Aubertine et. al (2004) examined the effects of wall roughness for two different rough-wall cases experimentally, involving flow over a ramp with separation and reattachment. For these cases, the roughness Reynolds number was matched at two different momentum thicknesses Reynolds numbers. Both flow conditions were fully rough. The effect of increasing
the wall roughness was observed to increase the friction velocity and hence increases the separation region. It was observed that roughness Reynolds number is not sufficient to characterize the roughness effects rather; another parameter such as the ratio of the roughness height to the boundary layer thickness is necessary. Klipp (2007) examined a variety of atmospheric boundary layer parameters as a function of wind direction in both urban and suburban settings in Oklahoma City, derived from measurements during the Joint Urban 2003 field campaign. He observed a heterogeneous surface characteristics result, significant differences in upwind fetch and, therefore, statistically significant differences in measured values, even for small changes in wind direction.

Cardillo et. al (2013) showed direct numerical simulation (DNS) of a turbulent boundary layer on rough surfaces with zero pressure gradient. Namboodiri et. al (2014) discussed the features of wind, turbulence, and surface roughness parameter over the coastal boundary layer of the Peninsular Indian Station, Thumba Equatorial Rocket Launching Station (TERLS).

Boundary layer, more or less depends upon different parameters, like velocity, roughness etc. With the help of these effects on the velocity profile on roughness and inclination, it will be easy to study the analysis of boundary layer along with these effects. Design of roof in the hilly areas, design of automobile, design of any open industry it will be helpful to use these results. Effect of boundary layer on smooth flat plates has been extensively studied. In this paper, the effect of roughness, distance from the leading edge and inclination of the plate on boundary layer parameters is analysed.

2. THEORETICAL CONSIDERATION

2.1 Boundary layer theory

The theory of boundary layer in fluids, plays the leading role in modern fluid dynamics especially for the low viscous fluids like air and water. The concept of boundary layer theory was introduced by Ludwig Prandtl, who conceived the idea of a layer characteristic of fluids in the region of a solid boundary. He proposed that even for a very small viscosity of a viscous fluid the no slip condition must be satisfied at the solid boundary. No slip condition implies that the velocity of the fluid at a solid boundary is same as that of the boundary itself. The boundary layer exists in the real fluid flow past a solid boundary.

2.2 Boundary layer parameters

2.2.1 Boundary layer thickness

The distance measured normal to a boundary layer from the wall or boundary, to a point where the flow velocity occupies the free stream velocity \( V \), is termed as boundary layer thickness \( \delta \). Basically the point where the flow velocity is that of the free stream is usually characterized as the point where;

\[
\delta = 0.99V
\]  

(1)

2.2.2 Displacement thickness

The distance by which a surface would have to be displaced in the direction normal to its normal vector away from the plane of reference in an inviscid fluid in motion, of velocity \( V \) to give the same rate of flow as takes place between the reference plane and the surface, in a real fluid. The displacement thickness can be calculated by;

\[
\delta^* = \int_0^\delta \left(1 - \frac{\nu}{V} \right) dy
\]  

(2)

The boundary layer's shape factor is calculated by the displacement thickness. A shape factor is used to determine the nature of the flow in boundary layer flow.

\[
H = \delta^* \theta
\]  

(3)
For value of $H$, stronger will be the adverse pressure gradient. The Reynolds number can be reduced by a high adverse pressure gradient at which transition into turbulence may occur.

2.2.3 Momentum thickness

The distance by which a surface is to be moved parallel to itself towards the plane of reference in an inviscid fluid of velocity $V$ giving the existence of same total momentum to compensate for the deficit of momentum loss between the reference plane and the surface, in the fluid is the momentum thickness ($\theta$). The momentum thickness can be calculated by:

$$\theta = \int_{0}^{\delta} \frac{\nu}{V}(1 - \frac{v}{V}) dy$$

(4)

2.2.3 Energy thickness

The distance normal to the boundary of the solid body, by which the boundary should be displaced in such way for the compensation of the depletion in kinetic energy of the fluid flow on the behalf of boundary layer formation is the energy thickness, $\delta_E$. The energy thickness can be determined by:

$$\delta_E = \int_{0}^{\delta} \frac{\nu}{V}(1 - \left(\frac{v}{V}\right)^2) dy$$

(5)

3. SOURCES OF DATA

Experimental investigation carried out by Gupta (2014) at the Hydrodynamics Laboratory of NIT Rourkela has been used in this paper. The experimentation was carried out on flat plates of 100cm length and 15cm width with four different roughness’ by using different grades of emery paper. The grades used were 40 grade (375μm), 50 grade (345μm), 60 grade (290μm) and 120 grade (125μm). Velocity gradient at various distances from the leading edge were analyzed to visualize the development of boundary layer. The experimental setup was modified by inclining these plates at three different angles i.e. (0°, 4° and 6°). A low speed wind tunnel is used for the experimentation. The speed of air in this wind tunnel can be varied from 10 to 25m/s. A constant speed of 12.3m/s is maintained throughout the experiment. In the study; 21 positions along the length of the plate were considered for velocity measurement in the vertical direction. However, data for 8 such sections at 15 cm interval from the leading edge are considered. At each such section, sets for positions 15, 30, 75 and 90m from the leading edge are analyzed in this paper.

![Fig. 1: Wind tunnel at NIT Rourkela (Gupta 2014)](image)

4. CALCULATION OF PARAMETRERS

From experiments and the graphical analysis, the equations obtained were used to determine the parameters of the boundary layer in the turbulent boundary layer zones. All the equations
obtained from the \( \frac{\nu}{\delta} \sim \frac{v}{V} \) plots were observed to be in the following quadratic form of, \((-ax^2+bx+c)\). This methodology was suggested by Singh (2015). Therefore, for easy and convenient calculations the integrations for different parameters are integrated for the above form and formulated.

4.1. Displacement thickness
For displacement thickness (\( \delta^* \));
\[
\delta^* = \int_0^\delta \left(1 - \frac{v}{V}\right) dy \\
\Rightarrow \delta^* = \int_0^\delta (-ax^2 + bx + c)dy \\
\Rightarrow \delta^* = \left[1 + \frac{a}{3} - \frac{b}{2} - c\right] \delta
\]

4.2. Momentum thickness
For momentum displacement thickness (\( \theta \));
\[
\theta = \int_0^\delta \frac{v}{V} \left(1 - \frac{v}{V}\right) dy \\
\Rightarrow \theta = \left[-\frac{a}{3} + \frac{b}{2} + c - \frac{a^2}{5} - \frac{b^2}{3} - c^2 + \frac{ab}{2} - bc + \frac{2ca}{3}\right] \delta
\]

4.2. Energy thickness
For energy thickness (\( \delta_E \));
\[
\delta_E = \int_0^\delta \frac{v}{V} \left(1 - \left(\frac{v}{V}\right)^2\right) dy \\
\Rightarrow \delta_E = \left[-\frac{a}{3} + \frac{b}{2} + c + \frac{a^3}{7} - \frac{b^3}{4} - c^3 - \frac{a^2b}{2} + \frac{3ab^2}{5} - \frac{b^2c}{2} - \frac{3bc^2}{5} + c^2a - \frac{3ca^2}{5} + \frac{3abc}{2}\right] \delta
\]

The above equations were used to obtain the boundary layer parameters at different conditions to analyze their dependence on the variables such as position from the leading edge, roughness of plate and the angle of inclination.

4. DISCUSSION

4.1. Variation in \( \delta^* \), \( \theta \) and \( \delta_E \) due to change in distance from leading edge
To study the variation of boundary layer parameters, the distance from the leading edge is taken as a reference. In Fig. 2, there are three sub-figures (a), (b) and (c); each illustrating the displacement, momentum and energy thickness values respectively.

For the variation of displacement thickness in Fig. 2(a), it consists of three insets for the three angle of incidence (0°, 4° and 6°) respectively. In each inset, the variation of \( \delta^* \) is illustrated with respect to the distance from the leading edge for different plate roughness’ in the X-axis. Similar illustration is provided for momentum and energy thickness variations in Fig. 2(b) and (c) respectively.
As observed from Fig. 2(a), at 0° inclination, the displacement thickness is observed to be in the range of 1 to 1.5 for distances up to 75cm from the leading edge, which then doubles itself at a distance of 90cm. Such an observation is consistent over different grades of roughness. An inclination of 4° for the plates does not show much variation to the above observation. However, for an inclination of 6°, the $\delta^*$ is observed to increase abruptly from 75cm distance from the leading edge.

Momentum and energy thickness coefficients in Fig. 2(b) and (c) also show similar trends. Although, the momentum thickness value varies from 0.4-2.8, whereas the energy thickness varies in the range of 0.8-4.2.

4.2. Variation in $\delta^*$, $\theta$ and $\delta_E$ due to change in roughness of plate

Undertaking the plate roughness as the reference, the boundary layer parameters are studied in Fig. 3. There are three sub-figures to Fig. 3 for the three thickness coefficients respectively.

For the variation of displacement thickness in Fig. 3(a), the figure consists of three insets for the three angle of incidence (0°, 4° and 6°) respectively. In each inset, the variation of $\delta^*$ is illustrated with respect to the grade of plate roughness, i.e. (40, 50, 60 and 120 grade) for different positions from the leading edge varied across the X-axis. Similar illustration is provided for momentum and energy thickness variations in Fig. 3(b) and (c) respectively.
As observed in Fig 3(a), at 0° inclination; the displacement thickness is seen to decrease gradually with respect to the roughness grade of the plate at a particular position from the leading edge. The ranges and trends of the values of $\delta$ seen to be more or less in variable at particular portions for different angle of inclination of the plate.

Similar observation is seen for $\theta$ and $\delta_E$, where the value of $\theta$ ranges from 0.8 to 2.5 and $\delta_E$ ranges from 0.8 to 4.2.

4.3. Variation in $\delta^*$, $\theta$ and $\delta_E$ due to change in angle of inclination

Taking the angle of inclination as the reference, the variation of boundary layer parameters is studied. Fig. 4 consists of three sub-figures (a), (b) and (c); illustrating the displacement, momentum and energy thickness coefficients respectively.

For the variation of displacement thickness in Fig. 4(a), it consists of four insets for the four distances from the leading edge of the flat plate (15cm, 30cm, 75cm and 90cm) respectively. In each inset, the variation of $\delta^*$ is illustrated with respect to the angles of inclination of the plates i.e. (0°, 4° and 6°) respectively with the grades of roughness in the X-axis. Similar illustration is provided for momentum and energy thickness variations in Fig. 4(b) and (c) respectively.
As observed in Fig 4(a), the displacement thickness at a particular position from the leading edge is shown for different angles of inclination keeping the roughness grade constant. At position 15cm and 30cm from the leading edge it is seen that $\delta^*$ is higher in the case of 4° inclination for all roughness grades, but at distances further from the edge, $\delta^*$ increases with increase of inclination angle.

The $\delta^*$ value is also observed to decrease with the increase in the roughness grade of the flat plates. Similar observation are seen for $\theta$ and $\delta_E$ where the value of $\theta$ ranges from 0.3 to 2.5 and $\delta_E$ where the value of $\theta$ ranges from 1 to 4.

5. CONCLUSIONS

The following conclusions is made from the experimental study;

i. The displacement, momentum and energy thickness coefficients are observed to depend upon the plate roughness, angle of elevation, and the distance from the leading edge.

ii. The boundary layer parameters are observed to undertake quadratic form which helps to form mathematical formulations.

iii. The boundary layer parameters are observed to increase with the distance from the leading edge for a particular roughness of plate inclined at an angle.

iv. The boundary layer parameters are observed to decrease with the decrease of plate roughness; which are also observed to increase with the distance from the leading edge.

v. The parameters are observed to decrease with the increase in the roughness grade of the plates. They are also observed to increase with the increase of plate inclination for higher bed roughness. For lower roughness of plates, these values are observed to decrease.

REFERENCES


