Prediction of Manning Roughness Coefficient in Meandering Compound flow

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ABSTRACT

During uniform flow in an open channel, the resistance is dependent on a number of flow and geometrical parameters. The usual practice in one dimensional flow analysis is to select an appropriate value of roughness coefficient for evaluating the carrying capacity of natural channels. This value of roughness is taken as uniform for the entire surface and for all depths of flow. However, it is observed that the resistance coefficient for meandering channels are found to vary with flow depths, aspect ratio, slope and sinuosity and are all linked to the stage-discharge relationship. Although much research has been done on Manning's *n* for straight channels, its dependence on the different parameters for a meandering channel is necessary to be studied. Factors affecting the roughness coefficient in a meandering compound channel are investigated and used in its prediction, particularly Manning's roughness coefficient by dimensional analysis. Observations of various researchers, on the laboratory study of meandering channels is used to investigate the factors affecting Manning' *n* for large scale as well as small scale channels.

Keywords Manning's roughness coefficient, meandering channel, conveyance estimation, dimensional analysis.

1. INTRODUCTION

It is so ubiquitous curves in rivers and therefore smooth meander forms so that they attracted the investigators of many field. The condition varied from one meandering loop to another meandering loop. Different meandering shows the different geometric characteristic and grain size distribution too. Water is the most obligatory things for mankind. Life can't be imaginable without water. River may return all the water to sea but in the occurrence of high rainfall, the river may overflow with possible danger to life. Rivers are the integral part of water cycle and due to this cycle the water available in landscape. Sometimes due to overbank flow in river it causes serious damage to the living beings. Therefore its very much necessary the accurate estimate the design capacity of meandering channel due to the flood protection, flood plain management, protection of bank, to understand the mechanism of sediment transport etc. Accurate prediction of roughness coefficient is also helpful in predicting discharge in open channels. Manning's, Chezy's and Darcy-Weisbach's, equations have been in use for obtaining discharge for uniform flows in simple channels but it fails to predict discharge for compound channels let alone for a meandering flow. These methods were typically developed for simple channels to find the characteristic of the bed material, called the roughness coefficient. The roughness coefficient in a meandering channel depends not only on the bed roughness but also on other geometric and hydraulic parameters. Therefore, an attempt is made to develop a model for predicting the roughness coefficient with respect to these parameters.

Manning's formula is primarily the most popular formula in open channel flow. The dimension of Manning's *n* was suggested to be as length to the one-sixth power by Rouse (1938) and Keulegan (1938) which was ambiguous and objectionable to many researchers. Further analysis of the coefficient was made frequently to comprehend the formula by various researchers. But, confusion lies among researchers regarding the dimension of n, which has been variously considered as length to the one-third power divided by time, dimensionless, or length to the one-sixth power.

Proper care need to be undertaken for implementing Manning's formula to non-uniform and compound channels. Manning's *n* is a roughness factor which measures n in terms of a geometric measure of the boundary roughness, reflecting the actual or effective unevenness of the boundary as suggested by Yen (1992) for simple uniform flows. Darcy-Weisbach's *f* on the other hand is defined as resistance coefficient reflecting the dynamic behaviour in terms of momentum or energy, of the boundary in resisting the flow of the fluid. In case of compound meandering channels, Manning's *n* is presumed to be a roughness coefficient which is affected not only by the boundary unevenness but also the dynamic behaviour of the channel.

Laboratory data sets for other investigators have been collected to find an improved model for predicting composite Manning's roughness coefficient *n* by using dimensional analysis. The analysis takes into account various geometric and hydraulic parameters such as, relative depth of flow, $\beta = (H-h)/H$ i.e. the ratio of water depth over the floodplain to that of the overall depth in the channel; width ratio, $\alpha = B/b$ i.e. the width of floodplain to that of the main channel; sinuosity, s; and bed slope, So. Manning's *n* estimated by other models is established and subsequent discharge capacity by all these models along with the proposed model is found. Error analysis for all the models for different data sets is attained, where the proposed model is observed to provide better results with respect to the other methods.

2. METHODOLOGIES FOR CALCULATING MANNING'S *n*

Computation of roughness coefficient is challenging due to the various hydraulic complexities in an open channel. There are various methods for estimating the roughness coefficient of a channel by use of tables, photographs or even equations.

Cowan (1956) established a procedure to approximate the value of Manning's n by use of flow retarding factors. Initially the base n value for the natural bed material is determined for a straight, uniform and smooth channel. Modifying values for channel-surface irregularity, channel-shape and size variation, obstructions, and type and density of vegetation is applied according to the factors affecting the particular channel. Then the sum of these factors is multiplied to the degree of channel meandering as an adjustment factor. The following equation portrays the postulation of Cowan,

Roughness coefficient have been suggested by books and manuals as in U.S. Department of Agriculture (1955), Chow (1959), Henderson (1966), Brater and King (1976) and U.S. Department of Transportation (1979) containing Manning's roughness coefficient for modified channels and streams. These provide an average estimate of roughness coefficient for a wide variety of channel types accumulated from laboratory and field computations.

Exhaustive study on natural and laboratory channels on roughness coefficient has aided researchers to identify and define by means of equations the relationship between flow resistance on hydraulic parameters and particle size distribution. Limerinos (1970), developed a relation of Manning's *n* to hydraulic radius and particle sizes by undertaking observations at 11 different sites for streams with straight reaches.

$$n = \frac{0.0926R^{1/6}}{1.16 + 2\log\left(R/d_{84}\right)} \tag{1}$$

where R is the hydraulic radius, in feet, and d84 is the intermediate particle diameter, in feet, that equals or exceeds that of 84 percent of the particles. If bed-material data needed for Limerinos's (1970) equation are unavailable, Bray (1979) provided a substitute that depends only on the water surface slope.

$$n = 0.1045S_w^{0.177} \tag{2}$$

where S_w is the slope of water surface in feet per foot.

Jarrett (1984) developed an equation for Manning's n using energy gradient and hydraulic radius by means of studying different mountain streams. This equation is applicable to channels with energy gradients from 0.002 to 0.09 and hydraulic radii from 0.5 to 7 ft.\

$$n = 0.395S_f^{0.38}R^{-0.16} \tag{3}$$

where Sf is the energy gradient and R is the hydraulic radius.

Coon (1998) assessed the roughness prediction method by V.B. Sauer (U.S. Geological Survey, Atlanta, Ga., written commun., 1990; this report, eq. 5), which suggested a relationship between water surface slope to that of the channel roughness. It was based on the observations of Barnes (1967) and is applicable to channels with water surface slopes between 0.0003 and 0.018 and with hydraulic radius up to 19 ft.

$$n = 0.11S_w^{0.18}R^{0.08} \tag{4}$$

The above equations have basically been derived from observations made by researchers in natural streams and rivers. Researchers have also developed relationships of Manning's n with respect to various parameter by controlled laboratory investigations.

U.S. Department of Agriculture (1963) suggested a model selecting roughness coefficient values for channels. The method known as the Soil Conservation Service (SCS) method was fairly adequate and provided better stage prediction than the other methods. The method proposed explanation for meander losses by adjusting the basic value of Manning's n on the basis of sinuosity, s, as

$$\begin{cases} \frac{n'}{n} = 1 \quad for\ s < 1.2 \\ \frac{n'}{n} = 1.15 \quad for\ 1.2 \le s < 1.5 \\ \frac{n'}{n} = 1.3 \quad for\ s \ge 1.5 \end{cases} \tag{5}$$

where, n' is the adjusted value and n is the base value.

The linearized SCS method (LSCS) given by James and Wark (1992) is derived for two ranges of sinuosity and is represented as,

$$\begin{cases} \frac{n'}{n} = 0.43s + 0.5 \quad for\, s < 1.5 \\ \frac{n'}{n} = 1.3 \quad for\, s > 1.5 \end{cases} \tag{6}$$

where, n' is the adjusted value and n is the base value.

Shiono, Al-Romaih and Knight (1999) carried out experimental investigation on meandering channels by varying the bed slope, So for different sinuosity. Consequently, they derived a model by dimensional analysis to illustrate that friction factor, f is mainly dependent on sinuosity. The relationship is shown below,

$$s = 10(f)^{1/2} \tag{7}$$

The aforementioned methodologies imply the existence of other contributing factors which affect the roughness coefficient or Manning's n of a channel other than being influenced by only the characteristic of the bed material. The above methods are predominantly developed for simple channels with different characteristics such as bed slope and sinuosity which are subsequently being used for prediction of discharge in compound meandering channels. However, in this paper an attempt has been made to compute the composite Manning's n for a compound meandering channel taking into account different geometric and hydraulic characteristics, and not just the bed slope and sinuosity.

2. SOURCES OF DATA

Table 1 also contains the channel dimensions and observations of other researchers, who have worked extensively on the laboratory investigation of meandering channels. Each investigator has studied on the various aspects of meandering channels due to the effect of one or two parameters. The data series' have been provided with references for later usage. The number in parenthesis denotes the number of runs for each series of experimentations.

The data sets taken into consideration in this paper are; the United States Army Corps of Engineers (1956) which conducted a series of experiments on meandering compound channels at Vicksburg. Two basic trapezoidal channels were constructed with 0.305m and 0.610m main channel widths for 1:0.5 trapezoidal channels. The overall floodplain width was varied to achieve various width ratios for different sinuosity as given in Table 1. Three different combinations of bed roughness' were carried out for each of the experimental channels, but only the channels with homogenous roughness are taken into consideration here. A total of 9 such experimental channels with different combinations of width ratio and sinuosity have been considered here.

Experimental investigations were carried out at the SERC Flood Channel Facility in 1990 and 1991 on large scale meandering channels in Phase B in Wallingford, UK, termed as FCF B (1990-1991). The data sets were obtained from the website http://www.birmingham.ac.uk/ and also from different reports and articles such as James and Wark (1992), Ervine, Willetts, Sellin and Lorena (1993), Greenhill and Sellin (1993). One set of rigid trapezoidal channel of 60° cross-over angle and two sets of natural channels with cross-over angles 60° and 110° were constructed. Different set of experiments were done on the natural channels by varying the total width of the compound channel, denoted as B. Various type of blocks and objects were introduced on the floodplains in order to vary the roughness.

Shiono et. al (1999) conducted analysis of compound meandering channels by varying the bed slope, So for different sinuosity. Four sets of channels have been considered here having different cross-sectional features. The details of the data sets are illustrated in Table 1. Each of the above investigators examined the effect of one or more geometric or hydraulic parameters on the flow analysis of a meandering channel. This extensive set of data series have been attained to aid in analysing the effect of various parameters that affect the roughness coefficient in such channels.

Table 1 Experimental runs for different researchers

Data Series		S_{sm}	h	b	α	s	So	β	Q (m^3s^{-1})
US Army	II (3)	1V:0.5H	0.1524	0.3048	30.00	1.33	0.001	0.167 - 0.375	0.037-0.433
	III (3)	1V:0.5H	0.1524	0.3048	16.00	1.33	0.001	0.167 - 0.375	0.027-0.227
	V (3)	1V:0.5H	0.1524	0.3048	30.00	1.17	0.001	0.167 - 0.375	0.038-0.441
	VI (3)	1V:0.5H	0.1524	0.3048	16.00	1.17	0.001	0.167 - 0.375	0.031-0.254
	XII (3)	1V:0.5H	0.1524	0.6096	8.00	1.57	0.001	0.167 - 0.375	0.040-0.223
	XIII (3)	1V:0.5H	0.1524	0.6096	8.00	1.4	0.001	0.167 - 0.375	0.044-0.243
	XIV (3)	1V:0.5H	0.1524	0.6096	8.00	1.2	0.001	0.167 - 0.375	0.048-0.277
	XV (3)	1V:0.5H	0.1524	0.6096	15.00	1.2	0.001	0.167 - 0.375	0.067-0.245
	XVI (3)	1V:0.5H	0.1524	0.6096	15.00	1.57	0.001	0.167 - 0.375	0.051-0.223
FCF Smooth	B 21 (16)	1V:1H	0.15	0.9	11.11	1.374	0.000996	0.08 - 0.48	0.082-0.989
	B 26 (16)	1V:1H	0.15	0.9	11.11	1.374	0.000996	0.017 - 0.49	0.040-1.093
	B 31 (14)	1V:1H	0.15	0.9	6.79	1.374	0.000996	0.06 - 0.47	0.039-0.571
	B 39 (14)	1V:1H	0.15	0.9	11.11	2.043	0.001021	0.09 - 0.50	0.038-0.943
	B 47 (14)	1V:1H	0.15	0.9	9.51	2.043	0.001021	0.09 - 0.49	0.036-0.751
FCF Rough	B 32 (13)	1V:1H	0.15	0.9	11.11	1.374	0.000996	0.05 - 0.49	0.043-0.918
	B 33 (12)	1V:1H	0.15	0.9	11.11	1.374	0.000996	0.05 - 0.51	0.042-0.765
	B 34 (18)	1V:1H	0.15	0.9	11.11	1.374	0.000996	0.05 - 0.53	0.040-0.455
	B 43 (15)	1V:1H	0.15	0.9	11.11	2.043	0.001021	0.06 - 0.52	0.033-0.433
Shiono-Al-Knight	1a (9)	1V:0.88H	0.0534	0.165	7.27	1.372	0.001	0.19 - 0.59	0.003-0.032
	1b (9)	1V:0.88H	0.0534	0.165	7.27	1.372	0.002	0.06 - 0.50	0.003-0.027
	1c (11)	1V:0.88H	0.0534	0.165	7.27	1.372	0.0005	0.13 - 0.64	0.002-0.033
	2a (8)	1V:0H	0.052	0.152	7.89	1.372	0.001	0.12 - 0.49	0.003-0.029
	2b (13)	1V:0H	0.052	0.152	7.89	1.372	0.002	0.014 - 0.43	0.002-0.020
	4a (10)	1V:0H	0.052	0.15	8.00	1.092	0.001	0.087 - 0.49	0.003-0.030
	4b (13)	1V:0H	0.052	0.15	8.00	1.092	0.002	0.07 - 0.43	0.002-0.028
	5a (9)	1V:0H	0.052	0.15	8.00	1.571	0.001	0.12 - 0.47	0.003-0.025
	5b (14)	1V:0H	0.052	0.15	8.00	1.571	0.002	0.10 - 0.47	0.002-0.023

4. FACTORS AFFECTING ROUGHNESS COEFFICIENT

Flow in open channels is generally subcritical and turbulent, hence the effect of Froude number, Fr and Reynold's number, Re is also taken into account. In subcritical state of flow, the role played by gravity forces is more pronounced; thus the flow has a low velocity and is often described as tranquil and streaming, Chow (1959). Manning roughness coefficient depends on Froude number, as Fr changes with hydraulic depth of flow. In turbulent flow, the water particles move in an irregular motion, which is neither smooth nor fixed, but a forward motion is obtained. In the natural channel practically all flows are turbulent. Hence the effect of Reynold's number is considered in evaluation of roughness coefficient. To verify the distinct effects of each of these parameters on the roughness coefficient, the variation of roughness with respect to the parameters need to be analysed. The roughness coefficient for the data sets of the compound meandering channels are back calculated from the actual discharges so as to obtain the composite Manning's n of the channel.

$$n = f(\alpha\beta\gamma s S_0) \qquad (8)$$

5. DIMENSIONAL ANALYSIS

To model the dependency of dependent and independent parameters, we need the parameters to be in non-dimensional groups. Here all the parameters are attempted to form non-dimensional groups. An improved method for calculating discharge of a meandering compound channel is carried out by using dimensional analysis. According to Manning's equation,

$$V = \frac{1}{n} R^{2/3} S_0^{1/2} \tag{9}$$

and $Q = VA$ (10)

where V is the mean velocity, R is the hydraulic radius and Q is the discharge of the channel.

It is evident that Manning's n is an important contributing factor to the discharge of a channel. Manning's equation is primarily the roughness coefficient for the bed material which is computed for a straight simple channel. Such calculation is not suitable to be used in compound channels and certainly not in meandering channels as it would provide spurious value for the bed roughness. Nonetheless the developed dimensional analysis model would provide with the overall n for the channel taking into account all the factors which affect the roughness coefficient and not just the bed roughness.

Therefore, by the use of dimensional analysis a relationship can be developed to predict roughness coefficient due to various factors. The factors affecting Manning's n as discussed are relative depth, β; Reynold's number, Re; Froude number, Fr; width ratio, α; bed slope, So and sinuosity s and can be functionally expressed as,

$$n = \phi(\beta, \mathrm{Re}, Fr, \alpha, S_o, s, \gamma) \tag{11}$$

Since, $\mathrm{Re} = \frac{VR}{\nu}$ and $Fr = \frac{V}{gR}$

The above functional relation can be expressed as

$$n = \phi(V, R, \nu, g, \beta, \alpha, S_o, s) \tag{12}$$

The Manning's equation can be rewritten as

$$\frac{VR}{\nu} = \frac{1}{n}\left(\frac{R^{10/3} S_o}{\nu^2}\right)^{1/2} \tag{13}$$

Therefore, the functional relation can be rewritten as

$$\frac{VR}{\nu} = \phi\left(\beta, \alpha, S_o, \frac{1}{s}, g, \frac{R^{10/3}}{\nu^2 m^{1/3}}\right) \tag{14}$$

The dimensionless group ($R^{10/3}/\upsilon^2$) is used because of its connection to the Manning's equation for one-dimensional flow in simple prismatic channels. A physical equation must be dimensionally homogeneous. Every correct physical equation that is, every equation that expresses a physically significant relationship between numerical values of physical quantities, must be dimensionally homogeneous. As manning's roughness coefficient is dimensionally non-homogenous, a length factor is $m^{1/3}$ considered to make it dimensionally homogenous which is take as unity in all calculations. As investigated, sinuosity (s) is inversely proportional to discharge, hence inversely proportional to mean velocity for which it is expressed in the denominator.

The functional relationship can be expressed in two dimensionless groups, represented as

$$\left(\frac{gR^{10/3}S_0}{\vartheta^2 m^{1/3}}\right) \sim \left(\frac{VRs\gamma}{\vartheta\alpha\beta}\right) \tag{15}$$

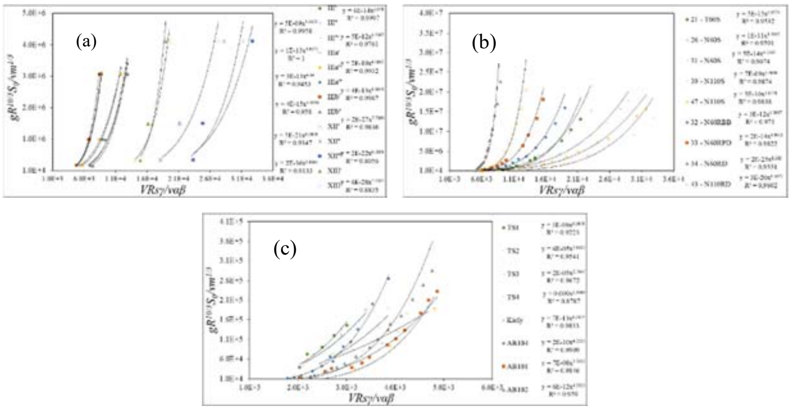

Figure 1: The functional relationship for (a) US Army, (b) FCF Phase B (c) Small scale channels

The dimensionless groups are plotted using the new experimental data along with the meandering compound channel data of other investigators in an attempt to find a simple relationship between the dimensionless groups

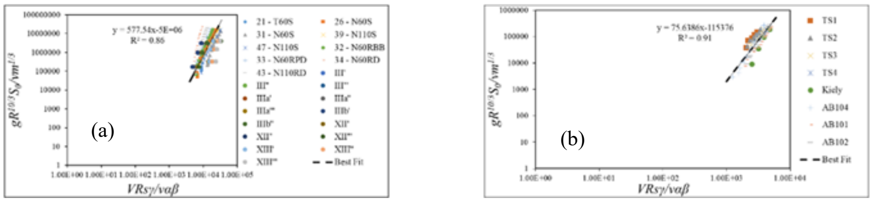

Figure 2: Calibration for large scale and small scale data sets

For Fig. 2(a),

$$Y=577.54X-5E+06 \tag{16}$$

$$\log\left(\frac{VRs\gamma}{\vartheta\alpha\beta}\right) = 0.0017\log\left(\frac{gR^{10/3}S_0}{\vartheta^2 m^{1/3}}\right) + 3.937 \tag{17}$$

The exponent of Eq. (8) could be taken as 0.002 and the coefficient as 8657. The developed model can thus be represented as,

$$\left(\frac{VRs\gamma}{\vartheta\alpha\beta}\right) = 8657\left(\frac{gR^{10/3}S_0}{\vartheta^2 m^{1/3}}\right)^{0.002} \tag{18}$$

The traditional Manning's equation in Eq. (11) can be rewritten as,

$$\left(\frac{VR}{\vartheta}\right) = \frac{1}{n}\left(\frac{R^{10/3}S_0}{\vartheta^2}\right)^{0.002}\left(\frac{R^{10/3}S_0}{\vartheta^2}\right)^{0.498} \tag{19}$$

On comparing Eq. (16) with the proposed model in Eq. (17), the generalized Manning's equation is represented as,

$$n = \frac{1}{8657}\left(\frac{R^{1.6}{S_0}^{0.498}m^{0.0006}\gamma s}{g^{0.002}\alpha\beta\vartheta^{0.996}}\right) \tag{20}$$

Similarly for the small scale data sets, the equation of Manning's *n* is derived as,

$$n = \frac{1}{1525}\left(\frac{R^{1.6}{S_0}^{0.48}m^{0.08}\gamma s}{g^{0.02}\alpha\beta\vartheta^{0.96}}\right) \tag{21}$$

6. RESULT AND DISCUSSION

The Manning's n predicted by the dimensional analysis model is used in predicting the conveyance of compound channels by using the traditional Manning's equation. It is essential to note that the n value predicted has taken into account the various geometric and hydraulic aspects of a compound channel and is different to that of the composite n computed backwards by the Manning's equation from the actual discharge.

There are various methods of estimating composite manning's n for meandering channels from the base n value. These methods namely, SCS (1963), LSCS (1992), Shiono-Al-Knight (1999), Jarrett (1984) and Sauer (1990) along with the new developed model are used to predict the modified n and subsequently predict the discharge capacity by considering the whole compound channel as a single unit. Error analysis for all the above models is performed to perceive the suitability of the developed model.

Due to the huge quantity of data sets, the error analysis for individual data series' is not realistic to be presented here. Hence, data sets of individual investigators are coupled together and the overall error analysis is carried out. Different types of error analysis, such as Coefficient of determination (R2), Mean Absolute Error (MAE), Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE), were computed as given in Eqs. (19) to (21) to find the acceptability of each of the models with respect to the data sets.

$$R^2 = \left(\frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}\right)^2 \tag{22}$$

$$MAE = \left(\frac{\sum |X-Y|}{p}\right) \tag{23}$$

$$RMSE = \left[\frac{\sum (X-Y)^2}{p}\right]^{1/2} \tag{24}$$

$$MAPE = \left[\frac{\sum\left(\frac{|X-Y|}{X}100\right)}{p}\right] \tag{25}$$

where $x = \left(X - \overline{X}\right)$; $y = \left(Y - \overline{Y}\right)$; *X* is the observed values; $\overline{X}$ is mean of *X*; *Y* is the predicted value; $\overline{Y}$ is mean of *Y*; and *p* is the number of samples

Figure 6 demonstrates the R^2 value of each of the models for different data sets. The value of R^2 closer to 1 suggests a better correlation between the actual and the predicted values for the models. It is observed that the developed model has quite a high R^2 value very close to 1 for all the data sets. The models by Shiono-Al-Knight (1999) and Sauer (1990) also show acceptable R^2 for some of the researchers. The MAE analysis shown in Fig. 4 shows lower error for the proposed model with respect to that of the other models. Similar observations are observed for RMSE and MAPE analysis where the proposed model provides acceptable results. Other methods like Sauer (1990), does provide with analogous observations with respect to the proposed model in some cases.

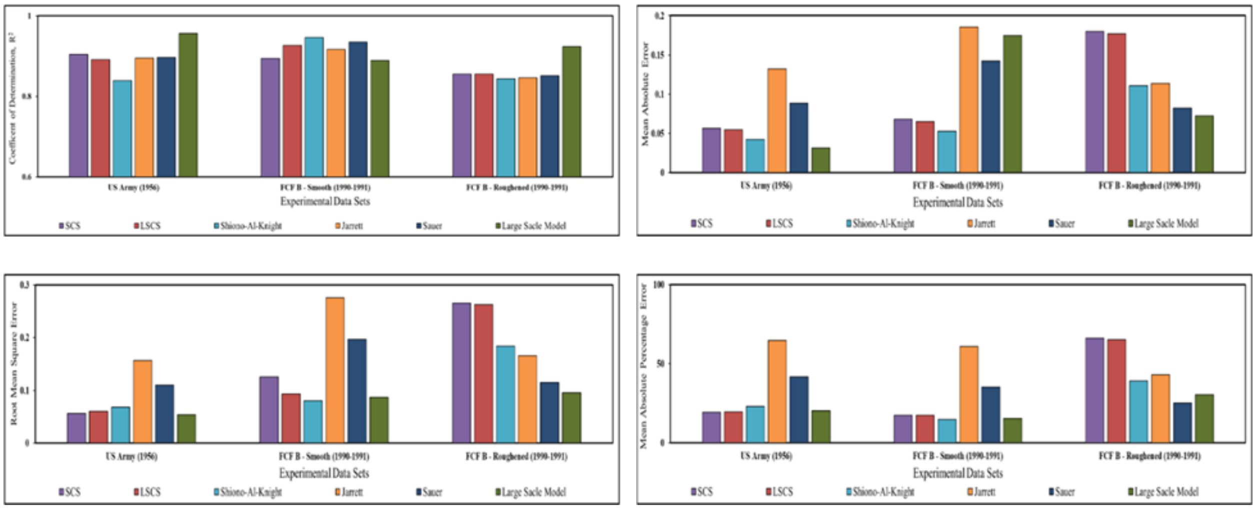

Figure 3. Error analysis for large scale data sets

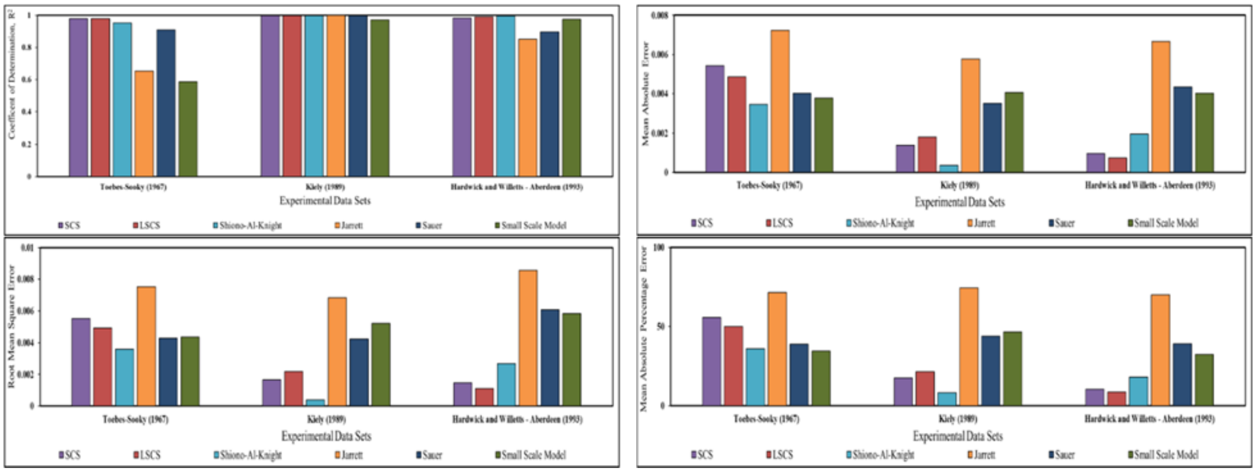

Figure 4. Error analysis for small scale data sets

Nonetheless, it is pertinent to imply here that the error analysis by the above procedure is not very conclusive for the validation of the different models. This might be because of categorizing the data sets according to the investigators. Each individual researcher has carried out experimental investigation on different types of meandering compound channels, by varying different parameters. Hence associating all those experimental observations as a single set, might provide with spurious results. Even the data sets are in different ranges i.e. some are large scale channels while others being small scale.

7. CONCLUSIONS

An empirical model to predict discharge in a meandering compound channel is proposed based on dimensional analysis. A new set of experimental data (15 runs each), along with a wide range of data sets of other researchers (i.e. 297 runs in total) with different channel parameters have been used in the development of the model. The data sets used have width ratio in the range 5 to 30 which are both small scale as well as large scale data. The data sets have different slopes and sinuosity with homogenous as well as heterogeneous roughness. The proposed model takes into account factors such as, relative depth of flow, width ratio of the channel, bed slope, sinuosity as well as relative roughness in computing the roughness coefficient of the channel. Previous methodologies were restricted to mostly on of the parameters or confined to a single range of data series. Therefore, the developed model on the basis of dimensional analysis can be used as a generalized formulation for a wide scope of channel dimensions, both geometric as well as hydraulic.

A selected number of models for predicting roughness coefficient were studied to estimate conveyance of compound meandering channels using the same data sets in order to investigate the advantages and disadvantages of the various methods. It was observed that the developed model provided satisfactory result as compared to the other models in terms of R^2, MAE, RMSE and MAPE for the different data series' of researchers. When observed in a more amplified approach, i.e. by considering the percentage of error along with the standard deviation for each individual data set, the proposed model showed noticeably better results proving to be a quite advanced model with respect to the others. It is essential to mention that while other models might provide acceptable results for some data sets but show large errors in others, the proposed model on the other hand gives satisfactory results for almost all of the data sets in the specified range.

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