Performance Assessment of Reparametrized K_G Distribution in Modelling SNR of Wireless Channel

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***Abstract*—Modelling composite wireless fading channel is crucial for design and performance analysis of communication system. In this regard, Nakagami/lognormal (NL) distribution is a widely acceptable composite channel model for characterizing SNR of wireless channel. However, probability density function of NL model is not in closed form which limits its applicability. Interestingly, K_G model is found to be a good approximation of NL distribution with relatively simpler mathematical expression. In this paper, we propose to reparameterize the K_G distribution suitably to approximate the NL distribution for SISO channel.The analytical expressions of outage probability, average symbol error rate and channel capacity are also derived for accessing the performance of reparametrized K_G model. Furthermore, analytical capacity bound is derived for the reparametrized K_G distribution to model SNR of MIMO wireless channel by exploiting majorization theory. Finally, experimental results are demonstrated to show the usefulness of the proposed K_G model which validates the applicability of reparametrized model in approximating NL distribution.**

I. Introduction

Modelling terrestrial wireless propagation has significant effect on performance of digital communication system. Typically, a wireless radio frequency (RF) channel experience joint effect of two independent random process such as small scale fading and large scale fading. Small scale fading that results from multipath effect leads to rapid fluctuations of signal. On the other hand, large scale fading that arise due to shadowing from various obstacles in the transmission link results into relatively slow variation of mean signal level [1]. The statistical model that encompasses simultaneous effect of multipath fading and shadowing is known as composite fading. Various composite models were proposed in [2] of which Nakagami-lognormal (NL) is the most common channel model.

The NL distribution assumes small scale and large scale fading statistics to be Nakagami and log-normal distributed, respectively. In formulating the above NL distribution, the mean square value of received carrier amplitude $E[\alpha^2]$ is considered to be unity. Here, the use of log-normal distribution as large scale fading component of NL distribution makes it more attractive in wireless channel modelling due to the applicability of log-normal distribution in characterizing fading channel for diverse scenario. However, the major drawback associated with NL model is that, the probability density function (PDF) is mathematically complicated and has no closed form expression making the further analysis of system performance more difficult. In view of that, several other models such as K, K_G, Weibull/gamma, G and $\eta - \mu$/gamma have been developed that approach NL distribution with having less mathematical complexity [3]. K_G distribution achieves good fit for both line of sight (LOS) and non-line of sight (NLOS) channel and is expressed in closed form mathematical expression. On the contrary, Weibull/gamma distribution has no experimental validation and less tractable. Similarly, $\eta - \mu$/gamma distribution is not as tractable as K_G model. Although G distribution has better fit to lognormal based model, it is less tractable as compared to K_G model. It can be noted that, K distribution can be expressed as a special case of K_G distribution by adjusting the shaping parameter.

In recent years, a good amount of research has been devoted towards performance analysis of K_G model [4]. For example, authors in [5] analyzed the statistical behavior of average symbol error rate (SER), capacity and outage probability. Similarly, performance analysis under various adaptive schemes are investigated in [6]. Essential statistical metrics of K_G distribution are derived in [7] considering different diversity schemes including equal gain combining (EGC), maximal ratio combining (MRC) and selection combining (SC). Furthermore, a multi input multi output (MIMO) channel is considered in [8] where, analytical capacity bound is obtained by using majorization theory. Recently, authors in [9], investigated on the physical-layer security over single input multiple output (SIMO) fading channel.

In the aforementioned literature, K_G model is obtained by averaging the instantaneous Nakagami-m distributed small scale received signal envelop over the conditional PDF of gamma shadowing. The authors in [4]–[9] assumed $E[\alpha^2]$ to be non-unity which deviates from the assumption considered in formulating NL distribution. This limits the accuracy of K_G model in approximating NL distribution and hence affect the system performance. Therefore, in this work we consider the mean square value of received carrier amplitude to be unity in modelling the fading channel and analyzed the consequences on system performance.

Researches regarding analysis on statistical characteristics of composite MIMO channel is limited because, the joint pdf of SNR distribution in simplified manner is quite difficult to achieve, specifically when the small scale fading is Nakagami-m distributed. However, the authors derived an closed form expression for upper bound of ergodic capacity considering small scale fading statistics to be Nakagami-m distributed [8], [10]. Therefore by using similar arguments as in [10] and [8], we formulate analytical bound for channel capacity of reparameterized K_G distribution to model SNR of MIMO channel.

The remainder of this paper is organized as follows. Section II introduces preliminaries on composite fading channel. Section III presents a detail explanation about distribution of K_G PDF. In section IV, capacity bound for K_G distributed MIMO channel is discussed. Finally, experimental results and conclusion are given in section V and VI respectively.

II. Composite Fading channel

A. Preliminaries

Radio wave propagation through wireless channels experience composite distribution due to shadowing and multipath fading. This fading scenario often suits for the case where the mobile users are either stationary or slowly moving. In such communication environment, instead of averaging out the multipath fading effect, the receiver operates on the instantaneous composite multipath/shadowed signal. According to Lee [11], the amplitude of signal envelop α can be expressed as a product of microscopic multipath fading and macroscopic shadowing as given below.

$$\alpha = \alpha_a \sigma_a \tag{1}$$

where α_a and σ_a are envelop of fading and shadowing, respectively. Nevertheless, SNR is considered as most common performance metric in digital communication system. Thus, the instantaneous SNR per symbol can be expressed by $\gamma = \alpha^2 E_s/N_0$ and average SNR per symbol by $\bar{\gamma} = E\left(\alpha^2\right) E_s/N_0$, where $E[\alpha^2]$ is the mean square value of α, E_s is the energy per symbol and N_0 denotes one-sided power spectral density of additive white Gaussian noise (AWGN). Thus, γ can be rewritten as

$$\gamma = \frac{\alpha^2 \bar{\gamma}}{E\left(\alpha^2\right)} \tag{2}$$

Here the multipath fading and shadowing are considered to be two independent random process. So, the distribution of composite fading is obtained by averaging the instantaneous small scale fading power over the PDF of shadowing as follows

$$f_\alpha(\alpha) = \int_0^\infty f_{\alpha|\sigma}\left(\alpha|\sigma\right) f_\sigma(\sigma)\, d\sigma \tag{3}$$

where $f_\sigma(\sigma)$ denotes PDF of macroscopic fading, $\sigma \left(= \sigma_a^2\right)$ and the conditional density $f_{\alpha|\sigma}\left(\alpha|\sigma\right)$ can be defined in terms of microscopic fading PDF $\left(f_{\alpha_a}(\alpha_a)\right)$ as

$$f_{\alpha|\sigma}\left(\alpha|\sigma\right) = \frac{1}{\sigma_a} f_{\alpha_a}\left(\frac{\alpha}{\sigma_a}\right) \tag{4}$$

As evident from (3), a multitude of composite models have been reported in the literature [3] for versatile $f_\sigma(\sigma)$ and $f_{\alpha|\sigma}\left(\alpha|\sigma\right)$. The NL distribution is found to be attractive in modelling SNR of wireless channel, details of which are given immediately next.

B. NL fading channel

The NL distribution considers α_a and σ to be Nakagami-m and log-normal distributed, respectively. The PDF of microscopic fading, α_a is expressed as

$$f_{\alpha_a}\left(\alpha_a\right) = \frac{2m^m \alpha_a^{2m-1} e^{-m\alpha_a^2}}{\Gamma(m)},\ \alpha_a > 0 \tag{5}$$

where $\Gamma(\cdot)$ is gamma function and m denotes fading parameter. Now, plugging (5) in (4), the conditional PDF can be obtained as

$$f_{\alpha|\sigma}\left(\alpha|\sigma\right) = \frac{2m^m \alpha^{2m-1} e^{-\frac{m\alpha^2}{\sigma}}}{\Gamma(m)\sigma^m}, \quad \alpha > 0 \tag{6}$$

Then the PDF of macroscopic shadowing, σ is given as

$$f_\sigma(\sigma) = \frac{1}{\sqrt{2\pi}\lambda\sigma} e^{-\frac{(\ln(\sigma)-\mu)^2}{2\lambda^2}}, \quad \sigma > 0 \tag{7}$$

where λ, μ denote scale and location parameter respectively. Substituting (6) and (7) in (3) and utilizing the relation (2), the SNR distribution of NL fading channel can be given as

$$f_\gamma(\gamma) = \int_0^\infty \frac{\gamma^{m-1} e^{-\frac{m\gamma}{\bar{\gamma}\sigma}}}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}\sigma}\right)^m \frac{e^{-\frac{(\ln(\sigma)-\mu)^2}{2\lambda^2}}}{\sqrt{2\pi}\lambda\sigma}\, d\sigma \tag{8}$$

Here the distribution of SNR given in (8) can be called gamma-lognormal (GL) distribution. Noticeably, the resulting SNR distribution of GL channel has no closed form expression, thereby limited to performance analysis of communication links.

III. The Reparametrized K_G Distribution for SISO channel

In this section, we formulate the PDF of K_G channel model by considering $E[\alpha^2]$ to be unity. Furthermore, analytical closed form expression for different statistical characteristics such as cumulative distribution function (CDF), moments and moment generating function (MGF) are derived.

A. Formulation of PDF

The small scale fading amplitude of K_G channel follows Nakagami-m distribution as given in (6), whereas shadowing is modelled using gamma distribution [12]. Accordingly, $f_\sigma(\sigma)$ of (3) can be given by

$$f_\sigma(\sigma) = \frac{\beta^\xi}{\Gamma(\xi)} \sigma^{\xi-1} e^{-\beta\sigma} \quad \xi, \beta > 0 \tag{9}$$

where, ξ, β denote shape and inverse scale parameter, respectively. By plugging (6) and (9) in (3), we obtain the PDF of K_G distribution as

$$f_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1} \beta^\xi}{\Gamma(m)\Gamma(\xi)} \int_0^\infty \sigma^{\xi-m-1} e^{-\frac{m\alpha^2}{\sigma} - \beta\sigma}\, d\sigma \tag{10}$$

Using the identity $\int_0^\infty t^{\nu-1} e^{-a/t-bt} dt = 2\left(\frac{a}{b}\right)^{\nu/2} K_\nu\left(2\sqrt{ab}\right)$ [13] in (10), we achieve the closed form PDF of α as

$$f_\alpha(\alpha) = \frac{4}{\Gamma(m)\Gamma(\xi)} (\beta m)^{\frac{m+\xi}{2}} \alpha^{m+\xi-1} K_{\xi-m}\left(2\sqrt{m\beta\alpha^2}\right) \tag{11}$$

where, $\beta = \xi/\bar{\gamma}$ and $K_\nu(\cdot)$ is kth order modified Bessel function of second kind. Now the SNR distribution $f_\gamma(\gamma)$ of K_G model can be obtained from (11) using (2) and expressed as

$$f_\gamma(\gamma) = \frac{2}{\Gamma(m)\Gamma(\xi)} \left(\frac{m\beta}{\bar{\gamma}}\right)^{\frac{m+\xi}{2}} \gamma^{\frac{m+\xi}{2}-1} K_{\xi-m}\left(2\sqrt{\frac{m\beta}{\bar{\gamma}}}\sqrt{\gamma}\right) \tag{12}$$

Interestingly, the use of gamma distribution to model shadowing leads to a closed form expression as given in (12). Here it is evident that, both the small scale and large scale fading follow gamma distribution. Thus $f_\gamma(\gamma)$ in (12) is termed as Gamma-Gamma distribution.

However, in the existing literature [4]–[9], the authors considered $E[\alpha^2]$ as non-unity. In such cases, the resulting PDF of γ is given by [4]

$$f_\gamma(\gamma) = \frac{2}{\Gamma(m)\Gamma(\xi)} (m\beta)^{\frac{m+\xi}{2}} \gamma^{\frac{m+\xi}{2}-1} K_{\xi-m}\left(2\sqrt{m\beta}\sqrt{\gamma}\right) \tag{13}$$

From (12) and (13), a notable difference in expression can be figured out. The PDF of (13) can be considered as special case of (12) where the average SNR $\bar{\gamma}$ makes the difference. Alternatively, (12) can be acknowledged as reparametrized form of (13). In the rest of the paper, the PDFs given in (12) and (13) are referred to as $K_{G(R)}$ and K_G model, respectively.

In order to illustrate the effectiveness of both $K_{G(R)}$ and K_G distributions in approximating GL channel model the relationship between their parameters is essential. Since, small scale fading statistics is considered same for both $K_{G(R)}$ and GL channel models, the fading parameter 'm' remains the same for both the cases. However, the large scale fading PDF being different for both the models, the relation between their parameters is established by employing moment matching method given in [14]. Here, the first and second-order moments of both log-normal and gamma distributions are equated which gives the relation between their parameters as

$$\beta = \frac{e^{\mu+\lambda^2/2}}{e^{2\mu+2\lambda^2} - \left(e^{\mu+\lambda^2/2}\right)^2}, \ \xi = \frac{\left(e^{\mu+\lambda^2/2}\right)^2}{e^{2\mu+2\lambda^2} - \left(e^{\mu+\lambda^2/2}\right)^2} \tag{14}$$

B. Formulation of CDF, Moments and MGF

The detailed formulation of CDF, moments and MGF for the $K_{G(R)}$ distribution is given in this subsection. The above analytical tools will be used in deriving performance metrics of fading channel discussed in next subsection.

1) Cumulative distribution function (CDF): For $K_{G(R)}$ distribution, closed form expression of CDF can be given as

$$\begin{aligned} F_\gamma(\gamma) &= \int_0^\gamma f_\gamma(\gamma) d\gamma \\ &= \int_0^\gamma \tfrac{2}{\Gamma(m)\Gamma(\xi)} \left(\tfrac{m\beta}{\bar{\gamma}}\right)^{\frac{m+\xi}{2}} \gamma^{\frac{m+\xi}{2}-1} K_{\xi-m}\left(2\sqrt{\tfrac{m\beta}{\bar{\gamma}}}\sqrt{\gamma}\right) d\gamma \end{aligned} \tag{15}$$

In (15), $K_\nu[\cdot]$ can be written in terms of Meijer's G-function [15] and can be further simplified by using relation [16, eq. 26] as

$$\begin{aligned} F_\gamma(\gamma) = &\tfrac{1}{\Gamma(m)\Gamma(\xi)} \left(\tfrac{m\beta}{\bar{\gamma}}\right)^{\frac{m+\xi}{2}} \gamma^{\frac{m+\xi}{2}} \\ &\times G_{1,3}^{2,1}\left[\tfrac{m\beta}{\bar{\gamma}}\gamma \,\middle|\, \begin{matrix} 1-(m+\xi)/2 \\ \frac{\xi-m}{2}, \frac{m-\xi}{2}, \frac{m+\xi}{2} \end{matrix}\right] \end{aligned} \tag{16}$$

where $G_{p,q}^{m,n}[\cdot]$ is the Meijer's G-function.

2) Moments: The nth order moment associated with $K_{G(R)}$ distribution can be evaluated as

$$E[\gamma^n] = \int_0^\infty \gamma^n f_\gamma(\gamma)\, d\gamma \tag{17}$$

where $E[.]$ denotes the expectation. Thus, by substituting (12) in (17) we obtain

$$\begin{aligned} E[\gamma^n] = &\tfrac{2}{\Gamma(m)\Gamma(\xi)} \left(\tfrac{m\beta}{\bar{\gamma}}\right)^{\frac{m+\xi}{2}} \\ &\times \int_0^\infty \gamma^{\frac{2n+m+\xi}{2}-1} K_{\xi-m}\left(2\sqrt{\tfrac{m\beta}{\bar{\gamma}}}\sqrt{\gamma}\right) d\gamma \end{aligned} \tag{18}$$

By substituting $t = \sqrt{\gamma}$ and using the integral representation $\int_0^\infty t^\mu K_\nu(at) dt = 2^{\mu-1} a^{-\mu-1} \Gamma\left(\frac{1+\mu+\nu}{2}\right) \Gamma\left(\frac{1+\mu-\nu}{2}\right)$, (18) can be reduced to its closed form expression given as

$$E[\gamma^n] = \frac{\Gamma(n+m)\Gamma(n+\xi)}{\Gamma(m)\Gamma(\xi)} \left(\frac{\bar{\gamma}}{m\beta}\right)^n \tag{19}$$

3) Moment generating function (MGF): MGF of γ can be defined by $M_\gamma(s) = E[e^{-s\gamma}]$. Therefore, MGF of $K_{G(R)}$ fading channel can be written using (12) as

$$\begin{aligned} M_\gamma(s) &= \int_0^\infty e^{-s\gamma} f_\gamma(\gamma) d\gamma \\ &= \tfrac{2}{\Gamma(m)\Gamma(\xi)} \left(\tfrac{m\beta}{\bar{\gamma}}\right)^{\frac{m+\xi}{2}} \\ &\quad \times \int_0^\infty e^{-s\gamma} \gamma^{\frac{m+\xi}{2}-1} K_{\xi-m}\left(2\sqrt{\tfrac{m\beta}{\bar{\gamma}}}\sqrt{\gamma}\right) d\gamma \end{aligned} \tag{20}$$

This integral can be reduced to its closed form expression by substituting $t = \sqrt{\gamma}$ and utilizing the identity [13, eq. (6.643.3)] as follows

$$\begin{aligned} M_\gamma(s) = &\left(\tfrac{\beta m}{\bar{\gamma}s}\right)^{\frac{\xi+m-1}{2}} e^{\beta m/(2\bar{\gamma}s)} \\ &\times W_{-(\xi+m-1)/2,\,(\xi-m)/2}\left(\tfrac{\beta m}{\bar{\gamma}s}\right) \end{aligned} \tag{21}$$

where, $W_{a,b}(\cdot)$ is the Whittaker function.

C. Performance Metrics

The wireless system design and its performance are highly influenced by the assumed channel model. Here we present performance metrics such as outage probability, average SER and average channel capacity for evaluating an M-ary quadrature amplitude modulation (M-QAM) wireless communication system.

1) Outage Probability : Outage probability is an essential performance criterion of communication link used over fading channel. It is defined as probability that the received SNR is below a given threshold γ_{th} . Hence, the outage probability, P_{out} can be expressed as

$$P_{out} = \int_0^{\gamma_{th}} f_\gamma(\gamma)\, d\gamma = F_\gamma\left(\gamma_{th}\right) \tag{22}$$

where the closed form expression for P_{out} can be evaluated using (16).

2) Average Symbol Error Rate: In this work, we are assuming a single user environment. Therefore, possibility of any interference is neglected here and we concentrate only on average SER performance. In practice, MGF is used to evaluate average SER in an accessible manner. In this work, a square M-QAM signaling scheme is considered where, $M = 2^k$. By utilizing [1, eq. 9.21], the average SER for M-QAM is given by

$$P_s(E) = \tfrac{4}{\pi}\left(\tfrac{\sqrt{M}-1}{\sqrt{M}}\right)\times \left[\int_0^{\pi/2} M_\gamma\left(\tfrac{g_{QAM}}{\sin^2\theta}\right)d\theta - \left(\tfrac{\sqrt{M}-1}{\sqrt{M}}\right)\int_0^{\pi/4} M_\gamma\left(\tfrac{g_{QAM}}{\sin^2\theta}\right)d\theta\right] \tag{23}$$

where $g_{QAM} = 3/(2(M-1))$, $q = 1 - 1\big/\sqrt{M}$ and $M_\gamma(\cdot)$ is obtained using (21). The above equation can be further simplified to its closed form with the use of hypergeometric function [17, eq. (12)].

3) Average Channel Capacity: According to Shannon′s theorem, when perfect channel state information (CSI) is available at receiver, the average channel capacity can be expressed as

$$C_{avg} = \frac{B}{\ln 2}\int_0^\infty \ln(1+\gamma) f_\gamma(\gamma)\, d\gamma \tag{24}$$

where B is channel bandwidth. Although the exact solution of (24) is intractable, an approximate solution obtained in [18], [19] where, $\ln(1+\gamma)$ is expanded using Taylor′s series about $E[\gamma]$. The resulting expression is given as

$$C_{avg} \approx \frac{B}{\ln 2}\left[\ln\left(1+E[\gamma]\right) - \frac{E[\gamma^2]-E^2[\gamma]}{2\left(1+E[\gamma]\right)^2}\right] \tag{25}$$

where $E[\gamma^n]$ can be obtained using (19).

IV. Capacity Analysis of Reparametrized K_G Distributed MIMO channel

In this section, we perform the capacity analysis for MIMO channel to evaluate the performance $K_{G(R)}$ distribution. Here, we consider a point to point MIMO system with Nt transmit and Nr receive antenna. The $Nr \times Nt$ composite channel matrix is modeled as $\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{\Xi}^{1/2}$ where, $\boldsymbol{H} \in C^{Nr\times Nt}$ and $\boldsymbol{\Xi} \in C^{Nr\times Nt}$ corresponds to small scale and large scale fading respectively. Individual elements of $\boldsymbol{H}$ are assumed as independent and identically distributed (i.i.d) random variable (RV) with uniform phase distribution in $[0, 2\pi)$ while , the amplitude $x = |h_{i,j}|$ following a Nakagami-m distribution as given in (5). Similarly, $\boldsymbol{\Xi}$ is defined by $\boldsymbol{\Xi} = \left(\frac{\sigma}{D^v}\right)\boldsymbol{I}_{Nt}$ where, D is the distance between transmitter and receiver, v denotes path loss exponent and σ is a gamma distributed RV $\sigma \sim Gamma(\xi, \beta)$ as given in (9).

A. Average MIMO channel capacity

In this paper, perfect channel state information (CSI) is assumed to be available at receiver side only that eventually leads to uniform power allocation across all transmit data streams. In this aspect, the average capacity of MIMO channel can be expressed as

$$C_{avg} = E\left[\log_2\left(\det\left(\boldsymbol{I}_{Nt} + \frac{\bar{\gamma}}{Nt}\boldsymbol{Z}^\dagger\boldsymbol{Z}\right)\right)\right] \tag{26}$$

where, det and $(\cdot)^\dagger$ respectively stands for determinant and Hermitian transpose of a matrix. Here the channel capacity upper bound of of MIMO $K_{G(R)}$ channel is deduced for the case $Nr > Nt$. According to [10], the upper bound for MIMO average capacity can be given as

$$C_{avg} \le C_{up} = E\left[\sum_{i=1}^{Nt}\log_2\left(1+\frac{\bar{\gamma}}{Nt}z_{i,i}\right)\right] \tag{27}$$

where, $z_{i,i},\ i = 1\cdots Nt$ are real, non-negative diagonal elements of $\boldsymbol{Z}^\dagger\boldsymbol{Z}$. It is to be noted that, sum of Nt i.i.d gamma RV with rate parameter $\{b_i\}_{i=1}^{Nt}$ and a common scale parameter a, is also a gamma distributed with parameters $\left(\sum_{i=1}^{Nt} 1/b_i,\, a\right)$. Thus (27) can be rewritten as

$$C_{up} = Nt \times E\left[\sum_{i=1}^{Nt}\log_2\left(1+\frac{\bar{\gamma}}{Nt}\frac{\sigma\eta}{D^v}\right)\right] \tag{28}$$

where, η is the sum of Nr i.i.d gamma RV, $\eta \sim Gamma(mNr, 1/m)$ given as

$$f(\eta) = \frac{\eta^{mNr-1}}{\Gamma(mNr)} m^{mNr} e^{-mNr}, \quad \eta > 0 \tag{29}$$

Thus, (29) can be represented as

$$C_{up} = \frac{Nt}{\ln 2}\int_0^\infty\int_0^\infty \ln\left(1+\frac{\bar{\gamma}}{Nt}\frac{\sigma\eta}{D^v}\right) f(\sigma) f(\eta) d\sigma d\eta \tag{30}$$

By expressing the logarithmic term in terms of Meijer′s $G-$function, (30) has the form

$$C_{up} = \frac{Nt}{\ln 2}\int_0^\infty\int_0^\infty G_{2,2}^{1,2}\left[\frac{\bar{\gamma}}{Nt}\frac{\sigma\eta}{D^v}\,\middle|\,\begin{matrix}1,1\\1,0\end{matrix}\right] f(\sigma) f(\eta) d\sigma d\eta \tag{31}$$

Applying (9), (29) in (31) and utilizing the integral relation [13, eq. (7.813.1)], (31) can be expressed in closed form as

$$C_{up} = \frac{Nt}{\Gamma(mNr)\Gamma(\xi)\ln 2} G_{4,2}^{1,4}\left[\frac{\bar{\gamma}}{mNtD^{v}\beta}\,\Big|_{1,0}^{1-\xi,1-mNr,1,1}\right] \tag{32}$$

B. High SNR Analysis

In the high SNR regime, $\log(1+ax)$ can be approximated to be $\log(ax)$. Thus, using the integration formula [13, eq. (4.352.1)] $\int_0^\infty x^{a-1}e^{-cx}\ln x dx = \frac{\Gamma(a)}{c^a}(\psi(a) - \ln(c))$, the upper bound can be approximated as

$$\begin{aligned} C_{up}^{\infty} = Nt\log_2\left(\tfrac{\bar{\gamma}}{Nt}\right) + \tfrac{Nt}{\ln 2}\left(\Psi(mNr) - \ln(m)\right) \\ +Nt\left(\Psi(k)/\ln 2 + \log_2(1/\beta) - v\log_2(D)\right) \end{aligned} \tag{33}$$

where, $\Psi(\cdot)$ is the digamma function.

V. Experimental Results

In this section, we present the experimental results to access the accuracy in approximating the GL model to existing and the proposed reparametrized K_G models. The performance of 4-QAM and 16-QAM systems in terms of outage probability, average SER and average channel capacity are demonstrated using GL, $K_{G(R)}$ and K_G channel models. Here, we consider the scenario where the environment experiences average shadowing that corresponds to parameters of GL to be $\lambda = 0.161$ and $\mu = -0.115$. The parameters of $K_{G(R)}$ and K_G models are evaluated using (14). For all distributions, parameter value m is set to 5. The SNR distribution of GL, $K_{G(R)}$ and K_G models are plotted in Fig.1. Clearly, Fig.1 shows that, $K_{G(R)}$ approximation fits well with GL model as compared to K_G approximation. Apparently, the PDF of K_G approximation deviates significantly towards the lower tail.

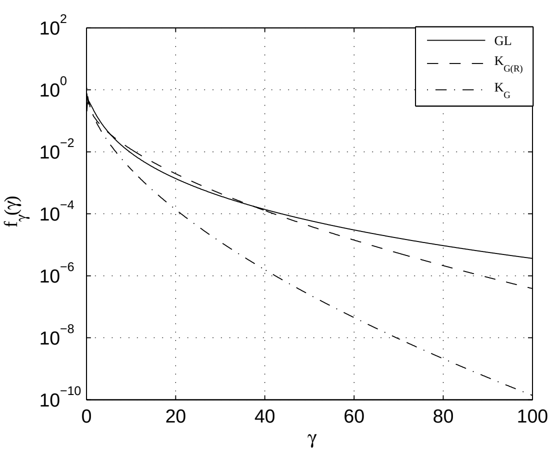

Fig. 1: Probability density function of GL(exact), $K_{G(R)}$ and K_G model

The accuracy is also measured by symmetric Kullback-Leibler (KL) divergence (D_{KL}) [20] defined by

$$D_{KL} = \sum_{\gamma} f_{Ext}(\gamma)\log\frac{f_{Ext}(\gamma)}{f_{App}(\gamma)} + \sum_{\gamma} f_{App}(\gamma)\log\frac{f_{App}(\gamma)}{f_{Ext}(\gamma)} \tag{34}$$

where $f_{App}(\gamma)$ and $f_{Ext}(\gamma)$ are approximated and exact distribution, respectively. In the given environmental condition, KL divergence of $K_{G(R)}$ and K_G distributions are found to be 0.1727 and 0.4735 respectively. Thus the lower value of D_{KL} verifies that, $K_{G(R)}$ model is a better substitute for GL distribution than K_G model.

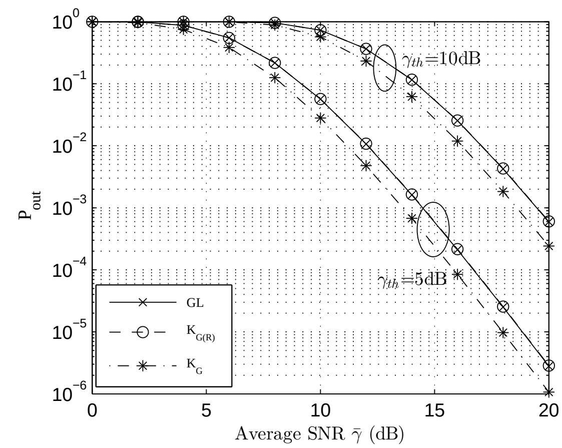

Fig. 2: Outage probability versus $\bar{\gamma}$ for γ_{th} = 5dB, 10dB over GL(exact), $K_{G(R)}$ and K_G fading channel

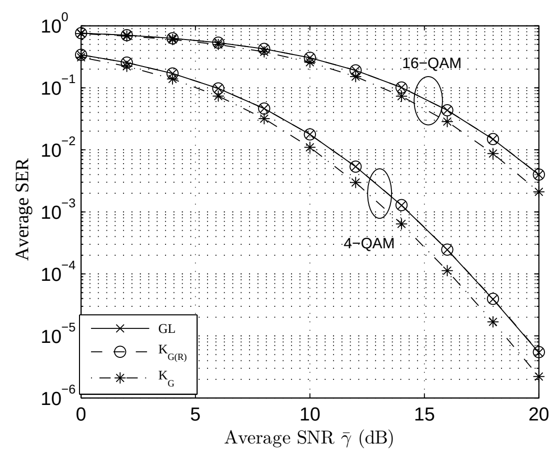

Fig. 3: Average SER of 4QAM and 16QAM over GL(exact), $K_{G(R)}$ and K_G fading channel

Fig.2 shows the outage probability with respect to average SNR $\bar{\gamma}$ for different values of threshold SNR γ_{th} . As shown in figure, outage probability of $K_{G(R)}$ distribution is more analogous to GL distribution as compared to K_G model. In Fig.3 and Fig.4 we have presented average SER and capacity analysis, respectively. 4-QAM and 16-QAM modulated symbols are considered for transmission to evaluate average SER. In both Fig.3 and Fig.4, average SER and capacity of $K_{G(R)}$ distribution are comparable to GL distribution whereas in case of K_G model, there is a certain deviation particularly in higher values of average SNR. Analytical upper bound and simulated output for average capacity of $K_G(R)$ MIMO channel is compared in Fig. 5. We also included the simulation result of GL distribution for comparative analysis. It is observed that, capacity increases with increase in number of receiving

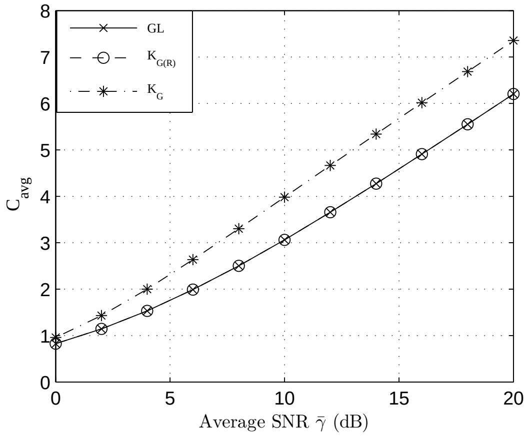

Fig. 4: Average channel capacity versus $\bar{\gamma}$ over GL(exact), $K_{G(R)}$ and K_G fading channel

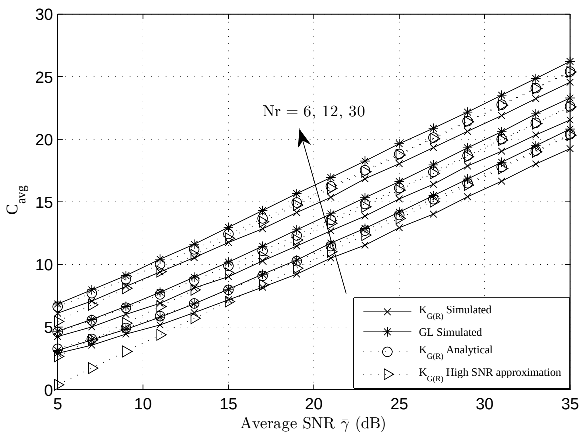

Fig. 5: Average channel capacity versus $\bar{\gamma}$ for MIMO $K_{G(R)}$ channel

antenna and the upper bound becomes more tighter. Moreover, the high SNR approximation is accurate even for moderate values of $\bar{\gamma}$.

VI. Conclusion

In this paper, we have proposed reparametrized K_G model that can be used as substitute for typical K_G distribution in order to approximate GL distribution. At the outset, we derive expression of SNR distribution for reparametrized K_G model and figure out the characteristic differences. The parameters of resulting distribution are obtained by moment matching method for comparative analysis. The accuracy of approximation is evaluated in terms of analytical results obtained from PDF and CDF expressions. It is found that, the proposed reparametrized K_G model has a better fit to GL model than the existing K_G model. The matching accuracy is also supported by KL divergence method. In this work, closed form expressions of performance metrics such as outage probability, average SER and capacity are deduced by capitalizing on the CDF, MGF and moment expressions derived earlier, respectively. Furthermore, upper bound of average channel capacity is deduced for MIMO channel. In addition to that, a simplified expression for high SNR approximation of upper bound is derived. Finally, experimental results illustrate that the reparameterized K_G model outperforms the existing K_G distribution in approximating GL distribution to model SNR of wireless channel.

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