

# Research Advances in the Dynamic Stability

## Behaviour of Plates and Shells: 1987-2005

### Part 1: Conservative Systems

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This paper reviews most of the recent research done in the field of dynamic stability/ dynamic instability/ parametric excitation /parametric resonance characteristics of structures with special attention to parametric excitation of plate and shell structures. The solution of dynamic stability problems involves derivation of the equation of motion, discretization and determination of dynamic instability regions of the structures. The purpose of this study is to review most of the recent research on dynamic stability in terms of the geometry (plates, cylindrical, spherical and conical shells), type of loading (uniaxial uniform, patch, point loading ....), boundary conditions (SSSS, SCSC, CCCC ....), method of analysis (exact, finite strip, finite difference, finite element, differential quadrature and experimental ....), the method of determination of dynamic instability regions (Lyapunovian, perturbation and Floquet's methods ), order of theory being applied (thin, thick, 3D, non-linear....), shell theory used (Sanders', Love's and Donnell's), materials of structures (homogeneous, bimodulus, composite, FGM....) and the various complicating effects such as geometrical discontinuity, elastic support, added mass, fluid structure interactions, non-conservative loading and twisting etc. The important effects on dynamic stability of structures under periodic loading have been identified and influences of various important parameters are discussed. Review on the subject for non-conservative systems in detail will be presented in Part-2.

## 1. INTRODUCTION

Plate and shell structures are extensively used in civil, mechanical and aerospace applications. Structural elements subjected to in-plane periodic forces may undergo unstable transverse vibrations, leading to parametric resonance, due to certain combinations of the values of load parameters and natural frequency of transverse vibration. Since the excitations when they are time dependent appear as parameters in the governing equations, these excitations are called parametric excitations. This instability may occur below the critical load of the structure under compressive loads over a range or ranges of excitation frequencies. Several means of combating parametric resonance such as damping and vibration isolation may be inadequate and sometimes dangerous with reverse results. A number of catastrophic incidents can be traced to parametric resonance. In contrast to the principal resonance, the parametric instability may arise not merely at single excitation frequency but even for small excitation amplitudes and combination of frequencies. Thus the dynamic stability characteristics are of great technical importance for understanding the dynamic systems under periodic loads. In structural mechanics, dynamic stability has received considerable attention over the years and encompasses many classes of problems. The distinction between ‘good’ and ‘bad’ vibration regimes of a structure, subjected to in-plane periodic loading can be distinguished through a simple analysis of dynamic instability region (DIR) spectra.

Dynamic instability was first observed by Faraday [1] in 1831. He observed that the liquid (wine) in a cylinder (wineglass) oscillated with half of the frequency of the exciting force movement of moist fingers around the glass edge. Rayleigh [2] gave the first mathematical explanation of the phenomenon in 1883. The general theory of dynamic stability of elastic systems of deriving the coupled differential equation of the Mathieu-Hill type and the determination of the regions of instability by seeking periodic solution using Fourier series expansion, was explained by Bolotin [3]. An extensive bibliography of the earlier works on parametric response of structures was presented by Evan-Iwanowski [4] in 1965. The survey of the theory of parametric vibration along with current and stochastic problems was given by Ibrahim [5] in review articles. The various phenomena under the heading of dynamic stability, with similarity and differences between them was discussed in detail by Simites [6] through 1987. He pointed out that parametric resonance characteristics are one of the best defined class of “dynamic stability” problems. Dynamic stability of structures was also discussed briefly by Nayfeh and Mook [7]. The present study mostly deals with recent investigations on dynamic stability of plates and shells after 1987 along with several early papers, which were inadvertently omitted, in the previous reviews.

This paper reviews the literature focusing on different aspects of research. The method of solution of dynamic stability class of problems involves first to reduce the equations of motion to a system of Mathieu-Hill equations having periodic coefficients and the parametric resonance characteristics are studied from the solution of the equations, that are obtained from different methods of solution. These methods may be grouped, with respect to their origins, into three main categories as Bolotin's approach using Floquet's theory, multiscale perturbation analysis and Lyapunovian exponents. In these analysis, the geometry, loading of the structural components as well as its boundary conditions play a major role in the choice of the methods of solution. The other aspects of research are the method of analysis. Dynamic stability of structures has been observed experimentally and analytically. The emergence of the digital computers with their enormous computing speed and core memory capacity has changed the outlook of the structural analysts and caused the evolution of various numerical methods such as finite strip method, finite difference method, method of multiple scales, finite element method (FEM), generalized differential quadrature method (GDQM) etc. Parametric excitation of plate and shell structures under periodic loads is investigated by classical thin plate theory, first order shear deformation theory (FSDT) considering shear deformation and using higher order shear deformation theory (HSDT). Dynamic instability studies are carried out on structures with homogeneous, transversely isotropic, bimodulus and orthotropic materials. Studies on parametric resonance characteristics of structures with cross-ply, angle-ply and sandwich configurations have also been conducted.

Most of the dynamic stability studies in literature are carried out on structures subjected to uniaxial uniform in-plane compressive periodic loads. However, studies have also been carried out on structures subjected to in-plane edge biaxial loads, patch loads, concentrated loads, random loads and even tensile loads. Plates and shells simply supported on four sides (SSSS) were considered by many investigators. Dynamic instability of structures under other boundary conditions such as clamped, elastic foundation, and multiple supports are also considered. Parametric excitation behaviour of plates of different geometry such as rectangular, annular, skew, polygonal, circular and isotropic stiffened plates has been studied in the literature. The parametric resonance characteristics of cylindrical, spherical, conical, elliptic and hyperbolic paraboloidal shells have been investigated. Complicating effects like geometrical discontinuity, plates supported on elastic foundations, optimization, visco-elasticity, twisting and non-linear theory have also been considered. The researchers have also investigated the problems involving combination resonance and the effect of longitudinal resonance on parametric excitation.

## 2. GENERAL THEORIES INVOLVING DYNAMIC STABILITY:

The basic configuration of the problem presented here is a laminated composite doubly curved panel with cutout subjected to in-plane periodic concentrated edge loading as shown in Fig.1. The choice of the laminated doubly curved panel geometry has been made as a basic configuration so that depending on the value of curvature parameter, plate, cylindrical panels and different doubly curved panels including twist can be considered as special cases. Specific problems can be explained from the general theory by proper choice of geometry, load, material and other parameters.

### 2.1 Governing differential equations

The governing differential equations, given by Bert and Birman [8] for dynamic stability of orthotropic cylindrical shells, modified for the parametric excitation of laminated composite shear deformable doubly curved panels, can be expressed as :

$$\begin{aligned}
 \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} - \frac{1}{2} C_2 \left( \frac{1}{R_y} - \frac{1}{R_x} \right) \frac{\partial M_{xy}}{\partial y} + C_1 \frac{Q_x}{R_x} + C_1 \frac{Q_y}{R_{xy}} &= P_1 \frac{\partial^2 u}{\partial t^2} + P_2 \frac{\partial^2 \theta_x}{\partial t^2} \\
 \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} + \frac{1}{2} C_2 \left( \frac{1}{R_y} - \frac{1}{R_x} \right) \frac{\partial M_{xy}}{\partial x} + C_1 \frac{Q_y}{R_y} + C_1 \frac{Q_x}{R_{xy}} &= P_1 \frac{\partial^2 v}{\partial t^2} + P_2 \frac{\partial^2 \theta_y}{\partial t^2} \\
 \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} - 2 \frac{N_{xy}}{R_{xy}} + N_x^0 \frac{\partial^2 w}{\partial x^2} + N_y^0 \frac{\partial^2 w}{\partial y^2} &= P_1 \frac{\partial^2 w}{\partial t^2} \\
 \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= P_3 \frac{\partial^2 \theta_x}{\partial t^2} + P_2 \frac{\partial^2 u}{\partial t^2} \\
 \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y &= P_3 \frac{\partial^2 \theta_y}{\partial t^2} + P_2 \frac{\partial^2 v}{\partial t^2}
 \end{aligned} \tag{1}$$

Where  $N_x^0$  and  $N_y^0$  are the external loading in the X and Y directions respectively. The constants  $R_x$ ,  $R_y$ , and  $R_{xy}$  identify the radii of curvature in the X and Y directions and the radius of twist respectively.  $N_x$ ,  $N_y$ , and  $N_{xy}$  are the internal membrane forces,  $Q_x$  and  $Q_y$  are the shearing forces and  $M_x$ ,  $M_y$  and  $M_{xy}$  are the moment resultants.  $C_1$  and  $C_2$  are tracers by which the analysis can be carried out by shear deformable version of the theories of Sanders' [9], Love's [10] and Donnell's [11] shallow shell theories. If  $C_1 = C_2 = 1$ , the equation corresponds to Sanders' theory. For the case,  $C_1 = 1$  and  $C_2 = 0$ , the equation reduces to Love's theory. For  $C_1 = C_2 = 0$ , the equation corresponds to Donnell's shallow shell theory.

The equation of motion for vibration of a laminated composite doubly curved panel with cutout as shown in Figure 1, subjected to in-plane periodic loads can be expressed in the form:

$$[M]\{\ddot{q}\} + [[K_e] - P[K_g]]\{q\} = 0 \quad (2)$$

Where  $q$  is the vector of degrees of freedoms ( $u, v, w, \theta_x, \theta_y$ ).

The in-plane load  $P$  is periodic and can be expressed in the form:

$$P(t) = P_s + P_t \cos \Omega t \quad (3)$$

where  $P_s$  is the static portion of load  $P(t)$ .  $P_t$  is the amplitude and  $\Omega$  is the frequency of the dynamic portion of  $P(t)$ . The static elastic buckling load of the shell  $P_{cr}$  may be considered as the measure of the magnitudes of  $P_s$  and  $P_t$  such that:

$$P_s = \alpha P_{cr}, \quad P_t = \beta P_{cr} \quad (4)$$

Where  $\alpha$  and  $\beta$  are the static and dynamic load factors respectively.

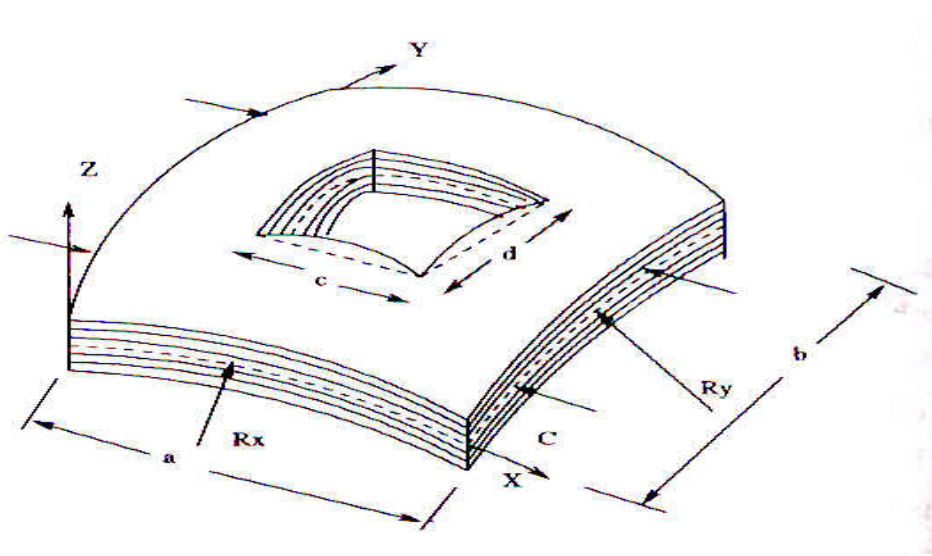


Fig. 1 Geometry of laminated composite doubly curved panel with cutout under periodic load

Using Eq. (4), the equation of motion in matrix form is obtained as:

$$[M]\{\ddot{q}\} + [[K_e] - \alpha P_{cr}[K_g] - \beta P_{cr}[K_g] \cos \Omega t]\{q\} = 0 \quad (5)$$

Eq. (5) represents a system of second order differential equations with periodic coefficients of the Mathieu-Hill type. The boundaries of the dynamic instability regions are formed by the periodic solutions of period  $T$  and  $2T$ ,

where  $T=2\pi/\Omega$ . The boundaries of the primary instability regions with period  $2T$  are of practical importance [3] and the solution can be achieved in the form of the trigonometric series

$$q(t) = \sum_{k=1,3,5,\dots}^{\infty} [\{a_k\} \sin(k\Omega t / 2) + \{b_k\} \cos(k\Omega t / 2)] \quad (6)$$

Putting this in Eq.(5) and if only first term of the series is considered, equating coefficients of  $\sin \Omega t/2$  and  $\cos \Omega t/2$ , the equation (5) reduces to

$$[[K_e] - \alpha P_{cr}[K_g] \pm \frac{1}{2} \beta P_{cr}[K_g] - \frac{\Omega^2}{4}[M]]\{q\} = 0 \quad (7)$$

Eq.(7) represents an eigenvalue problem for known values of  $\alpha$ ,  $\beta$  and  $P_{cr}$ . The two conditions under a plus and minus sign correspond to two boundaries of the dynamic instability region (DIR). The eigenvalues are  $\Omega$ , which give the boundary frequencies of the instability regions for given values of  $\alpha$  and  $\beta$ . In this analysis, the computed static buckling load of the panel may be considered as the reference load for numerical computations

## 2.2 Constitutive relations

The basic doubly curved laminated shell is considered to be composed of composite material laminate (typically thin layers). The material of each lamina consists of parallel continuous fibers embedded in a matrix material. Each layer may be regarded as on a microscopic scale as being homogenous and orthotropic. The stress resultants are related to the mid-plane strains and curvatures for a laminated shell element as:

$$\{F\} = [D]\{\mathcal{E}\}$$

or

$$\begin{Bmatrix} N_i \\ M_i \\ Q_i \end{Bmatrix} = \begin{bmatrix} A_{ij} & \dots & B_{ij} & \dots & 0 \\ B_{ij} & \dots & D_{ij} & \dots & 0 \\ 0 & \dots & 0 & \dots & S_{ij} \end{bmatrix} \begin{Bmatrix} \mathcal{E}_j \\ k_j \\ \gamma_m \end{Bmatrix} \quad (9)$$

The extensional, bending-stretching coupling and bending stiffnesses are expressed as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^n \int_{Z_{k-1}}^{Z_k} (\overline{Q_{ij}})_k (1, z, z^2) dz \quad i, j = 1, 2, 6 \quad (10)$$

The transverse shear stiffness is expressed as :

$$(S_{ij}) = \sum_{k=1}^n \int_{Z_{k-1}}^{Z_k} \kappa (\overline{Q_{ij}})_k dz \quad i, j = 1, 2, 6 \quad (11)$$

Where  $\kappa$  is the transverse shear correction factor and  $\overline{Q_{ij}}$  terms are the conventional off axis stiffness values, which depend on the material constants, and ply orientations.

### 2.3 Strain displacement relations

Green-Lagrange's strain displacement relations are presented in general throughout the structural analysis. The linear part of the strain is used to derive the elastic stiffness matrix and the non-linear part of the strain is used to derive the geometric stiffness matrix. The total strain is given by

$$\{\varepsilon\} = \{\varepsilon_l\} + \{\varepsilon_{nl}\} \quad (12)$$

The linear strain displacement relations are

$$\begin{aligned} \varepsilon_{xl} &= \frac{\partial u}{\partial x} + \frac{w}{R_x} + zk_x \\ \varepsilon_{yl} &= \frac{\partial v}{\partial y} + \frac{w}{R_y} + zk_y \\ \gamma_{xyl} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{2w}{R_{xy}} + zk_{xy} \\ \gamma_{xzl} &= \frac{\partial w}{\partial x} + \theta_x \\ \gamma_{yzl} &= \frac{\partial w}{\partial y} + \theta_y \end{aligned} \quad (13)$$

Where the bending strains  $k_j$  are expressed as,

$$k_x = \frac{\partial \theta_x}{\partial x}, \quad k_y = \frac{\partial \theta_y}{\partial y} \quad (14)$$

$$k_{xy} = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x}$$

The non-linear strain components are as follows:

$$\begin{aligned} \varepsilon_{xnl} &= \frac{1}{2} \left( \frac{\partial u}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial v}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial x} - \frac{u}{R_x} \right)^2 + \frac{1}{2} z^2 \left[ \left( \frac{\partial \theta_x}{\partial x} \right)^2 + \left( \frac{\partial \theta_y}{\partial x} \right)^2 \right] \\ \varepsilon_{ynl} &= \frac{1}{2} \left( \frac{\partial u}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial v}{\partial y} \right)^2 + \frac{1}{2} \left( \frac{\partial w}{\partial y} - \frac{v}{R_y} \right)^2 + \frac{1}{2} z^2 \left[ \left( \frac{\partial \theta_x}{\partial y} \right)^2 + \left( \frac{\partial \theta_y}{\partial y} \right)^2 \right] \\ \gamma_{xynl} &= \frac{\partial u}{\partial x} \left( \frac{\partial u}{\partial y} \right) + \frac{\partial v}{\partial x} \left( \frac{\partial v}{\partial y} \right) + \left( \frac{\partial w}{\partial x} - \frac{u}{R_x} \right) \left( \frac{\partial w}{\partial y} - \frac{v}{R_y} \right) \\ &\quad + z^2 \left[ \left( \frac{\partial \theta_x}{\partial x} \right) \left( \frac{\partial \theta_x}{\partial y} \right) + \left( \frac{\partial \theta_y}{\partial x} \right) \left( \frac{\partial \theta_y}{\partial y} \right) \right] \end{aligned} \quad (15)$$

### 3. DYNAMIC STABILITY OF PLATES

General theories involving dynamic stability presented in section 2 can be appropriately recast to study the instability behaviour of both isotropic and composite plates.

#### 3.1 Isotropic Plates

The dynamic stability of plates under periodic in-plane loads was considered first by Einaudi [12] in 1936. A comprehensive review of early developments in the parametric instability of structural elements including plates was presented in the review articles [4-7]. Simons and Leissa [13] explained the stability behaviour of homogeneous plates subjected to in-plane acceleration loads. Yamaki and Nagai [14] treated rectangular plates under in-plane periodic compression. The dynamic stability of clamped annular plates is studied theoretically by Tani and Nakamura [15] using the Galerkin procedure. It was found that principal resonance was of most practical importance, but that of combination resonance cannot be neglected when the static compressive force was applied. Dixon and Wright [16] studied experimentally the parametric instability behaviour of flat plates by normal or shear periodic in-plane forces. Oscillating tensile in-plane load at the far end causing parametric instability effects around the free edge of the cutout is an interesting phenomenon in structural instability. Carlson [17] conducted experiments on the parametric response characteristics of a tensioned sheet with a crack

like opening. Cutouts, cracks and other kinds of discontinuities are inevitable in structures due to practical considerations. Datta [18] investigated experimentally the buckling behaviour and parametric resonance behaviour of tensioned plates with circular and elliptical openings. Datta [19] later studied the parametric instability of tensioned panels with central openings and edge slot. The parametric resonance experiments for different opening parameters indicate that the dynamic instability effects are more significant for narrow openings than for wider openings. The studies on the dynamic stability of plates by Ostiguy *et al.* [20] showed good agreement between theory and experiment. The emergence of digital computers caused the evolution of various numerical methods besides analytical and experimental procedures. Hutt and Salam [21] used the finite element method for the dynamic stability analysis of homogeneous plates using a thin plate 4-noded finite element model. Extensive results were presented on dynamic stability of rectangular plates subjected to various types of uniform loads with/without consideration of damping. Prabhakara and Datta [22] explained the parametric instability characteristics of rectangular plates subjected to in-plane periodic load using finite element method, considering shear deformation. Plates and shells are seldom subjected to uniform loading at the edges. Cases of practical interest arise when the in-plane stresses are caused by localized or any arbitrary in-plane forces. Deolasi and Datta [23] studied the parametric instability characteristics of rectangular plates subjected to localized tension and compression edge loading using Bolotin's approach. The effect of damping on dynamic stability of plates subjected to non-uniform in-plane loads was investigated by Deolasi and Datta [24] using the Method of Multiple Scales (MMS). They further extended the work [25] to explain the combination resonance characteristics of rectangular plates subjected to non-uniform loading with damping. It was observed that under localized edge loading, combination resonance zones were important as simple resonance zones and the effects of damping on the combination resonances may be destabilizing under certain conditions. Deolasi and Datta [26] verified experimentally the parametric response of plates under tensile loading.

Floquet's theory was used by most of the investigators [21-23] to study the dynamic stability of plates. The regions of dynamic instability regions were determined by Bolotin's method. Aboudi *et al.* [27] studied the instability of viscoelastic plates subjected to periodic loads on the basis of Lyapunov exponents. The viscoelastic behaviour of the plate was given in terms of the Boltzmann superposition principle, allowing any viscoelastic character. Square and rectangular plates were the subject of research for many investigators [21-25, 28]. Shen and Mote [29] discussed the parametric excitation of circular plates subjected to a rotating spring. The analytical

works on dynamic stability analysis of annular plates [15] got new direction with the use of finite element method. Chen *et al.* [30] investigated the parametric excitation of thick annular plates subjected to periodic uniform radial loading along the outer edge, using higher order plate theory and axi-symmetric finite element. The dynamic stability of annular plates of variable thickness was studied by Mermertas and Belek [31]. The Mindlin plate finite element model was used to handle both the thin and thick annular plates. Young *et al.* [32] presented results on the dynamic stability of skew plates acted upon simultaneously by an aerodynamic force in a chordwise direction and a random in-plane force in spanwise direction. The dynamic instability of simply supported thick polygonal plates was analyzed by Baldinger *et al.* [33] and the corresponding stability regions of the first and second order are calculated, considering shear and rotatory inertia. Structures consisting of plates are often attached with stiffening ribs for achieving greater strength with relatively less material. Srivastava *et al.* [34] investigated the parametric instability of stiffened plates using the 9-node isoparametric plate element and stiffener element. The results showed that location, size and number of stiffeners have a significant effect on the location of the boundaries of the principal instability regions. As far as loading is concerned, many studies involved dynamic stability of plates subjected to uniform [21, 28] in-plane periodic loading. The dynamic stability of plates subjected to partial edge loading and concentrated in-plane compressive edge loading was considered by Deolasi and Datta [23-25]. Srivastava *et al.* [35-36] investigated the dynamic stability of stiffened plates subjected to non-uniform in-plane periodic loading. Takahashi and Konishi [37] analyzed the parametric resonance as well as combination resonance of rectangular plates subjected to in-plane dynamic force. Takahashi and Konishi [38] further investigated the dynamic stability of rectangular plates subjected to in-plane moments. Langley [39] examined the response of two-dimensional periodic structures to point harmonic loading. The study has extensive application to all types of two-dimensional periodic structures including stiffened plates and shells and it raises the possibility of designing a periodic structure to act as a spatial filter to isolate sensitive equipment from a localized excitation source. Young *et al.* [32] studied the parametric excitation of plates subjected to aerodynamic and random in-plane forces. The numerical studies involving dynamic stability behaviour of plates with openings are relatively complex due to non-uniform in-plane load distribution and are relatively new. Prabhakara and Datta [40] investigated the parametric instability behaviour of plates with centrally located cutouts subjected to tension or compression in-plane edge loading. Srivastava *et al.* [41] analyzed the dynamic stability of stiffened plates with cutouts subjected to uniform in-plane periodic loading.

The study considered stiffened plates with holes possessing different boundary conditions, cutout parameters, aspect ratios neglecting the in-plane displacements. The interaction of forced and parametric resonance of imperfect rectangular plates was explained by Sassi *et al.* [42]. In this study, the temporal response and the phase diagram were used besides the frequency response and FFT curves to study the transition zones. The effect of one particular spatial mode of imperfection on a different mode of vibration was investigated for the first time. Cederbaum [43] through a finite element formulation studied the effect of in-plane inertia on the dynamic stability of non-linear plates. Ganapathi *et al.* [44] investigated the non-linear instability behaviour of isotropic as well as composite plates, subjected to periodic in-plane load through a finite element formulation. The analysis brought out the existence of beats, their dependency on the forcing frequency, the influence of initial conditions, load amplitude and the typical character of vibrations in different regions. Touati and Cederbaum [45] analyzed the dynamic stability of non-linear visco-elastic plates.

### **3. 2 Complicating effects**

Most of the investigators considered the dynamic stability of plates on classical simply supported edges. Saha *et al.* [46] studied the dynamic stability of a rectangular plate on non-homogeneous Winkler foundation. The effects of stiffness and geometry of the foundation and aspect ratio on the stability boundaries of the plate for first and second order simple and combination resonance were investigated. Lee and Ng [47] presented results on the dynamic stability of plate on multiple line and point supports subjected to pulsating conservative in-plane loads. The effects of sinusoidal perturbations are examined by Bolotin's method. The dynamic stability of electrically conducting beam-plates in transverse magnetic fields was considered by Lee [48], considering the concise theory of flexural vibration of magnetoelastic plates immersed in transverse magnetic fields. A variational formulation of optimization problems for mechanical elements including plates subjected to parametric excitation force, in-plane periodic loading was presented by Forsys [49]. The examples of variational optimization against loss of stability were solved and analyzed in the state of parametric periodic resonance. Kim *et al.* [50] analyzed the parametric resonance of the sheet metal in a model of a plate subjected to time varying and non-uniform edge tension. Theoretical results for plate vibration were compared to experimental measurements of sheet metal vibration in a production facility. Datta and Deolasi [51] investigated the dynamic instability of plates subjected to partially distributed follower edge loading with damping.

### 3. 3 Composite Plates

The increasing use of fibre-reinforced composite materials in automotive, marine and especially aerospace structures, has resulted in interest in problems involving dynamic instability of these structures. The effects of number of layers, ply lay-up, orientation and different types of materials introduce material couplings such as stretching-bending and twisting-bending couplings etc. All these factors interact in a complicated manner on the vibration frequency spectrum of the laminates affecting the dynamic instability region. The stability behaviour of laminates was essential for assessment of the structural failures and optimal design. As per Evan-Iwanowski [4], the earliest works on dynamic stability of anisotropic plates were done by Ambartsumian and Khachaturian [52] in 1960. Considerable progress has been made since the survey [4-7] in this subject. There is a renewed interest on the subject after Birman [53] studied analytically the dynamic stability of rectangular laminated plates, neglecting transverse shear deformation and rotary inertia. The effect of unsymmetrical lamination on the distribution of the instability regions was investigated in the above study. Mond and Cederbaum [54] analyzed the dynamic stability of antisymmetric angle ply and cross ply laminated plates within the classical lamination theory, using the method of multiple scales. It was observed that besides the principal instability regions, other cases could be of importance in some cases. Srinivasan and Chellapandi [55] analyzed thin laminated rectangular plates under uniaxial loading by the finite strip method. The transverse shear deformation and in-plane inertia as well as rotatory inertia were neglected and the region of parametric instability was derived using Bolotin's procedure. Bert and Birman [56] investigated the effect of shear deformation on dynamic stability of simply supported anti-symmetric angle-ply rectangular plates neglecting in-plane and rotary inertia. The parametric studies on the effects of the number of layers, aspect ratio and thickness-to-edge length ratio were investigated. The dynamic instability of composite plates subjected to in-plane loads was investigated by Cederbaum [57] within the shear deformable lamination theory, using the method of multiple scales. Chen and Yang [58] investigated on the dynamic stability of thick anti-symmetric angle-ply laminated composite plates subjected to uniform compressive stress and/or bending stress using Galerkin's finite element. The thick plate model included the effects of transverse shear deformation and rotary inertia. The effects of number of layers, lamination angle, static load factor and boundary conditions were investigated. Moorthy *et al.* [59] considered the dynamic stability of uniformly uniaxially loaded laminated plates without static component of load and the instability regions were obtained using finite element method. Extensive results were presented on the effects of different

parameters on dynamic stability of angle-ply plates. Kwon [60] studied the dynamic instability of layered composite plates subjected to biaxial loading using a high order bending theory. Chattopadhyay and Radu [61] used the higher order shear deformation theory to investigate the dynamic instability of composite plates by using the finite element approach. The first two instability regions were determined for various loading conditions using both first and second order approximations. Pavlovic [62] investigated the dynamic stability of anti-symmetrically laminated angle-ply rectangular plates subjected to random excitation using Lyapunov direct method. Tylikowski [63] studied the dynamic stability of non-linear anti-symmetric cross-ply rectangular plates. The parametric results on biaxial loading were compared with those obtained by classical theory. Cederbaum [64] has investigated on the dynamic stability of laminated plates with physical non-linearity. Librescu and Thangjitham [65] analyzed the dynamic stability of simply supported shear deformable composite plates along with a higher order geometrically non-linear theory for symmetrical laminated plates. Gilai and Aboudi [66] obtained results on the dynamic stability of non-linearly elastic composite plates using Lyapunov exponents. The non-linear elastic behaviour of the resin matrix was modelled by the generalized Ramberg-Osgood representation. The instability of laminated composite plates considering geometric non-linearity was also reported using  $C^0$  shear flexible QUAD-9 element by Balamurugan *et al.* [67]. The non-linear governing equations were solved using the direct iteration technique. The effect of a large amplitude on the dynamic instability was studied for a simply supported laminated composite plate. The non linear dynamic stability was also carried out using  $C^1$  eight-noded element by Ganapathi *et al.* [68]. Numerical results were presented to study the influences of ply angle and lay-up of laminate. The parametric resonance characteristics of composite plates for different lamination schemes were also studied. Certain fiber reinforced materials, especially those with soft matrices exhibit quite different elastic behaviour depending upon whether the fiber direction strain is tensile or compressive. The dynamic stability of thick annular plates with such materials, called the bimodulus materials was studied by Chen and Chen [69]. The annular element with Lagrangian polynomials and trigonometric functions as shape function was developed. The non-axisymmetric modes were shown to have significant effects in the annular bimodulus plates. The dynamic stability of thick plates with such bimodulus materials were examined by Jzeng *et al.* [70]. The finite element method was used to investigate the stability of bimodulus rectangular plates subjected to periodic in-plane loads. The effects of shear deformation and rotatory inertia were considered using first order shear deformation theory. The dynamic stability of bimodulus thick

circular and annular plates was analyzed by Chen and Juang [71]. Chen and Hwang [72] studied the axisymmetric dynamic stability of orthotropic thick circular plates. Cederbaum [73] investigated on the dynamic stability of viscoelastic orthotropic plates. The stability boundaries were determined analytically by using the multiple scale method. Time dependent instability regions and minimum load parameter were derived together with an expression for the critical time at which the system, with a given load amplitude, would turn unstable. Cederbaum *et al.* [74] studied the dynamic instability of shear deformable viscoelastic laminated plates by Lyapunov exponents. Librescu and Chandiramani [75] analyzed the dynamic stability of transversely isotropic viscoelastic plates subjected to in-plane biaxial edge load system. The effects of transverse shear deformation, transverse normal stress and rotatory inertia effects are considered in this study. Sahu and Datta [76] have investigated the dynamic stability of composite plates subjected to non-uniform loads including patch and concentrated loads using finite element method. The dynamic stability of laminated composite stiffened plates/shells due to periodic in-plane forces at boundaries was discussed by Liao and Cheng [77]. The 3-D degenerated shell element and 3-D degenerated curved beam element were used to model plates/shells and stiffeners respectively. The method of multiple scales was used to analyze the dynamic instability regions.

### **3. 4 Complicating effects**

Most of the studies on parametric excitation are for structures without any geometrical discontinuity. However, delaminations are inevitable in composite structures due to practical considerations. Chattopadhyay *et al.* [78] investigated the instability associated with delaminated composite plates subjected to dynamic loads. Wang and Chen [79-80] explained the dynamic instability behaviour of non-rotating [79] and rotating [80] sandwich circular plates using finite element method. It was observed that the effects of constraint layer tend to stabilize the circular plate system. The width of instability regions increased with increase of rotational speeds. Patel *et al.* [81] studied the dynamic instability of layered anisotropic composite plates on elastic foundations. Yeh and Chen [82] investigated the parametric excitation of a rectangular orthotropic sandwich plate with electro-rheological fluid core. Yeh and Chen [83] also analyzed on the dynamic stability of a sandwich plate with a constraining layer and electro-rheological fluid core. However, studies concerning the dynamic stability characteristics of the plate in thermal environments are scarce. Marcus *et al.* [84] examined the dynamic stability of symmetrically laminated orthotropic rectangular plates due to a thermally oscillating load by using an extension of Bolotin's theory. As the advanced inhomogeneous composite materials mainly used for thermal

resistant components, functionally graded materials (FGMs) attracted much more research effort because of their multifunctional properties. The first contribution to the dynamic stability of FGM structures was made by Ng *et al.*[85], who studied the parametric resonance of simply supported FGM rectangular thin plates under pulsating in-plane loading and a fixed temperature by the normal mode expansion technique. Yang *et al.*[86] investigated the dynamic stability of laminated functionally graded materials (FGM) plates based on higher-order shear deformation theory. Recently, Wu *et al.*[87] analyzed the dynamic stability of thick FGM plates subjected to aero-thermo-mechanical loads using the moving least squares differential quadrature method. The influences of various factors such as gradient index, temperature, mechanical and aerodynamic loads, thickness and aspect ratios as well as boundary conditions were studied. Shukla and Nath [88] dealt with the non-linear dynamic buckling of laminated composite rectangular plates subjected to uniform time dependent in-plane temperature induced loading. The non-linear governing equations of motion were solved analytically using fast Chebyshev series technique. The numerical results for various boundary conditions were presented in this study. Chen and Chen [89] studied the parametric resonance of polar orthotropic sandwich annular plates with a viscoelastic core layer subjected to a periodic uniform radial stress using the finite element method. The axisymmetric discrete layer annular element and Hamilton's principle were employed to derive the equations of motion for a sandwich plate including the transverse shear effect. The viscoelastic material in the core layer was assumed to be incompressible, and the extentional and shear moduli were described by complex quantities. Kim and Kim [90] analyzed the dynamic stability of laminated plates under follower forces. Ravi Kumar *et al.*[91] examined the dynamic instability characteristics of laminated composite plates subjected to partial follower edge load with damping and showed certain aspects of destabilizing behaviour of damping.

#### **4. DYNAMIC STABILITY OF SHELLS**

The widespread use of shell structures in civil, aerospace and hydrospace applications has stimulated many researchers to study various aspects of their structural behaviour. The dynamic stability analysis of shells is more complicated due to the addition of curvature in the panel.

##### **4.1 Isotropic Shells**

The widespread use of shell structures in aerospace and hydrospace applications has stimulated many researchers to study various aspects of their structural behaviour. Instability in shells under periodic loads occurs when there exists certain relationships between the frequency of axial loads and the natural frequencies of the shell. As per

Evan-Iwanowski [4], an early publication on the parametric stability of cylindrical shell filled with liquid was made by Bublik and Merkulov [92] in 1942. The dynamic stability of simply supported cylindrical shells under periodic axial and radial loads was treated by Yao [93]. The dynamic stability of circular cylindrical and spherical shell subjected to uniform axial and radial pressure was studied by Bolotin [3]. Parametric resonance in shell structures under periodic loads had been of considerable interest since the subject was studied by him. The Lyapunov direct method was used to define the stability of a cylindrical shell under radial pressure by Bieniek, Fan and Lackman [94] and the solutions for the pre-buckling motion and the perturbed motion were obtained by the use of Galerkin method. Evensen and Evan-Iwanowski [95] investigated the dynamic response and stability of completely clamped, shallow, thin elastic spherical shells under uniformly distributed periodic loads both analytically and experimentally. Parametric instability of thin, cantilevered circular cylindrical shells subjected to in-plane longitudinal inertia loading arising from sinusoidal base excitation was investigated by Vijayraghavan and Evan-Iwanowski [96] analytically and experimentally. The linear bending theory used in the analysis was found adequate in predicting the incipience of instability. Excellent agreement was obtained between the analytical and experimental results, in determining the principal instability regions. The effect of longitudinal resonance on dynamic stability was examined by Koval [97] for simply supported cylindrical shells under axial excitation. A detailed study of resonances was carried out in the above study. The dynamic stability of clamped, truncated conical shells under periodic axial load was studied by Tani [98] using the Donnell type equations. Two principal instability regions were determined by combining Bolotin's method and a finite difference method. The effects of static axial load and end plate mass on the principal instability regions were also investigated. Yamaki and Nagai [99] investigated the dynamic stability of circular cylindrical shell subjected to periodic shearing forces, on the basis of Donnell type equations modified with the transverse inertia force. Yamaki and Nagai [100] also studied the dynamic stability of circular cylindrical shell under four types of boundary conditions, with the effect of the static compressive load using Galerkin procedure and Hsu's method. It was found that the effect of longitudinal resonance was generally negligible for thin shells. The stability of the steady state response of simply-supported circular cylinders subjected to harmonic excitation was investigated by Radwin and Genin [101] using variational equations. The dynamic stability of supported cylindrical pipes conveying fluid was examined by Ariaratnam and Namachchivaya [102]. The effects of the mean flow velocity, dissipative forces, boundary conditions, and virtual mass on the extent of the parametric instability regions were

then discussed. Chiba *et al.*[103] performed experimental studies on the dynamic stability of cantilever cylindrical shells partially filled with liquid, under horizontal excitation. It was found that a combination instability resonance of sum type could occur, involving two natural vibrations with the same axial mode vibration number but with the circumferential wave number differing by one. Tylikowski [104] investigated the stability of circular cylindrical viscoelastic shells subjected to time varying axial compression and uniformly distributed radial loading, using the direct Lyapunov method. The change of membrane loading direction from the axial direction to the circumferential one on the stability regions was also discussed. Kratzig and Eller [105] developed numerical procedures for the dynamic stability analysis of non-linear, dynamically excited shell structures. Special algorithms were deduced for the treatment of dynamic snap-through phenomena, dynamic quasi-bifurcations and parametric resonances. The dynamic stability and non-linear parametric vibration of isotropic cylindrical shells with added mass were considered by Kovtunov [106] using finite element method. Ye [107] investigated the effects of static load and static snap through buckling on the instability for spherical and conical shells were investigated using Galerkin method. Nawrotzki *et al.*[108] presented a unified concept for the dynamic stability of shells subjected to conservative and non-conservative forces, using finite rotation theory and finite element method involving elasto-plastic material behaviour. For the characterization of kinematic instability phenomena, such as parametric resonances, flutter, dynamic quasi bifurcations, or kinetic snap-through behaviour, special classes of qualitative techniques for neighboring orbits were considered. Turhan [109] presented a boundary tracing method as an extension of Bolotin's method to cover combination resonance for parametrically excited systems. The applicability of a uniform stability theory to shell structures undergoing elastic or elasto-plastic deformations was demonstrated by Nawrotzki, Kratzig and Montag [110] using FEM with the help of Lyapunov exponents. Gilat and Aboudi [111] studied the dynamic buckling of viscoelastic plates and shells under cylindrical bending. The method of solution relied on an incremental process in conjunction with the finite difference method with respect to the special co-ordinate and the Runga-Kutta method with respect to time. The parametric resonance of cylindrical shells under combined static and periodic loading was investigated using four different thin shell theories by Lam and Ng [112] using Bolotin's method. The effects of various thin shell theories on parametric instability were based on Donnell's, Love's, Sanders' and Flugge's theories. The contribution of the stresses due to the external forces was accounted for according to Donnell's theory. The parametric resonance of cylindrical shells under combined static and periodic loading was

studied using Donnell's, Love's, and Flugge's thin shell theories by Lam and Ng [113]. The contribution of stresses due to external forces was considered in this study according to the assumptions made in that particular theory unlike the previous work. The parametric resonance of a rotating cylindrical shell subjected to periodic axial loads was investigated by Ng *et al.* [114]. Popov [115] demonstrated the use of bifurcation theory and non-linear dynamics for the understanding of structural buckling under dynamic loads and vibration of circular cylindrical shells under parametric excitation.

Most of the investigators studied the dynamic stability of uniformly loaded closed cylindrical shells with a simply supported boundary condition, using analytical approach. Popov, Thompson and Croll [116] investigated the stability of periodic solutions of parametrically excited cylindrical panels, neglecting transverse shear and rotary inertia. The dynamic stability of uniformly loaded cylindrical panels was studied by Ng, Lam and Reddy [117] using an extension of Donnell's shell theory to a first order shear deformation theory (FSDT) and Bolotin's approach. The dynamic instability of conical shells was studied by Ng *et al.* [118] using Generalized Differential Quadrature method. Sahu and Datta [119] studied the dynamic stability of singly and various doubly curved panels including elliptic paraboloids and hyperbolic paraboloids, subjected to non-uniform in-plane harmonic loading, using finite element method, considering transverse shear deformation and rotary inertia. The effect of cutout on parametric excitation of doubly curved panels was investigated by Sahu and Datta [120]. The effects of static and dynamic load factors, geometry, boundary conditions and the cutout parameters on the principal instability regions of curved panels were investigated in detail using Bolotin's approach.

#### **4. 2 Complicating effects**

Noah and Hopkins [121] studied the effect of support flexibility on the dynamic behaviour of pipes conveying fluid both for steady and pulsatile flows. The numerical results illustrated the effects of the amount of translational and rotational resiliences at the elastic support on the regions of parametric and combination resonances of pipes. Popov *et al.* [122] analyzed the internal auto- parametric instabilities in the free non-linear vibrations of cylindrical shells. Direct numerical integration was employed to examine chaotic motions. It was observed that the chaotic motions near a homoclinic separatrix appeared immediately after the bifurcation, giving an irregular exchange of energy. This chaos occurred at arbitrarily low amplitude, with approaching of perfect tuning. Mcrobie *et al.* [123] presented on the auto parametric instabilities in the free non-linear vibrations of cylindrical shells, focused on two modes i.e. a concertina mode and chequerboard mode, whose non-linear

interaction breaks the in-out symmetry of the linear vibration theory. Ganesan and Kodali [124] discussed on the dynamic instability of cylindrical shell conveying a pulsatile flow of hot fluid. The semi analytical finite element formed the basis for the modeling the structural continuum under the influence of temperature and flowing fluid. Beroulli's principle and impermeable conditions of the fluid were the basis for the coupled fluid structure interaction analysis. The effect of fluid temperature and excitation parameter on the behaviour of dynamic stability system was examined. Javidruzi *et al.*[125] presented a finite element study on the vibration, buckling and dynamic stability behaviour of a cracked cylindrical shell with fixed supports and subjected to an in-plane compressive/tensile periodic edge load. The effects of crack length and orientation were analyzed. Zhu *et al.*[126] investigated on the three dimensional analysis of the dynamic stability of piezoelectric circular cylindrical shells. Bolotin's method was employed to determine the dynamic instability regions. It was observed that both the piezoelectric effect and electric field had minor effect on the instability regions. Djondjorov *et al.* [127] investigated on the dynamic stability of fluid conveying straight cantilevered pipes lying on variable elastic foundations. Tao and Zhang [128] studied the dynamic stability of a rotor partially filled with a viscous liquid. Most *et al.*[129] presented the stochastic dynamic stability analysis of non-linear structures with geometrical imperfections under random loading, by the maximum Lyapunov exponent. This exponent turns positive for unstable systems and can be computed by non-linear time integration with simultaneous stability analysis. Fluid structure interaction problems were investigated by Jung *et al.* [130] to study the dynamic stability of liquid filled projectiles under a thrust. The projectile was modeled as a flexible cylindrical shell, and the constant and pulsating follower force modeled the thrust. Park and Kim [131] investigated the dynamic stability of completely free cylindrical shell under a follower force by using finite element method. Recently Ravi Kumar *et al.* [132] analyzed the dynamic instability characteristics of doubly curved panels subjected to partially distributed follower edge loading with damping using finite element method.

#### **4. 3 Composite Shells**

As per Ibrahim [5], the dynamic stability of anisotropic cylindrical shells was first considered by Markov [133] in 1949. The earlier studies on parametric resonance of laminated shell structures were found from the review papers by Evan-Iwanowski [4], Ibrahim [5], and Simites [6]. The parametric instability of thick orthotropic cylindrical shells was studied analytically by Bert and Birman [8]. The theory used is a general first-order shear deformable shell theory and can be considered to be the thick shell version of the popular Sanders' thin shell

theory for cylindrical shells. By means of tracers, this theory can be reduced to thick shell versions of the theories of Love's and of Donnell's theories. Extensive results were presented for thick isotropic (short and long) shells and for two layered cross-ply and angle-ply cylindrical shells. The principal instability regions of thick and thin two layered cross-ply cylindrical shells were also compared. Ray and Bert [134] studied the dynamic stability of suddenly heated thick composite shells. The dynamic instability of shear deformable laminated composite simply supported circular cylindrical shell was analyzed by the Method of Multiple Scales (MMS) by Cederbaum [135]. The simply supported laminated shell of finite length was examined within Love's first approximation theory, with the addition of transverse shear deformation and rotary inertia. It was shown that, besides the principal instability region, other cases of resonances i.e combination resonance can be of importance. A perturbation technique was employed by Argento and Scott [136-137] to study the instability regions of layered anisotropic circular cylindrical shells subjected to axial loading. The studies discussed the theoretical development [136] and numerical results [137] of the variation of instability regions with the circumferential wave number and also the magnitude of the external axial load. Results were presented for a three layered  $0^0/90^0/0^0$  graphite epoxy shell. The same technique was used again later by Argento [138] to determine the instability regions of a composite circular cylindrical shell subjected to axial loading and torsional loading. The main emphasis of this study was the comparison of effects of pure axial loading, pure torsional loading and combined axial and torsional loading on the dynamic stability of the laminated shells. Ganapathi and Balamurugan [139] studied the dynamic instability of laminated composite circular cylindrical shells using a  $C^0$  shear flexible two noded axisymmetric shell element. The effects of various parameters such as ply angle, thickness, aspect ratio, axial and circumferential wave numbers on dynamic stability were studied. The dynamic stability of thin cross-ply laminated composite cylindrical shells under combined static and periodic axial force was investigated by Ng, Lam and Reddy [140] using Love's classical theory of thin shells. The effects of different lamination scheme and the magnitude of the axial load on the instability regions were examined using Bolotin's method. Lam and Ng [141] investigated on the dynamic stability analysis of thin laminated composite cylindrical shells under combined static and periodic loads, using Love's theory of thin shells. The effects of the length-to-radius and thickness-to-radius ratios of the cylinder on the instability regions were examined. Most of the above mentioned investigators studied the dynamic stability of uniformly loaded closed cylindrical shells with a simply supported boundary condition. The study of the parametric instability behaviour of curved

panels is new. The effects of curvature and aspect ratio on dynamic instability for a uniformly loaded laminated composite thick cylindrical panel were studied by Ganapathi *et al.*[142] using finite element method. The effectiveness of a nine-noded shear flexible shell element, based on the field consistency principle was demonstrated in this study by examining the dynamic stability of laminated curved panels due to periodic in-plane load. Sahu and Datta [143] investigated on the dynamic stability of singly and various doubly curved laminated composite panels including spherical, elliptic and hyperbolic paraboloids using finite element method. Zhang and Campen [144] studied the dynamic stability of doubly curved orthotropic shallow shells under impact. The non-linear governing differential equations were derived based on a Donnell type shallow shell theory. The non-linear behaviour was investigated by neglecting the influence of inertia and damping, and the results showed that two saddle node bifurcation would occur under certain conditions. Ravi Kumar *et al.*[145] examined the tension buckling and dynamic stability behaviour of laminated composite doubly curved panels subjected to partial edge loading. The investigation showed the presence of pockets of compression region causing instability effects. Sahu and Datta [146] also analyzed recently the dynamic stability of singly and doubly curved panels with cutouts. The effects of sizes of cutouts, ply orientation, curvature on parametric excitation of different curved panels including spherical, elliptic and hyperbolic paraboloidal shells were considered in this investigation. Ravi Kumar *et al.* [147] investigated on the tension buckling and parametric instability characteristics of doubly curved panels with circular cutout subjected to non-uniform tensile edge loading. The concept of local buckling effects was discussed. Ganapathi and the co-researchers [148-149] studied the dynamic instability analysis of truncated circular conical shells, using  $C^0$  two noded shear flexible shell element. Kamat *et al.*[150] analyzed the parametrically excited laminated composite joined conical-cylindrical shells. The formulation was based on first order shear deformation theory and the effect of in-plane and rotary inertia was considered. The influence of various parameters such as orthotropicity, cone angle, lay up, combination of different sections, thickness ratio, static load and external pressure on the dynamic stability regions of cross ply laminates was studied in this investigation.

#### **4. 4 Complicating effects**

Birman and Bert [151] also investigated the dynamic stability of torsionally reinforced composite cylindrical shells in thermal fields. It was found in this study that a shell subjected to high temperature became dynamically

unstable at smaller values of the magnitude and frequencies of the driving force when compared to shells excited at room temperature. Ng *et al.* [152] studied the parametric resonance of simply supported FGM cylindrical thin shells under pulsating in-plane loading and at a fixed temperature by the normal mode expansion technique. Yang and Shen [153] presented semi analytical solutions of dynamic stability problems for shear deformable FGM cylindrical panels, which has tremendous aerospace applications, subjected to periodic in-plane force and thermal load due to temperature change. Kadoli and Ganesan [154] studied the parametric resonance of a composite cylindrical shell containing pulsatile flow of hot fluid. A coupled fluid structure interaction model in conjunction with uncoupled thermomechanical model was used for the analysis. Recently, Ravi Kumar *et al.* [155-156] studied the dynamic stability of doubly curved panels subjected to follower edge load. These studies involved non-conservative load cases, which will be further discussed in the Part-2 of the review.

## 5. CONCLUDING REMARKS

This paper surveyed the dynamic stability of plates and shells subjected to conservative forces. On the whole, the focus of research on dynamic stability was changing towards new materials, methods of analysis and towards complicated geometry, loading and boundary conditions. Recently more studies were conducted on laminated composites than homogeneous materials. Functionally graded materials (FGMs) are the new generation of composite materials in extreme high temperature environments. The shell-type piezoelectric smart structures have become the focus of research in recent years. The structural configuration shifted from plates to closed cylindrical shells and then towards curved panels including cylindrical, spherical, hyperbolic paraboloidal and elliptical paraboloidal panels. As regards methodology, the focus was shifted from analytical methods to numerical methods and with the advent of high speed computers, more studies were made using the finite element method. The study revealed that recent investigations on dynamic stability were concentrating more on complicated aspects, such as non-uniform loading, stiffened plates, plates on Winkler foundations, boundary conditions, combination resonance, damping, fluid structure interaction, non-linearity, doubly curved panels etc. than plates or closed cylindrical shells. The studies involving cutouts have been dealt up to bare and stiffened plates and curved panels subjected to uniform loading. Limited attention was given towards experimental verification on the parametric instability behaviour of homogeneous plates. On a literature study by the authors, more investigations were reported in the literature on dynamic stability of turbomachinery blades which were modeled as beams. However, no research was reported on dynamic stability of blades as twisted plates. More

attention should be given for dynamic stability analysis of shells containing fluids, non-linearity, damping, non-conservative loading, non-classical curvature and boundary conditions. Attention is also needed for dynamic stability of surface structures with varying thickness to simulate more towards practical applications. More research is also needed for rotating structures subjected to in-plane periodic loading. Considerable attention is also needed towards experimental verification of the computational models.

## ACKNOWLEDGEMENT

The authors thank Professor AW Leissa for his encouragement and valuable suggestions.

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