Incremental Modified Leaky LMS

Sowjanya Modalavalasa, Upendra Kumar Sahoo, Ajit Kumar Sahoo
Electronics and Communication Department
National Institute of Technology
Rourkela, Odisha
India, 769008
Email: sowjanya.modalavalasa@gmail.com,(sahooupen,ajitsahoo@nitrkl.ac.in)

***Abstract*—Incremental Least means squares algorithm(ILMS) is one of the simplest algorithm for parameter estimation in distributed wireless networks, which find a wide range of applications from monitoring environmental parameters to satellite positioning. Digital implementation of adaptive filters results in quantization errors and finite precision errors, which makes the ILMS algorithm to suffer from drift problem. Incremental Leaky LMS algorithm(ILLMS) introduces a leakage factor in the update equation and overcomes the drift problem. But the overall performance of ILLMS is similar to ILMS in terms of convergence speed. To overcome this, an Incremental Modified Leaky LMS(IMLMS) is proposed based on MLLMS algorithm which in turn derived from the Least Sum of Exponentials(LSE) algorithm. LSE algorithm employs sum of exponentials of errors in its cost function and it results in convex and smooth error surface with more steepness, which results in faster convergence rate. Simulation results prove that the proposed IMLLMS outperforms the ILLMS.**

***Keywords*—Parameter Drift, Finite precision effects, Incremental MLLMS, Incremental Leaky LMS**

I. Introduction

The distributed sensor signal processing deals with collection and processing of local noisy observations of a parameter of interest in a geographical area where the micro sensors or nodes are deployed[1][2]. All the nodes share their information according to the network topology and estimate the parameter of interest in a collaborative manner by utilizing their local noisy observations and the shared information from their immediate neighbors. In the traditional centralized processing, all the nodes will collect its noisy local data and send them to a centralized processor, which will perform the job of parameter estimation and broadcasts the result back to all the individual nodes. This involves a very powerful centralized processor and huge communication burden. Whereas in distributed adaptive solution all the nodes will have processing capabilities and perform the job of parameter estimation individually using their local data and information received form neighbor nodes. This saves a lot of energy and bandwidth. The strategy of cooperation between the nodes will decide the data bandwidth and energy consumption. Basically there are three modes of collaboration namely incremental mode, diffusion mode, and probabilistic diffusion mode [3][4]. Each node in the adaptive distributed network is adaptive and the efficiency depends on the adaptation algorithm used and the mode of cooperation between the nodes. Least Mean Square(LMS) algorithm is the most popularly used due to its less computational complexity and ease of implementation. Incremental mode of cooperation requires less power and communication and hence incremental mode is considered throughout the paper. The convergence speed of the LMS algorithm depends on the eigen value spread of the input fed to the nodes. Eigen value spread is defined as the ratio of largest eigen value to the smallest eigen value of the autocorrelation matrix of the input sequence. The largest eigen value limits the allowable range of step size for stability assurance and the smallest eigen value accounts for slow convergence rate[5] [6]. So the best convergence rate is achieved when all the eigen values are equal, which can be achieved by pre-whitening the data before processing. LMS algorithm suffers from drift problem, where the parameter estimate will go unbounded even though the input sequence and error quantities are bounded[7]. The accuracy and stability of LMS cannot be assured in non-ideal or practical scenarios where finite precision effects, quantization errors, inadequate input excitation comes into picture [6][8][5].

A modification of the LMS algorithm is the Leaky LMS algorithm(LLMS) primarily developed to overcome the drift problem [6]. LLMS employs a leakage factor in its weight update equation and so bounds the weights within limits by leaking some energy out[9]. So ILLMS is developed to overcome the drift problem, accuracy and stability issues arising in ILMS. Though ILLMS overcomes drift issue,the performance of ILLMS is similar to ILMS both in terms of convergence speed and MSE. A modified Leaky LMS (MLLMS) is the enhanced version of the LLMS, which is developed to obtain superior performance compared to ILLMS. MLLMS is based on LSE algorithm[10][11], which is a generalization of the mixed norm gradient descent algorithms and employs sum of exponentials of errors in its cost function. So the cost function will have sum of even powers of error and so will have the combined effect of the second order statistic (SOS) algorithms like LMS, NLMS and under Higher order statistic (HOS) algorithms like LMF[10][11]. This results in a convex and smooth error surface with more steepness assuring faster convergence rate and better MSE performance. Both LLMS and MLLMS are implemented in incremental case of the distributed network and are compared in terms of convergence speed and MSE.

A. Modified LLMS

The LMS algorithm is one of the most famous adaptive algorithms for linear estimation due to its simplicity and ease of implementation. This has led to the development of variations of LMS algorithm, which are available in the literature. Some of the improved versions of LMS include NLMS, sign LMS, variable step size LMS, sign error LMS, sign regressor LMS etc. All these improved versions are developed to achieve faster convergence and better MSE. One of the main difficulties facing with LMS algorithm is the drift problem, where the parameter estimate will diverge despite of the bounded input conditions[5][8]. The leaky LMS (LLMS) is a modified version of conventional LMS algorithm and it overcomes the drift problem by bounding the parameter estimate using the leakage factor in the weight update equation. LLMS also solves the problems like stalling and improves stability, tracking capability[6]. The main drawback of LLMs is its convergence speed. Though LLMS solves the drift problem, the convergence speed and MSE performance is almost same as that of LMS algorithm. A novel algorithm is proposed in the literature to improve the convergence speed based on the Least Sum of Exponentials algorithm (LSE) [11]. LMS uses the second order error function, such Second Order Statistic (SOS) algorithms are very easy to implement and have less computational complexity. Higher order error power algorithms like Least Mean Fourth (LMF) algorithm comes under Higher Order Statistic (HOS) algorithms, which have high computational complexity, faster convergence rate and instability issues. To make use of both the advantages of SOS and HOS, mixed norm gradient descent algorithms have been developed [11]. LSE is one of such mixed norm gradient descent algorithm, which considers infinite number of error powers in the cost function. LSE algorithm employs sum of exponentials of errors in the cost function, which is the generalization of the mixed norm stochastic gradient algorithms. The cost function of the Modified Leaky LMS (MLLMS) algorithm is defined as below:

$$J\left(k\right) = \left(\exp\left(e\left(k\right)\right) + \exp\left(-e\left(k\right)\right)\right)^2 + \gamma w^T\left(k\right) w\left(k\right) \quad (1)$$

So the error surface of the cost function defined in eq.1 is smooth and convex, which improves the convergence speed. MLLMS is the modification of LSE[12]. The error surface will be steeper and so the convergence speed is faster than the LLMS algorithm[11] Where $e\left(k\right)$ is the error defined as below:

$$e(k) = d(k) - u^T(k)w(k-1) \quad (2)$$

Differentiating above equation with respect to $w\left(k\right)$, we get

$$\frac{\delta J\left(k\right)}{\delta w\left(k\right)} = 2\left(-u\left(k\right)\exp\left(e\left(k\right)\right) + u\left(k\right)\exp\left(-e\left(k\right)\right)\right) + 2\gamma w\left(k\right) \quad (3)$$

Now the weight update equation is given as:

$$w\left(k\right) = w\left(k-1\right) - \frac{\mu}{2}\frac{\delta J\left(k\right)}{\delta w\left(k\right)} \quad (4)$$

Substituting eq.3 in eq.4, the resulting weight update equation for Modified leaky LMS is as below:

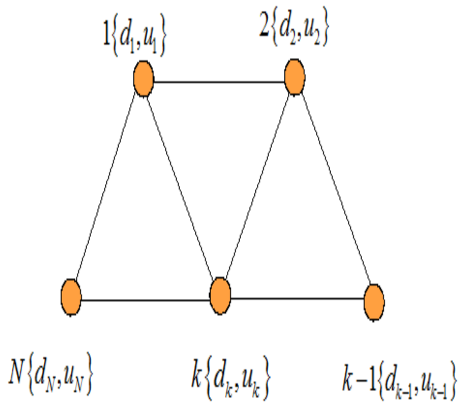

Fig. 1. Data Flow and updation in Incremental modified leaky LMS

$$w\left(k+1\right) = \left(1-\mu\gamma\right)w\left(k\right) + 2\mu u\left(k\right)\sinh\left(e\left(k\right)\right) \quad (5)$$

II. Problem Formulation

Consider a distributed network with N nodes as shown in Fig 1. Each and every node k has access to local noisy data realizations $\{d_k(i), u_{k,i}\}$ of the zero mean spatial data $\{d_k, u_k\}$ for $k = 1, 2, ..., N$ where d_k is the desired scalar and u_k is a regression input vector of size $1 \times M$. The main intention is to estimate the vector w of size $1 \times M$ by using the above data collected from all N nodes and it should solve

$$\min_w J\left(w\right)$$

Where $J\left(w\right)$ indicates the cost function which signifies the MSE, given as bellow:

$$J\left(w\right) = E\|d - uw\|^2$$

The solution to the above optimization problem is solved using many approaches in the literature, Steepest Descent Solution, Incremental LMS, Incremental Leaky LMS and so on[3][6][12][13].

III. Incremental Modified Leaky LMS

Incremental Modified Leaky LMS algorithm is proposed to improve the performance of ILLMS. The proposed IMLLMS will overcome drift problem with improved performance compared to ILMS.

A. Data Model and Assumptions

The data model and the assumptions followed for the IMLLMS algorithm are listed below:

- The desired unknown vector w^0 relates $\{d_k(i), u_{k,i}\}$ as $d_k(i) = u_{k,i}w^0 + v_k(i)$ Where $v_k(i)$ is white noise sequence with variance $\sigma^2_{v,k}$ and independent of $\{d_l(j), u_{l,j}\}$ for all l, j
- $u_{k,i}$ is independent of $u_{l,i}$ for $k \neq l$ (Spatial independence).
- $u_{k,i}$ is independent of $u_{k,j}$ for $i \neq j$ (Time independence).

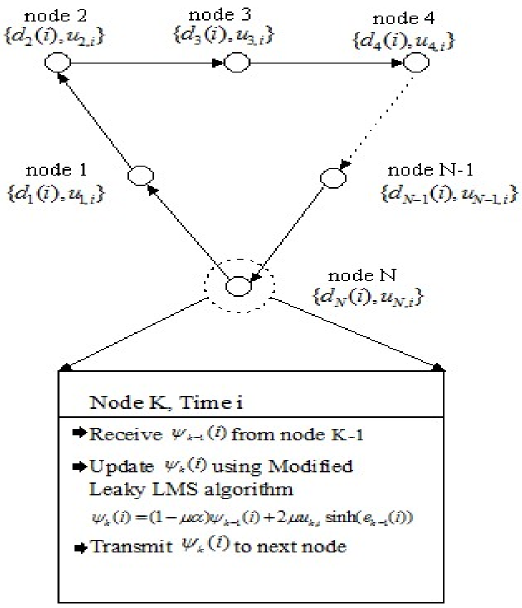

Fig. 2. Data Flow and updation in Incremental modified leaky LMS

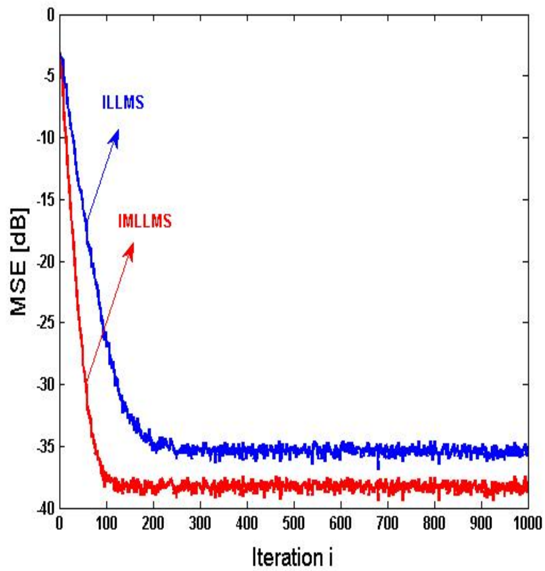

Fig. 3. MSE at node 1 for Incremental LLMS and Incremental MLLMS

Fig.2. shows the data flow and the weight updation in IMLLMS strategy in a distributed adaptive sensor network with N nodes. Incremental Modified LLMS algorithm can be depicted as in Table.1:

Table1.Algorithm for Incremental Modified LLMS Solution
Let $\psi_k^{(i)}$ denote a local estimate of w^0 at node k at time i Start with $w_{-1} = 0$ Let $\psi_0^{(i)} = w_{i-1}$ For nodes to $k = 1, 2, .., N$, repeat : Receive $\psi_{k-1}^{(i)}$ from previous node $\psi_k^{(i)} = (1 - \mu_k \alpha)\,\psi_{k-1}^{(i)} + 2\mu_k u_{k,i}^* \sinh\left(d_k(i) - u_{k,i}\psi_{k-1}^{(i)}\right)$, $k = 1, 2, .., N$ End $w_i = \psi_N^{(i)}$ Send w_i to node 1 End

IV. Simulation Results and Discussion

The computer simulations are provided by performing 300 independent experiments and averaging. ILLMS and IMLLMS are implemented and then compared in terms of convergence speed, MSE, MSD and EMSE. The input at each node is considered as shift structure in order to cope up with the realistic scenarios like correlated data. The regression vectors are filled up as below:
$u_{k,i} = col\,\{u_k(i), u_k(i-1), ...u_k(i-M+1)\}$

The measured data $d_k^{(i)}$ are generated at each node by using the regular data model as mentioned before and the desired $MX1$ vector to be estimated is set as

$$w^0 = col\,\{1, 1, ..., 1\}\Big/\sqrt{M}$$

where M is the tap size and taken as M=5. The quantities of interest are defined as below:

- EMSE (Excess Means quare error) = $\left|u_{k,i}\left(\psi_k^{(i)} - w^0\right)\right|^2$
- MSE (Mean square error) = $\left|d_k(i) - u_{k,i}\psi_{k-1}^{(i)}\right|^2$
- MSD (Mean square deviation) = $\left|\left(\psi_k^{(i)} - w^0\right)\right|^2$

A network of 20 nodes is considered in this experiment i.e. N=20 with each input regressor of size (1×5) . The input is created by a first order auto regressive model given as below:

$$u_k(i) = 0.2u_k(i-1) + \eta_{0k}(i)$$

Where η_{0k} is a WGN with mean zero and variance $\sigma_{\eta_0}^2 = 0.36$. The input signal is assumed to be corrupt with white Gaussian noise with zero mean and variance $\sigma_{v_0}^2 = 0.0001$. Step size taken as $\mu = 0.003$ and leakage factor considered as $\alpha = 0.01$. The learning curves for MSE, EMSE, MSD for IMLLMS and ILLMS at node 1 are shown in Fig.3, Fig.4, Fig.5.

V. Conclusion

A new algorithm has been proposed which improves the performance of ILLMS by implementing MLLMS in place of LLMS, which is obtained by slight modification of cost function of LLMS according to the LSE algorithm. Simulation results show that the IMLLMS algorithm outperforms the ILLMS in terms of convergence rate and the steady state performance in the presence of white Gaussian noise with a penalty of slight increase in computational complexity. The algorithms implemented in this paper can be implemented in

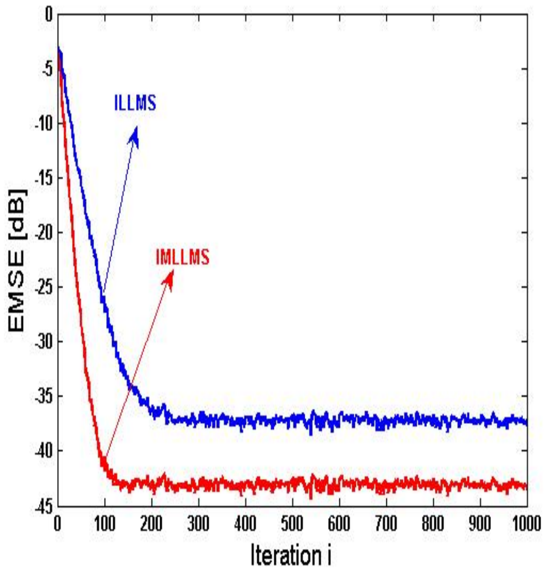

Fig. 4. EMSE at node 1 for Incremental LLMS and Incremental MLLMS

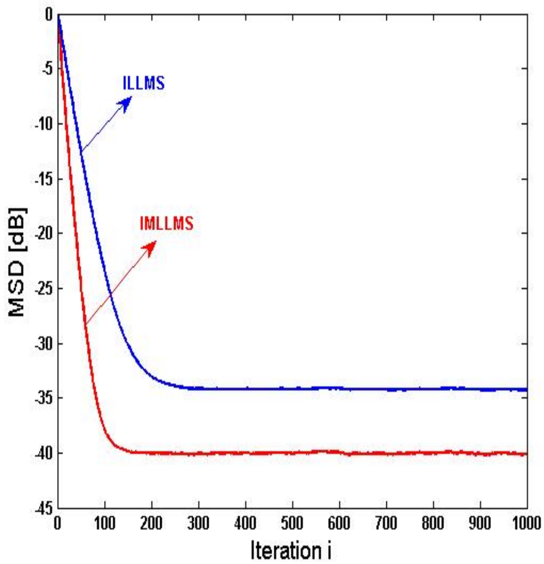

Fig. 5. MSD at node 1 for Incremental LLMS and Incremental MLLMS

other modes of cooperation like diffusion mode which could be the future work.

REFERENCES

[1] D. Estrin, L. Girod, G. Pottie, and M. Srivastava, "Instrumenting the world with wireless sensor networks," in *Acoustics, Speech, and Signal Processing, 2001. Proceedings.(ICASSP'01). 2001 IEEE International Conference on*, vol. 4. IEEE, 2001, pp. 2033–2036.

[2] L. A. Rossi, B. Krishnamachari, and C.-C. Kuo, "Distributed parameter estimation for monitoring diffusion phenomena using physical models," in *Sensor and Ad Hoc Communications and Networks, 2004. IEEE SECON 2004. 2004 First Annual IEEE Communications Society Conference on*. IEEE, 2004, pp. 460–469.

[3] C. G. Lopes and A. H. Sayed, "Incremental adaptive strategies over distributed networks," *IEEE Transactions on Signal Processing*, vol. 55, no. 8, pp. 4064–4077, 2007.

[4] R. Abdolee, B. Champagne, and A. H. Sayed, "A diffusion lms strategy for parameter estimation in noisy regressor applications," in *Signal Processing Conference (EUSIPCO), 2012 Proceedings of the 20th European*. IEEE, 2012, pp. 749–753.

[5] J. Cioffi, "Limited-precision effects in adaptive filtering," *IEEE Transactions on circuits and systems*, vol. 34, no. 7, pp. 821–833, 1987.

[6] M. Sowjanya, A. Sahoo, and S. Kumar, "Distributed incremental leaky lms," in *Communications and Signal Processing (ICCSP), 2015 International Conference on*. IEEE, 2015, pp. 1753–1757.

[7] A. H. Sayed, *Fundamentals of adaptive filtering*. John Wiley & Sons, 2003.

[8] W. Sethares, D. Lawrence, C. Johnson, and R. Bitmead, "Parameter drift in lms adaptive filters," *IEEE transactions on acoustics, speech, and signal processing*, vol. 34, no. 4, pp. 868–879, 1986.

[9] K. Mayyas and T. Aboulnasr, "Leaky lms algorithm: Mse analysis for gaussian data," *IEEE Transactions on Signal Processing*, vol. 45, no. 4, pp. 927–934, 1997.

[10] E. Walach and B. Widrow, "The least mean fourth (lmf) adaptive algorithm and its family," *IEEE Transactions on Information Theory*, vol. 30, no. 2, pp. 275–283, 1984.

[11] C. Boukis, D. Mandic, and A. Constantinides, "A generalised mixed norm stochastic gradient algorithm," in *2007 15th International Conference on Digital Signal Processing*. IEEE, 2007, pp. 27–30.

[12] J.-P. Montillet and K. Yu, "Modified leaky lms algorithms applied to satellite positioning," in *2014 IEEE 80th Vehicular Technology Conference (VTC2014-Fall)*. IEEE, 2014, pp. 1–5.

[13] M. Kamenetsky and B. Widrow, "A variable leaky lms adaptive algorithm," in *Signals, Systems and Computers, 2004. Conference Record of the Thirty-Eighth Asilomar Conference on*, vol. 1. IEEE, 2004, pp. 125–128.