Thin Coal Layer Thickness Estimation using MUSIC Algorithm

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***Abstract*— In coal mining, thickness of thin coal layer is measured for maintaining a defined coal mining horizon. Researchers working in the geotechnical field for detection and thickness measurement of near surface interface address challenges using recent development in radar signal processing. This paper addresses challenge in measuring thickness of thin coal layer left on mine haulage way roof for mine safety. Here, step frequency continuous wave ground penetrating radar (SFCW GPR) signal processing is given for measuring thickness of thin coal layer in presence of interfaces as coal-shale and coal-shale-clay. We use multiple signal classification (MUSIC) algorithm for detecting the interfaces of dissimilar material. In order to improve the resolving power, MUSIC with spatial smoothing process (SSP) and modified spatial smoothing process (MSSP) are applied. Experimental results on thickness measurement using synthetic data models, full wave model (FWM), plane wave model (PWM) and modified plane wave model (MPWM) are demonstrated to compare the effectiveness of estimation algorithms.**

Keywords—*Coal layer, interface, ground penetrating radar, multiple signal classification algorithm, resolution.*

I. INTRODUCTION

Ground penetrating radar (GPR) has found numerous applications for near surface interface detection in coal mining. A thin layer of low quality coal is typically found between the actual coal seam and surrounding layers. Mining anything other than coal lead to a reduction in productivity and consequential loss of profit due to contamination of coal into the roof or floor material [1], [2]. The most common overburden material in underground coal mine is shale and clay. Therefore, geological scenarios that can occur in an underground coal mine are the coal-shale, coal-clay and coal-shale-clay interfaces [3]. An existing growth of GPR signal processing is found to be promising for estimating the thickness of the thin coal seam [4].

In GPR signal processing, coal layer thickness is measured using echo detection and then time delay estimation (TDE). Calculation of TDE is based on information available in echoes from interface of layers with dissimilar characteristics. The technique to separate two echoes demands, either the improvement of time resolution, $\Delta\tau$ by signal processing [5]-[8] or broadening the GPR effective bandwidth, B [9], [10]. The purpose of the work in this paper is to develop an efficient signal processing technique to improve the time resolution of GPR signal (lowering the value of $\Delta\tau$), so that thickness of thin coal layer can be estimated.

A conventional signal processing based method such as fast Fourier transform (FFT) or cross correlation based methods (matched filter), can directly provide TDE for thickness measurement from the frequency domain observation vector by simple inverse FFT (IFFT). But limitation of IFFT is that it cannot separate the overlapped echoes [11]. Several signal processing algorithms such as Capon [12], MUSIC [13], Estimation of Signal Parameter via Rotational Invariance Technique (ESPRIT) [14] have been proposed over the years for improving time resolution. Capon's method is theoretically simple but it incurs inferior performance in terms of resolution and it fails if signals that are correlated with the signal of interest are present [15]. However, when only single source is to be recognized resolution edge is not of main concern but for more than one source, high resolution techniques (MUSIC, ESPRIT, etc.) may be used proficiently. The MUSIC algorithm used for TDE can process coherent signal of GPR by using smoothing processes or averaging techniques and also, have ability to separate overlapping echoes [8], [11]. In [8], MUSIC is applied to measure poly vinyl chloride (PVC) slab of thickness 1.52 cm using 2 GHz GPR and relative root mean square error (RRMSE) is found to be less than or equal to 5%. To the best of our knowledge, there is no off-the-shelf MUSIC algorithm for such thin layer of coal measurement.

In literature, techniques available for coal seam thickness measurement are pattern recognition method [3], Passive Gamma Ray instruments, electromagnetic techniques and radar sensor [16]-[18]. However, the minimum value of coal seam thickness measured by low power 1.4 GHz impulse GPR is 5 cm [3]. Therefore, the purpose of this paper is to examine the GPR performance to measure thin coal seam thickness in different geological scenarios of coal mines. In this paper, we use MUSIC algorithm with smoothing process for TDE and claim ability of GPR to measure thin coal seam. The permittivity and conductivity of coal are assumed to be known.

This paper is organized as follows. Section II gives the details of GPR signal model used for signal processing involved in high resolution algorithm for TDE. Section III explain different geological scenarios in coal mining and also give the methods for synthetic data generation. In section IV experimental results are discussed elaborately. Finally conclusion is made in section V.

II. HIGH RESOLUTION TDE (MUSIC) ALGORITHM

The subsurface layer of coal mining is supposed to be the parallel stratified medium with K layer from which, K

backscattered echoes are reflected. The amplitude of the echo depends on the dielectric dissimilarity between interfaces through the Fresnel coefficients [5]-[8].

A. Signal Model for Time Delay Estimation (TDE)

The received signal from K subsurface layer depends on the reflection coefficients observed from a reference plane at, $f_i\ (i = 1,2,\ldots,N : N > K)$ uniformly sampled N frequency points. Here the time delay corresponding to k^{th} reflection point is $\tau_k\ (k = 1,2,\ldots,K)$ and the measured value of received signal corresponding to the operating frequency, f_i is given by [11]

$$y_i = \sum_{k=1}^{K} s_{ik}.\exp(-j2\pi f_i \tau_k) + n_i \tag{1}$$

where $s_{ik}\ (i = 1,2,\ldots,N, k = 1,2,\ldots,K)$ is the complex reflection coefficient of the k^{th} reflection point at frequency, f_i and n_i is additive white Gaussian noise (AWGN) with mean zero and variance σ^2. The reflection coefficients are generally a continuous function of frequency. The narrower the measurement bandwidth $(f_N - f_1)$ is, the smaller the change of reflection coefficient is in the band. Here, let us assume that the measurement bandwidth is narrow. Then, s_{ik} does not rely on f_i, and we may express s_{ik} as s_k.

From (1) the received signal for N discrete frequency over bandwidth, B can be given in vector notation as follows:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{s} + \boldsymbol{n} \tag{2}$$

where

$$\boldsymbol{y} = [y_1\ y_2 \cdots y_N]^T \tag{3}$$

$$\boldsymbol{s} = [s_1\ s_2 \cdots s_K]^T \tag{4}$$

$$\boldsymbol{n} = [n_1\ n_2 \cdots n_N]^T \tag{5}$$

$$\boldsymbol{A} = [a(\tau_1)\ a(\tau_2) \cdots a(\tau_K)] \tag{6}$$

$$\boldsymbol{a}(\tau_k) = [\exp(-j2\pi f_1 \tau_k) \cdots \exp(-j2\pi f_N \tau_k)]^T \tag{7}$$

Here T denotes the transpose, $\boldsymbol{A}$ is a $N \times K$ delay parameter matrix and $\boldsymbol{a}(\tau_k)$ is the mode vector of each signal.

B. MUSIC Algorithm

The high resolution algorithm, MUSIC for TDE uses the eigen structure of the received radar data (snapshots) correlation matrix, which can be given using (2) as

$$\begin{aligned} \boldsymbol{R} &= E[\boldsymbol{y}\boldsymbol{y}^H] \\ &= \boldsymbol{A}\boldsymbol{S}\boldsymbol{A}^H + \sigma^2 I \end{aligned} \tag{8}$$

where H denotes the complex conjugate transpose and $E[\cdot]$ denotes an ensemble average. $\boldsymbol{S} = E[\boldsymbol{s}\boldsymbol{s}^H]$ and $E[\boldsymbol{n}\boldsymbol{n}^H] = \sigma^2 I$ are the signal and noise correlation matrices respectively. If ensembling is done over P number of snapshots, $\boldsymbol{R}$ can be obtained as

$$\boldsymbol{R} = \frac{1}{P}\sum_{p=1}^{P} \boldsymbol{y}^{(p)} \boldsymbol{y}^{(P)H} \tag{9}$$

where $\boldsymbol{y}^{(p)}\ (p = 1,2,\cdots P)$ is the data vector at p^{th} snapshot.

At low signal to noise ratio (SNR), more number of snapshots are needed to achieve good time resolution and delay estimates, whereas at high SNR, around 100 snapshots are enough for an acceptable performance in terms of resolution [19]. Therefore, implementation of MUSIC algorithm needs either more data or higher SNR to resolve closely spaced signals.

This algorithm defines a cost function, $\boldsymbol{a}^H(\tau)\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{a}(\tau)$, where $\boldsymbol{E}_n = [e_{K+1}\ e_{K+2} \cdots e_N]$ are the noise eigen vectors which, measures the orthogonality between noise subspace and signal vector [13]. The distribution of eigen value allows differentiating between noise and the signal subspace if the full rank of data correlation matrix is present and its size is larger than the number of echoes [13]. The K eigen vectors, accompanying with K maximum eigen values of $\boldsymbol{R}^p$ and N-K lowest eigen values, generate signals and noise subspace respectively, which are perpendicular to each other because of the Hermitian property of the matrix. Then we can calculate time delay of each reflection point by searching the peak position of the following function [11]

$$P_{music}(\tau) = \frac{\boldsymbol{a}^H(\tau)\boldsymbol{a}(\tau)}{\boldsymbol{a}^H(\tau)\boldsymbol{E}_n\boldsymbol{E}_n^H\boldsymbol{a}(\tau)} \tag{10}$$

Above relation in (10) holds when the signal, $\boldsymbol{s}$ is non-singular. However, the signals are coherent in measurement with network analyser (NA). Therefore, to reduce the signal coherence, we employ pre-processing for de-correlation. Here we examine two techniques known as spatial smoothing process (SSP) and modified spatial smoothing process (MSSP) to reduce the correlation between the echoes. Basically both the SSP and MSSP involve averaging technique. The SSP and MSSP techniques differ in their methodology adopted for averaging as discussed next.

C. SSP and MSSP Techniques

Correlation is present if the rank of source correlation matrix is less than the number of echoes, that deteriorate the performance of MUSIC algorithm [8]. In this process the total frequency bandwidth of L data has been arranged into M overlapping sub-bands with N data in each band. We regard $\boldsymbol{y}$ in (2) as the data vector corresponds to the first sub-band. The range of N, L and M are related to each other for the maximum overlapping ratio between sub-bands by following expressions:

$$L = N + M - 1 \tag{11}$$

The averaging requires the averaging of M correlation matrices calculated from each sub-band of N data. Its application is called SSP if it is in direct order and MSSP if data are averaged in forward and reverse order both. Now the size of the final correlation matrix after smoothing will reduce to $N \times N$. The reduction in correlation matrix depends on the number of sub-bands, M. As M increases, N decreases, as does the resolution power. So the balance between correlation magnitude and resolution power has to be set. The data correlation matrix for SSP and MSSP is given as [11]

$$\boldsymbol{R}_{SSP} = \frac{1}{M} \cdot \sum_{N_1=1}^{M} \boldsymbol{R}_{N_l}^{p} \quad (12)$$

and

$$\boldsymbol{R}_{MSSP} = \frac{1}{2M} \cdot \sum_{N_1=1}^{M} \left(\boldsymbol{R}_{N_1}^{p} + J\boldsymbol{R}_{N_1}^{p*} J \right) \quad (13)$$

where $\boldsymbol{R}_{N_1}^{p}$ is the data correlation matrix at N_1^{th} sub-band of p^{th} snapshot, asterisk denotes the complex conjugate and J is the $N \times N$ exchange matrix given by

$$J = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ \vdots & \cdots & 1 & 0 \\ 0 & 1 & \iddots & \vdots \\ 1 & 0 & \cdots & 0 \end{bmatrix} \quad (14)$$

The data pre-processed by SSP and MSSP promises to hold good in the relation given by (10) which gives TDE of dissimilar layer interfaces for thickness measurement. However, the effectiveness of above thickness measurement technique needs to use an accurate signal model which is still developing for portraying multi-layer GPR signal. Therefore, in this work the thin coal layer thickness is measured by MUSIC with SSP and MUSIC with MSSP using signal models known as full wave model (FWM) [20], plane wave model (PWM) [21] and modified plane wave model [22].

III. SYNTHATIC DATA GENERATION

In GPR signal processing, the modelling assumptions and synthetic data generation for FWM, PWM and MPWM are discussed here. The model configurations for multi-layer media are also given.

A. Modelling Assumptions

We consider a monostatic step frequency continuous wave ground penetrating radar (SFCW GPR) combined with a TEM horn antenna is setup using a vector network analyzer (VNA), configured in off-ground condition. In far field measurement, the antenna is expected to be point source and receiver is positioned in its phase centre. The signal is assumed to be transmitting in vertical, i.e., z direction [20]. The antenna, VNA and subsurface are modeled as linear transfer function (LTF) model as shown in Fig. 1, representing the antenna behaviour, which accounts the effect of gain and delay due to frequency dependent antenna face central location through the calibration process [23].

The complex reflection coefficient $S_{11}(\omega)$ from Fig. 1 is given by

$$S_{11}(\omega) = \frac{Y(\omega)}{X(\omega)} = H_i(\omega) + \frac{H_t(\omega) G_{xx}^{\uparrow}(\omega) H_r(\omega)}{1 - H_f(\omega) G_{xx}^{\uparrow}(\omega)} \quad (15)$$

where $X(\omega)$ and $Y(\omega)$ are respectively the signals transmitted and received at reference plane of VNA, $\omega = 2\pi f$ and f is an operational frequency in it. $H_i(\omega), H_r(\omega), H_t(\omega)$ and $H_f(\omega)$ are transfer functions corresponding to return loss, receive power, transmit power and feedback loss respectively of the antenna. $G_{xx}^{\uparrow}(\omega)$ is the air-subsurface system transfer function, modelled as multi-layered medium. All these frequency dependent transfer functions are possible to estimate by suitable calibration process [23].

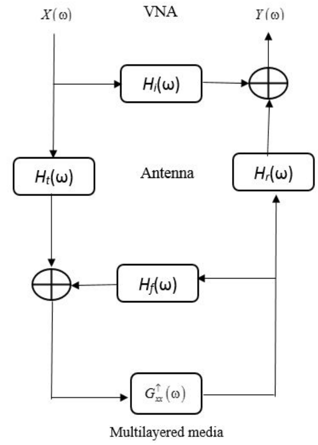

Fig. 1 Block diagram representing the VNA-antenna-multi-layered medium system

B. Data Generation

According to the condition of geological scenario present in underground coal mine, two different model configurations are used for data generation i.e., air-coal-shale and air-coal-shale-clay case as shown in Fig. 2(a) and 2(b) respectively, where Tx/Rx (Transmitter/Receiver) is the VNA with monostatic antenna. Coal, shale and clay layer permittivity and conductivity are taken as $\varepsilon_1 = 4.4 - j0.26$, $\sigma_1 = 20.5 mS/m$, $\varepsilon_2 = 9.1 - j2.1$, $\sigma_2 = 154.6 mS/m$, $\varepsilon_3 = 24.9 - j6.9$ and $\sigma_2 = 517.6 mS/m$ respectively [3]. Permeability of the layer with height h_1, h_2, h_3 and h_4 respectively are μ_1, μ_2, μ_3 and μ_4 which can be assumed equal and constant free space value, μ_0 for nonmagnetic materials, which mostly occurs in the earth surface.

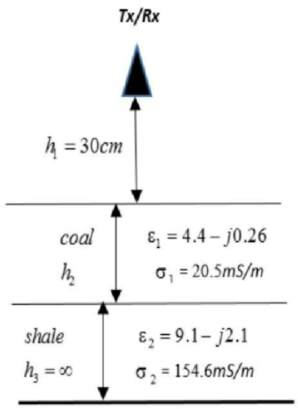

Fig. 2(a) Model configuration for three layer media

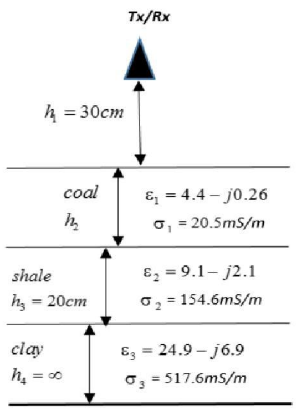

Fig. 2(b) Model configuration for four layer media

Synthetic data are generated by three different GPR signal modelling schemes named by FWM, PWM and MPWM for the signal, $\boldsymbol{y}$ given in (2). Here we assume that, any k^{th} layer is homogeneous and the dispersive nature of the materials (coal, shale and clay) are described by Debye relaxation equation as [24]

$$\varepsilon_e(f) = \varepsilon_{e,\infty} + \frac{\varepsilon_{e,0} - \varepsilon_{e,\infty}}{1 + j\frac{f}{f_r}} \quad (16)$$

1) Full Wave Model (FWM): FWM of GPR signal is a promising approach for accurate characterization of multi-layer media. In this, the Green's function due to multi-layered media is determined by the 3D-Maxwell's equations. In the area of off-ground monostatic SFCW GPR, multi-layer media characterization has done with a FWM scheme by using LTF model [20].

The spatial domain Green's function in monostatic mode at source point ((x, y, z) = 0) can be found from spectral domain Green's function given as [20]

$$G_{xx}^{\uparrow FWM}(\omega) = \frac{1}{4\pi}\int_0^{+\infty} \tilde{G}_{xx}^{\uparrow}(\kappa_\rho, \omega)\kappa_\rho d\kappa_\rho \quad (17)$$

where the spectral domain Green's function involving the spectral domain parameter, κ_ρ is given as

$$\tilde{G}_{xx}^{\uparrow}(\kappa_\rho, \omega) = \left[Q_k^{TM}\frac{\Psi_k}{\eta_k} - Q_k^{TE}\frac{\zeta_k}{\Psi_k}\right]e^{-2\Psi_k h_k} \quad (18)$$

The transverse magnetic global reflection coefficient, Q_k^{TM} and transverse electric global reflection coefficient, Q_k^{TE} considering for whole reflection from the multi-layered boundaries. Here $\Psi_k \left(= \sqrt{\kappa_\rho^2 - \kappa_k^2}\right)$ is the vertical wavenumber of k^{th} layer and κ_k is the free space propogation constant, where $\kappa_k^2 = -\xi_k\eta_k$, $\xi_k = j\omega\mu_k$ and $\eta_k = \sigma_k + j\omega\varepsilon_k$.

2) Plane Wave Model (PWM): Implementation of FWM makes it difficult for real time applications because of large computation time. To overcome this problem PWM of GPR signal based on common reflection method has been suggested [21]. Formulation of PWM is based on the simplified expression for spectral domain Green's function, by assuming TEM horn antenna as an infinitesimal x-directed electric diople, is given by [21]

$$\tilde{G}_{xx}^{\uparrow}(\kappa_\rho, \omega) = \left[\Gamma_k^{TE} - \Gamma_k^{TM}\right]e^{-2\Psi_k h_k} \quad (19)$$

For three layer media, an m^{th} order reflections from 2^{nd} layer interface by PWM can be found as follows,

$$Q_2^m = \frac{\hat{r}_{2,3}^1}{2\pi i}\left(r_{2,1}r_{2,3}\right)^{(m-1)} e^{-(m-1)2\gamma_2 h_2}\left\{\frac{1}{2\left(\frac{h_1}{\gamma_1} + m\frac{h_2}{\gamma_2}\right)} + \frac{\left(\frac{h_1}{\gamma_1^3} + \frac{h_2}{\gamma_2^3}\right)}{4\left(\frac{h_1}{\gamma_1} + m\frac{h_2}{\gamma_2}\right)^3}\right\} \quad (20)$$

Here reflection coefficient $\left(r_{k,k+1}\right)$ for plane wave propagation at k^{th} layer interface is given by,

$$r_{k,k+1} = \frac{Z_{k+1} - Z_k}{Z_{k+1} + Z_k} \quad (21)$$

where Z_k represents the impedance of k^{th} layered media and it is given by

$$Z_k = \sqrt{\frac{\zeta_k}{\eta_k}} = \sqrt{\frac{i\omega\mu_k}{\sigma_k + i\omega\varepsilon_k}} \quad (22)$$

For k^{th} layer media propagation parameter γ_k is represented as

$$\gamma_k = \alpha_k + i\beta_k = \sqrt{i\omega\mu_k\left(\sigma_k + i\omega\varepsilon_k\right)} = \sqrt{\zeta_k\eta_k} = i\kappa_k \quad (23)$$

where α_k and β_k are k^{th} layer media attenuation and phase constant respectively. The first order reflection from k^{th} layer interface is given by

$$\left(\hat{r}_{k,k+1}^1\right) = r_{k,k+1}\prod_{j=1}^{k-1}\left(1 - \left(r_{j,j+1}\right)^2\right)\prod_{j=1}^{k}\exp\left(-2\gamma_j h_j\right) \quad (24)$$

For K-layered media, the overall Green's function with highest reflection of order N_0 can be written as [21],

$$G_{xx}^{\uparrow PWM}(\omega) = \sum_{b=1}^{N_0}\sum_{q=1}^{K-1} Q_{q}^b \quad (25)$$

3) Modefied Plane Wave Model (MPWM): It is an extension of PWM which accounts higher order reflections from multi-layered media, espetially when the number of layers are more than three [22]. The m^{th} order reflections from k^{th} layer interface is given by a generalized relationship as

$$Q_k^m = \sum_{a=1}^{F_k^m} L_{R_k}^{m,a} L_{S_k}^{m,a}\frac{\hat{r}_{2,3}^1}{2\pi i}\exp\left(\sum_{j=1}^{k} -2\gamma_j b_j h_j\right) \quad (26)$$

where

$$L_{R_k}^{m,a} = r_{k,k+1}\prod_{j=1}^{k-1}\left(1 - \left(r_{j,j+1}\right)^2\right) \quad (27)$$

and

$$L_{S_k}^{m,a} = (1/2\pi i)\left(1\Big/\left(2\sum_{j=1}^{k} b_j h_j/\gamma_j\right)\right) + (1/2\pi i)\left(\left(\sum_{j=1}^{k} b_j h_j/\gamma_j^3\right)\Big/4\left(\sum_{j=1}^{k} b_j h_j/\gamma_j^3\right)^3\right) \quad (28)$$

Here $L_{R_k}^{m,a}$ indicates the losses due to reflections and refractions at different interfaces, $L_{S_k}^{m,a}$ shows the spreading loss for travelling path and b_j is defined as a positive integer constant whose value is more than one, and are showing the following inequality:

$$k + m - 1 \le \sum_{j=1}^{k} b_j \le (k-1)m + 1 \quad (29)$$

F_k^m is the total number of possible path in which inequality (28) satisfied for m^{th} order reflection from k^{th} interface. The overall Green's function with highest reflection of order N_0 from K-layered media is represented as

$$G_{xx}^{\uparrow MPWM}(\omega) = \sum_{b=1}^{N_0}\sum_{q=1}^{K-1} Q_q^b \quad (30)$$

The Green's functions given in (17), (25) and (30) respectively are correspond FWM, PWM and MPWM. Therefore, measured value of received signal, y_i given in (1) can be generated using (17), (25) and (30) for obtaining GPR signal for FWM, PWM and MPWM respectively. Before using the above Green's functions in (10) pre-processing by SSP and MSSP is done. Then the obtained $P_{music}(\tau)$ is used for measuring thickness of coal layer in geological conditions as coal-shale and coal-shale-clay. Then effectiveness in measurement is elaborate discussed in the next section.

IV. Results And Discussion

Experimental results on thin coal layer thickness measurement using MUSIC with SSP and MUSIC with MSSP are presented here. The synthetic data is generated for monostatic GPR where data set comprises 251 equispaced frequency points for B = 2 GHz. In our experiment, thickness of coal layer is varied from 2 cm to 20 cm with step size 0.5 cm and are considered as true thickness of coal layers.

In order to detect the interfaces of layers in air-coal-shale and air-coal-shale-clay, the signal subspace dimension is set as the exact number of expected echoes, i.e., K=2 and K=3 respectively for the above geological scenarios. Fig. 3(a) and Fig 3(b) are showing the peaks corresponds to 3.5 cm coal seam for four layered media by FWM and PWM respectively and their respective time measurement gives the layer thickness estimate of coal seam true thickness 3.5 cm.

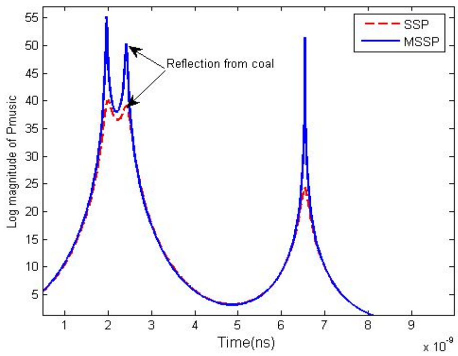

Fig. 3(a) Peaks resolve through MUSIC algorithm for 3.5 cm coal seam by FWM (four layer media)

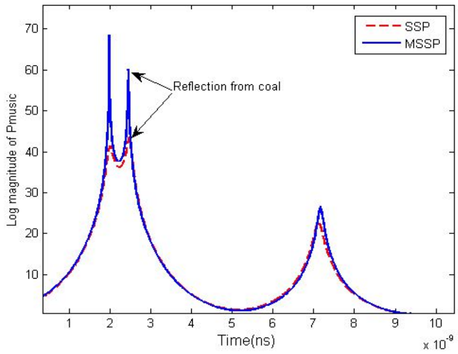

Fig. 3(b) Peaks resolve through MUSIC algorithm for 3.5 cm coal seam by PWM (four layer media)

There are various causes of error in TDE and few of those can be given as follows. In practice, we compute ***R*** using a finite number of snapshots for ensembling, measured data contains uncertainties because measurement system is not completely perfect and also dependency of reflection coefficient, ***s*** on frequency is still there if required bandwidth is not narrow.

In order to assess the performance of MUSIC with SSP and MUSIC with MSSP algorithms in estimating coal layer thickness, the relative root mean square error (RRMSE) in estimation is given by

$$RRMSE(\%) = 100 \times \frac{\sqrt{\frac{1}{U}\sum_{i=1}^{U}\left(\hat{h}_{2i} - h_2\right)^2}}{h_2} \tag{31}$$

Where, U = number of independent trials, $\hat{h}_2$ = estimated thickness of coal layer and h_2 = true thickness of coal layer

TABLE I
ASSESSMENT OF MODELS IN TERMS OF RELATIVE BIAS IN % RRMSE, ESTIMATED COAL SEAM THICKNESS, $\hat{h}_2$ AND TIMING EFFICIENCY FOR $N_0 = 10$

Modell-ing schemes	h_2 (cm)	$\hat{h}_2$ (cm)		Relative bias (% RRMSE)		Time needed to compute single Green's function (sec.)
		SSP	MSSP	SSP	MSSP	
FWM (3 layer)	3	3.1	3.1	3.33	3.33	124.9
FWM (4 layer)	3.5	3.520	3.517	0.57	0.49	232.3
PWM (3 layer)	3	3.109	3.1	3.63	3.33	0.2615
MPWM (4 layer)	3.5	3.523	3.524	0.65	0.69	0.491

The estimated thickness values are presented in Table-I which also includes the information regarding time required to compute Green's function by 3.4 GHz core i7 computer. A performance is considered to be reasonably acceptable whenever RRMSE lies below 5%. The minimal value of thickness measured by three layered and four layered media are respectively 3 cm and 3.5 cm by both PWM and FWM corresponds to RRMSE $\le$ 5%. There is no prior information to the estimator about typical thickness of coal layer. The results demonstrated in Table-I give overall assessment on estimation accuracy which depends on signal model used for GPR data set.

The effectiveness of estimation by MUSIC with SSP and MUSIC with MSSP is presented for varying true thickness value for 2 cm to 20 cm in Fig. 4(a). The respective %RRMSE in estimation of thickness is shown in Fig. 4(b).

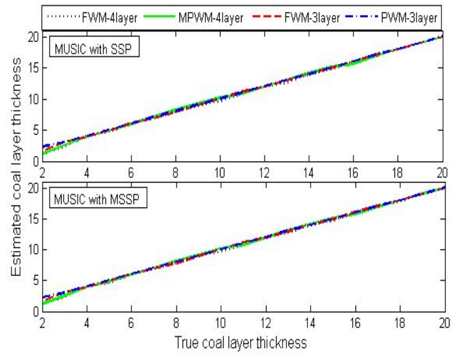

Fig. 4(a) True versus estimated layer thickness by MUSIC with SSP and MUSIC with MSSP

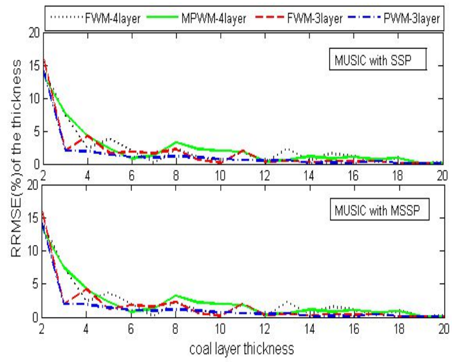

Fig. 4(b) RRMSE variation with estimated thickness by MUSIC with SSP and MUSIC with MSSP

V. CONCLUSION

In this paper, an experimental analysis is done on thin layer coal seam thickness measurement by MUSIC with SSP and MUSIC with MSSP. The synthetic data is generated using recently developed Green's functions available in FWM, PWM and MPWM for finding their applicability in above estimation algorithms and also %RRMSE in estimation. The %RRMSE corresponds to minimum value of thickness estimated by all the modelling schemes are found to be almost similar for both three layer and four layer media respectively. However, PWM and MPWM have significantly less computational burden than FWM.

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