New Exact Solutions of Fractional Davey-Stewartson Equation with Power-Law Nonlinearity and new Integrable Davey-Stewartson Type Equation

S. Saha Ray
National Institute of Technology
Department of Mathematics
Rourkela-769008, India
Email: santanusaharay@yahoo.com

Abstract

In this article, the Jacobi elliptic function method viz. mixed dn-sn method has been presented for finding the travelling wave solutions of the Davey-Stewartson Equations. As a result, some solitary wave solutions and doubly periodic solutions are obtained in terms of Jacobi elliptic functions. Moreover, solitary wave solutions are obtained as simple limits of doubly periodic functions. These solutions can be useful to explain some physical phenomena. The proposed Jacobi elliptic function method is efficient, powerful and can be used in order to establish more newly exact solutions for other kinds of nonlinear fractional partial differential equations arising in mathematical physics.

Key words: mixed dn-sn method; fractional complex transform; fractional Davey-Stewartson equation; local fractional calculus; Jacobi elliptic function.

1. Introduction

In this paper, we present the travelling wave solutions of the fractional (2+1)-dimensional Davey-Stewartson equation and doubly periodic solutions of new integrable Davey-Stewartson type equation. We employ the mixed dn-sn method [1] approach via fractional complex transform in order to obtain exact solutions to the fractional (2+1)-dimensional Davey-Stewartson equation and new integrable Davey-Stewartson type equation. Davey-Stewartson (DS) equations have been used for various applications in fluid dynamics. Davey-Stewartson equations were proposed initially for the evolution of weakly nonlinear pockets of water waves in the finite depth by Davey and Stewartson [2].

2. Algorithm of the mixed dn-sn method with fractional complex transform

In this present analysis, we deal with the determination of explicit solutions of fractional (2+1)-dimensional Davey-Stewartson equation by using the mixed dn-sn method. The main steps of this method are described as follows:

Step 1: Suppose that coupled nonlinear FPDEs, say in three independent variables x, y and t is given by

$$F(u,v,u_x,v_x,u_y,v_y,u_t,v_t,iD_t^{\alpha}u,D_t^{\alpha}v,D_x^{2\beta}u,D_x^{2\beta}v,D_y^{2\gamma}u,D_y^{2\gamma}v,...)=0\,,\ 0<\alpha,\beta,\gamma\le 1 \quad (2.1a)$$

$$G(u,v,u_x,v_x,u_y,v_y,u_t,v_t,D_t^{\alpha}u,D_t^{\alpha}v,D_x^{2\beta}u,D_x^{2\beta}v,D_y^{2\gamma}u,D_y^{2\gamma}v,...)=0\,,\ 0<\alpha,\beta,\gamma\le 1 \quad (2.1b)$$

where $u=u(x,y,t)$ and $v=v(x,y,t)$ are unknown functions, F and G are polynomials in u, v and its various partial derivatives in which the highest order derivatives and nonlinear terms are involved.

Step 2: We use the fractional complex transform [3-6]:

$$u(x,y,t)=e^{i\theta}u(\xi)\,,\quad v(x,y,t)=v(\xi)\,,$$

$$\theta=\frac{\theta_1 x^{\beta}}{\Gamma(1+\beta)}+\frac{\theta_2 y^{\gamma}}{\Gamma(1+\gamma)}+\frac{\theta_3 t^{\alpha}}{\Gamma(1+\alpha)} \text{ and } \xi=\frac{\xi_1 x^{\beta}}{\Gamma(1+\beta)}+\frac{\xi_2 y^{\gamma}}{\Gamma(1+\gamma)}+\frac{\xi_3 t^{\alpha}}{\Gamma(1+\alpha)} \quad (2.2)$$

where θ_1, θ_2, θ_3, ξ_1, ξ_2 and ξ_3 are real constants to be determined later.

By using the chain rule in [3, 6], we have

$$D_t^{\alpha}u=\sigma_t u_{\xi} D_t^{\alpha}\xi\,,$$
$$D_x^{\alpha}u=\sigma_x u_{\xi} D_x^{\alpha}\xi\,,$$
$$D_y^{\alpha}u=\sigma_y u_{\xi} D_y^{\alpha}\xi\,,$$

where σ_t, σ_x and σ_y are the fractal indexes [5, 6], without loss of generality we can take $\sigma_t = \sigma_x = \sigma_y = \kappa$, where κ is a constant.

Using fractional complex transform eq. (2.2), the FPDE (2.1) can be converted to couple nonlinear ordinary differential equations (ODEs) involving $\Phi(\xi) = u(x, y, t)$ and $\Psi(\xi) = v(x, y, t)$. Then eliminating $\Psi(\xi)$ between the resultant coupled ODEs, the following ODE for $\Phi(\xi)$ is obtained

$$F(\Phi, \theta_3\Phi', \theta_3^2\Phi'', \theta_3^3\Phi''', \xi_3\Phi', \xi_3^2\Phi'', \xi_3^3\Phi, \ldots) = 0, \tag{2.3}$$

where prime denotes the derivative with respect to ξ.

Step 3:

Let us assume that the exact solution of eq. (2.3) is to be defined in the polynomial $\phi(\xi)$ of the following form:

$$\Phi(\xi) = \sum_{i=0}^{N} c_i \phi^i(\xi) + \sqrt{k^2 - \phi^2(\xi)} \sum_{i=0}^{N-1} d_i \phi^i(\xi), \tag{2.4}$$

where $\phi(\xi)$ satisfies the following elliptic equation:

$$\phi_\xi = \sqrt{(k^2 - \phi^2)(\phi^2 - k^2(1-m))}. \tag{2.5}$$

The solutions of eq. (2.5) are given by

$$\phi(\xi) = k dn(k\xi | m),$$

$$\phi(\xi) = k\sqrt{1-m}\, nd(k\xi | m), \tag{2.6}$$

where $dn(k\xi|m)$ and $nd(k\xi|m) = \dfrac{1}{dn(k\xi|m)}$ are the Jacobi elliptic functions with modulus m $(0 < m < 1)$.

Step 4:

According to the proposed method, we substitute $\Phi(\xi) = \xi^{-p}$ in all terms of eq. (2.3) for determining the highest order singularity. Then the degree of all terms of eq. (2.3) has been taken into study and consequently the two or more terms of lower degree are chosen. The maximum value of p is known as the pole and it is denoted as " N ". If " N " is an integer then the method only can be implemented and otherwise if " N " is a non-integer, the above eq. (2.3) may be transferred and the above procedure is to be repeated.

Step 5:

Substituting eq. (2.4) into eq. (2.3) yields the following algebraic equation

$$P(\phi) + \sqrt{k^2 - \phi^2}\, Q(\phi) = 0 \tag{2.7}$$

where $P(\phi)$ and $Q(\phi)$ are the polynomials in $\phi(\xi)$. Setting the coefficients of the various powers of ϕ in $P(\phi)$ and $Q(\phi)$ to zero will yield a system of algebraic equations in the unknowns c_i, d_i, k and m. Solving this system, we can determine the value of these unknowns. Therefore, we can obtain several classes of exact solutions involving the Jacobi elliptic functions *sn*, *dn*, *nd* and *cd* functions.

3. Implementation of the Jacobi elliptic function method

In this section, the new exact analytical solutions of fractional (2+1)-dimensional Davey-Stewartson equation and new integrable Davey-Stewartson type equation have been obtained using the mixed dn-sn method.

3.1 Exact solutions of fractional (2+1)-dimensional Davey-Stewartson equation

Let us consider the fractional (2+1)-dimensional Davey-Stewartson equation [7]

$$iD_t^\alpha q + a(D_x^{2\beta} q + D_y^{2\gamma} q) + b|q|^{2n} q - \lambda q r = 0, \tag{3.1}$$

$$D_x^{2\beta} r + D_y^{2\gamma} r + \delta D_x^{2\beta}\left(|q|^{2n}\right) = 0, \tag{3.2}$$

where $0 < \alpha, \beta, \gamma \le 1$, $q \equiv q(x, y, t)$ and $r \equiv r(x, y, t)$. Also a, b, λ and δ are all constant coefficients. The exponent n is the power law parameter. It is necessary to have $n > 0$. In eqs. (3.1) and (3.2), $q(x, y, t)$ is a complex valued function which stands for wave-amplitude, while $r(x, y, t)$ is a real valued function which stands for mean-flow. This system of equations is completely integrable and is often used to describe the long-time evolution of a two-dimensional wave packet [8-10].

We first transform the fractional (2+1)-dimensional Davey-Stewartson equations (3.1)-(3.2) to a system of nonlinear ordinary differential equations in order to derive its exact solutions.
By applying the following fractional complex transform

$$q(x,y,t)=e^{i\theta}u(\xi)\,,\quad r(x,y,t)=v(\xi)\,,$$

$$\theta=\frac{\theta_1 x^{\beta}}{\Gamma(1+\beta)}+\frac{\theta_2 y^{\gamma}}{\Gamma(1+\gamma)}+\frac{\theta_3 t^{\alpha}}{\Gamma(1+\alpha)} \text{ and } \xi=\frac{\xi_1 x^{\beta}}{\Gamma(1+\beta)}+\frac{\xi_2 y^{\gamma}}{\Gamma(1+\gamma)}+\frac{\xi_3 t^{\alpha}}{\Gamma(1+\alpha)}\,,$$

eqs. (3.1) and (3.2) can be reduced to the following couple nonlinear ODEs:

$$-(\theta_3+a\theta_1^2+a\theta_2^2)u+(a\xi_1^2+a\xi_2^2)u_{\xi\xi}+bu^{2n+1}-\lambda uv=0\,, \tag{3.3}$$

$$\xi_1^2 v_{\xi\xi}+\xi_2^2 v_{\xi\xi}+\delta\xi_1^2\left(u^{2n}\right)_{\xi\xi}=0\,, \tag{3.4}$$

where ξ_3 has been set to $-2a\xi_1\theta_1-2a\xi_2\theta_2$. Eq. (3.4) is then integrated term by term twice with respect to ξ where integration constants are considered zero. Thus we obtain

$$v=-\frac{\delta\xi_1^2 u^{2n}}{\xi_1^2+\xi_2^2}\,. \tag{3.5}$$

Substituting eq. (3.5) into eq. (3.3) yields

$$-(\theta_3+a\theta_1^2+a\theta_2^2)u+(a\xi_1^2+a\xi_2^2)u_{\xi\xi}+bu^{2n+1}+\lambda\frac{\delta\xi_1^2 u^{2n+1}}{\xi_1^2+\xi_2^2}=0\,. \tag{3.6}$$

Using the transformation

$$u(\xi)=\Phi^{\frac{1}{n}}(\xi)\,,$$

eq. (3.6) further reduces to

$$-(\theta_3+a\theta_1^2+a\theta_2^2)n^2\Phi^2+(a\xi_1^2+a\xi_2^2)(1-n)\Phi_{\xi}^2++(a\xi_1^2+a\xi_2^2)n\Phi_{\xi\xi}+bn^2\Phi^4+\lambda\frac{\delta\xi_1^2 n^2\Phi^4}{\xi_1^2+\xi_2^2}=0 \tag{3.7}$$

By balancing the terms $\Phi\Phi_{\xi\xi}$ and Φ^4 in eq. (3.7), the value of N can be determined, which is $N=1$ in this problem.

Therefore the solution of eq. (3.7) can be written in the following ansatz as

$$\Phi(\xi)=c_0+c_1\phi(\xi)+d_0\sqrt{k^2-\phi^2(\xi)}\,, \tag{3.8}$$

where c_0 , c_1 and d_0 are constants to be determined later and $\phi(\xi)$ satisfies eq. (2.5).

Now substituting eq. (3.8) alongwith eq. (2.5) into eq. (3.7) and then equating each coefficients of $\phi^i(\xi)$, $i=0,1,2,...$ to zero, we can get a set of algebraic equations for c_0, c_1, d_0, θ_3 and m and solving the algebraic equations, we have the set of coefficients for the non-trivial solutions of eq. (3.7) as given below:

Case 1:

$$c_0=0\,,\ c_1=-\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}}\,,\ d_0=0\,,\ m=1\,,\ \theta_3=-\frac{a(n^2\theta_1^2+n^2\theta_2^2-k^2\xi_1^2-k^2\xi_2^2)}{n^2}\,, \tag{3.9}$$

where $\xi_3=-2a\xi_1\theta_1-2a\xi_2\theta_2$ and k is the free parameter.

Substituting eqs.(3.9) into eq.(2.4) and using the special solutions (2.6) of eq. (2.5), we obtain

$$\Phi(\xi)=-\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)k\sec h(k\xi)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}}$$

which yields the following solitary wave solutions of eqs. (3.1) and (3.2):

$$u(x,y,t)=\Phi(\xi)^{\frac{1}{n}}=\left(-\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)k\sec h(k\xi)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}}\right)^{\frac{1}{n}}, \tag{3.10a}$$

$$v(x,y,t)=-\frac{a(1+n)\delta\xi_1^2(\xi_1^2+\xi_2^2)k\sec h^2(k\xi)}{(bn^2\xi_1^2+n^2\delta\lambda\xi_1^2+bn^2\xi_2^2)}. \tag{3.10b}$$

Case 2:

$$c_0=0,\ c_1=\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}},\ d_0=0,\ m=1,\ \theta_3=-\frac{a(n^2\theta_1^2+n^2\theta_2^2-k^2\xi_1^2-k^2\xi_2^2)}{n^2}, \tag{3.11}$$

where $\xi_3=-2a\xi_1\theta_1-2a\xi_2\theta_2$ and k is the free parameter.

Substituting eqs.(3.11) and using the special solutions (2.6) of eq.(2.5), we obtain

$$\Phi(\xi)=\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)k\sec h(k\xi)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}}$$

which yields the following solitary wave solutions of eqs. (3.1) and (3.2):

$$u(x,y,t)=\Phi(\xi)^{\frac{1}{n}}=\left(\frac{i\sqrt{a}\sqrt{1+n}(\xi_1^2+\xi_2^2)k\sec h(k\xi)}{\sqrt{-bn^2\xi_1^2-n^2\delta\lambda\xi_1^2-bn^2\xi_2^2}}\right)^{\frac{1}{n}}, \tag{3.12a}$$

$$v(x,y,t)=-\frac{a(1+n)\delta\xi_1^2(\xi_1^2+\xi_2^2)k\sec h^2(k\xi)}{(bn^2\xi_1^2+n^2\delta\lambda\xi_1^2+bn^2\xi_2^2)}. \tag{3.12b}$$

3.2 Exact solutions of the fractional (2+1)-dimensional new integrable Davey–Stewartson-type equation

Let us consider the fractional (2+1)-dimensional new integrable Davey–Stewartson-type equation

$$\begin{aligned} &iD_\tau^\alpha\Psi+L_1\Psi+\Psi\Phi+\Psi\chi=0,\\ &L_2\chi=L_3|\Psi|^2,\\ &D_\xi^\beta\Phi=D_\eta^\gamma\chi+\mu D_\eta^\gamma\left(|\Psi|^2\right), \qquad \mu=\mp1,\ 0<\alpha,\beta,\gamma\le1 \end{aligned} \tag{3.13}$$

where the linear differential operators are given by

$$L_1\equiv\left(\frac{b^2-a^2}{4}\right)D_\xi^{2\beta}-aD_\xi^\beta D_\eta^\gamma-D_\eta^{2\gamma},$$

$$L_2\equiv\left(\frac{b^2+a^2}{4}\right)D_\xi^{2\beta}+aD_\xi^\beta D_\eta^\gamma+D_\eta^{2\gamma},$$

$$L_3\equiv\pm\frac{1}{4}\left(b^2+a^2+\frac{8b^2(a-1)}{(a-2)^2-b^2}\right)D_\xi^{2\beta}\pm\left(a+\frac{2b^2}{(a-2)^2-b^2}\right)D_\xi^\beta D_\eta^\gamma\pm D_\eta^{2\gamma},$$

where $\Psi\equiv\Psi(\xi,\eta,\tau)$ is complex while $\Phi\equiv\Phi(\xi,\eta,\tau)$, $\chi\equiv\chi(\xi,\eta,\tau)$ are real and a, b are real parameters. The above equation in integer order was devised firstly by Maccari [11] from the Konopelchenko-Dubrovsky (KD) equation [12].

According to algorithm discussed in Section 3, let us consider the following fractional complex transform

$$\Psi(\xi,\eta,\tau)=\Psi(X)e^{i\theta}\,,\ \ \Phi(\xi,\eta,\tau)=\Phi(X)\,,\ \ \chi(\xi,\eta,\tau)=\chi(X)\,,$$

$$X=k\left(\frac{\xi^{\beta}}{\Gamma(1+\beta)}+l\frac{\eta^{\gamma}}{\Gamma(1+\gamma)}+\lambda\frac{\tau^{\alpha}}{\Gamma(1+\alpha)}\right),\quad \theta=\frac{\theta_1\xi^{\beta}}{\Gamma(1+\beta)}+\frac{\theta_2\eta^{\gamma}}{\Gamma(1+\gamma)}+\frac{\theta_3\tau^{\alpha}}{\Gamma(1+\alpha)}\,, \tag{3.14}$$

where k , l , λ , θ_1, θ_2 and θ_3 are constants.

By applying the fractional complex transform (3.14), eq. (3.13) can be reduced to the following couple nonlinear ODEs:

$$k^2M_1\frac{d^2\Psi(X)}{dX^2}+M_0\Psi(X)+\Psi(X)\Phi(X)+\Psi(X)\chi(X)=0 \tag{3.15}$$

$$k^2M_2\frac{d^2\chi(X)}{dX^2}=k^2M_3\frac{d^2\Psi^2(X)}{dX^2} \tag{3.16}$$

$$k\frac{d\Phi(X)}{dX}=kl\frac{d\chi(X)}{dX}+\mu kl\frac{d\Psi^2(X)}{dX} \tag{3.17}$$

where λ has been set to $a(l\theta_1+\theta_2)+2l\theta_2-\dfrac{\theta_1(b^2-a^2)}{2}$.

Here,

$$M_0=-\theta_3-\frac{(b^2-a^2)}{4}\theta_1^2+a\theta_1\theta_2+\theta_2^2\,,$$

$$M_1=-al-l^2+\frac{(b^2-a^2)}{4}\,,$$

$$M_2=al+l^2+\frac{(b^2+a^2)}{4}\,,$$

$$M_3=\pm l^2\pm\left(a+\frac{2b^2}{(a-2)^2-b^2}\right)l\pm\frac{1}{4}\left(b^2+a^2+\frac{8b^2(a-1)}{(a-2)^2-b^2}\right).$$

Now, eqs. (3.17) and (3.16) are integrated once and twice term by term with respect to X where integration constants are considered zero. Thus we obtain

$$\chi(X)=\frac{M_3}{M_2}\Psi^2(X)\,,$$

$$\Phi(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi^2(X)\,. \tag{3.18}$$

Eliminating $\chi(X)$, $\Phi(X)$ from eqs. (3.15) and (3.18), we arrive at

$$k^2M_1\frac{d^2\Psi(X)}{dX^2}+M_0\Psi(X)+\left(\frac{lM_3}{M_2}+\mu l+\frac{M_3}{M_2}\right)\Psi^3(X)=0 \tag{3.19}$$

By balancing the nonlinear term $\Psi^3(X)$ and highest order derivative term $\dfrac{d^2\Psi(X)}{dX^2}$ in eq. (3.19), the value of N can be determined, which is $N=1$ in this problem.

Therefore the solution of eq. (3.19) can be written in the following ansatz as

$$\Psi(X)=c_0+c_1\phi(X)+d_0\sqrt{p^2-\phi^2(X)}\,, \tag{3.20}$$

where c_0 , c_1 and d_0 are constants to be determined later and $\phi(X)$ satisfies the elliptic equation:

$$\frac{d\phi(X)}{dX} = \sqrt{(p^2 - \phi^2(X))(\phi^2(X) - p^2(1-m))} \tag{3.21}$$

whose solutions are given by

$$\phi(X) = p\,dn(pX|m),$$
$$\phi(X) = p\sqrt{1-m}\,nd(pX|m), \tag{3.22}$$

Now substituting eq. (3.20) alongwith eq. (3.21) into eq. (3.19) and then equating each coefficients of $\phi^i(X)$, i = 0,1,2,... to zero, we can get a set of algebraic equations for c_0, c_1, d_0, p and m and solving the algebraic equations, we have the set of coefficients for the non-trivial travelling wave solutions of eq. (3.19) as given below:

Case 1:

$$c_0 = 0,\ c_1 = -\frac{k\sqrt{2M_1M_2}}{\sqrt{l\mu M_2 + (l+1)M_3}},$$

$$d_0 = 0,\ m = \frac{M_0 + 2M_1k^2p^2}{M_1k^2p^2}.$$

$$\Psi_{11}(X) = -\frac{kp\sqrt{2M_1M_2}}{\sqrt{l\mu M_2 + (l+1)M_3}}\,dn(pX|m),$$

$$\Phi_{11}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{11}^2(X),$$

$$\chi_{11}(X) = \frac{M_3}{M_2}\Psi_{11}^2(X),$$

$$\Psi_{12}(X) = -\frac{kp\sqrt{2M_1M_2}\sqrt{1-m}}{\sqrt{l\mu M_2 + (l+1)M_3}}\,nd(pX|m),$$

$$\Phi_{12}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{12}^2(X),$$

$$\chi_{12}(X) = \frac{M_3}{M_2}\Psi_{12}^2(X).$$

Case 2:

$$c_0 = 0,\ c_1 = \frac{k\sqrt{2M_1M_2}}{\sqrt{l\mu M_2 + (l+1)M_3}},$$

$$d_0 = 0,\ m = \frac{M_0 + 2M_1k^2p^2}{M_1k^2p^2}.$$

$$\Psi_{21}(X) = \frac{kp\sqrt{2M_1M_2}}{\sqrt{l\mu M_2 + (l+1)M_3}}\,dn(pX|m),$$

$$\Phi_{21}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{21}^2(X),$$

$$\chi_{21}(X) = \frac{M_3}{M_2}\Psi_{21}^2(X),$$

$$\Psi_{22}(X) = \frac{kp\sqrt{2M_1M_2}\sqrt{1-m}}{\sqrt{l\mu M_2 + (l+1)M_3}}\,nd(pX|m),$$

$$\Phi_{22}(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi_{22}^2(X),$$

$$\chi_{22}(X)=\frac{M_3}{M_2}\Psi_{22}^2(X).$$

Case 3:

$$c_0=0,\ c_1=-\frac{k\sqrt{M_1M_2}}{\sqrt{2l\mu M_2+2(l+1)M_3}},$$

$$d_0=-\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},\ m=\frac{2M_0+M_1k^2p^2}{2M_1k^2p^2}.$$

$$\Psi_{31}(X)=-\frac{kp\sqrt{M_1M_2}}{\sqrt{2l\mu M_2+2(l+1)M_3}}dn(pX|m)-p\sqrt{m}sn(pX|m)\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},$$

$$\Phi_{31}(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi_{31}^2(X),$$

$$\chi_{31}(X)=\frac{M_3}{M_2}\Psi_{31}^2(X),$$

$$\Psi_{32}(X)=-\frac{kp\sqrt{M_1M_2}\sqrt{1-m}}{\sqrt{2l\mu M_2+2(l+1)M_3}}dn(pX|m)-p\sqrt{1-(1-m)nd^2(pX|m)}\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},$$

$$\Phi_{32}(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi_{32}^2(X),$$

$$\chi_{32}(X)=\frac{M_3}{M_2}\Psi_{32}^2(X).$$

Case 4:

$$c_0=0,\ c_1=\frac{k\sqrt{M_1M_2}}{\sqrt{2l\mu M_2+2(l+1)M_3}},$$

$$d_0=-\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},\ m=\frac{2M_0+M_1k^2p^2}{2M_1k^2p^2}.$$

$$\Psi_{41}(X)=\frac{kp\sqrt{M_1M_2}}{\sqrt{2l\mu M_2+(l+2)M_3}}dn(pX|m)-\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},$$

$$\Phi_{41}(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi_{41}^2(X),$$

$$\chi_{41}(X)=\frac{M_3}{M_2}\Psi_{41}^2(X),$$

$$\Psi_{42}(X)=\frac{kp\sqrt{M_1M_2}\sqrt{1-m}}{\sqrt{2l\mu M_2+(l+2)M_3}}nd(pX|m)-\frac{k\sqrt{M_1M_2}}{\sqrt{-2l\mu M_2-2(l+1)M_3}},$$

$$\Phi_{42}(X)=\left(l\frac{M_3}{M_2}+\mu l\right)\Psi_{42}^2(X),$$

$$\chi_{42}(X)=\frac{M_3}{M_2}\Psi_{42}^2(X).$$

Case 5:

$$c_0 = 0\,,\ c_1 = -\frac{k\sqrt{M_1 M_2}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,,$$

$$d_0 = \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,\ m = \frac{2M_0 + M_1 k^2 p^2}{2M_1 k^2 p^2}\,.$$

$$\Psi_{51}(X) = -\frac{kp\sqrt{M_1 M_2}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,dn(pX|m) + \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,$$

$$\Phi_{51}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{51}^2(X)\,,$$

$$\chi_{51}(X) = \frac{M_3}{M_2}\Psi_{51}^2(X)\,,$$

$$\Psi_{52}(X) = -\frac{kp\sqrt{M_1 M_2}\sqrt{1-m}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,nd(pX|m) + \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,$$

$$\Phi_{52}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{52}^2(X)\,,$$

$$\chi_{52}(X) = \frac{M_3}{M_2}\Psi_{52}^2(X)\,.$$

Case 6:

$$c_0 = 0\,,\ c_1 = \frac{k\sqrt{M_1 M_2}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,,$$

$$d_0 = \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,\ m = \frac{2M_0 + M_1 k^2 p^2}{2M_1 k^2 p^2}\,.$$

$$\Psi_{61}(X) = \frac{kp\sqrt{M_1 M_2}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,dn(pX|m) + \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,$$

$$\Phi_{61}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{61}^2(X)\,,$$

$$\chi_{61}(X) = \frac{M_3}{M_2}\Psi_{61}^2(X)\,,$$

$$\Psi_{62}(X) = \frac{kp\sqrt{M_1 M_2}\sqrt{1-m}}{\sqrt{2l\mu M_2 + 2(l+1)M_3}}\,nd(pX|m) + \frac{k\sqrt{M_1 M_2}}{\sqrt{-2l\mu M_2 - 2(l+1)M_3}}\,,$$

$$\Phi_{62}(X) = \left(l\frac{M_3}{M_2} + \mu l\right)\Psi_{62}^2(X)\,,$$

$$\chi_{62}(X) = \frac{M_3}{M_2}\Psi_{62}^2(X)\,.$$

4. Conclusion

In this paper, the Jacobi elliptic function method has been used to determine the exact solutions of time fractional (2+1)-dimensional Davey-Stewartson equation and new integrable Davey-Stewartson type equation. In both the problems, with the help of fractional complex transform, the Davey-Stewartson system was first transformed to a system of nonlinear ordinary differential equations, which were then solved to obtain the exact solutions. In this paper, the fractional complex transform has been considered which is derived from the local fractional calculus defined on fractals. Using this proposed method, some new solitary wave solutions and double-periodic solutions have been obtained. To the best information of the author, these solitary wave solutions of the fractional Davey-Stewartson equation are new exact solutions which are not reported earlier. Being concise and powerful, this current method can also be extended to solve many other fractional partial differential equations arising in mathematical physics.

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