Decision Boundary for Underwater Acoustic Communication with Generalized Gaussian Noise Model

Snigdha Bhuyan, Siddharth Deshmukh
Department of Electronics and Communication Engineering
National Institute of Technology, Rourkela, Odisha 769008
Email: bhuyan.snigdha09@gmail.com, deshmukhs@nitrkl.ac.in

***Abstract*—In this paper, we consider Generalized Gaussian (GG) distribution to model the additive noise source in underwater acoustic (UWA) communication. Since communication in oceanic medium is dominated by both prevailing and spontaneous noise sources, we model the resultant noise distribution as mixture of GG distribution. Owing to the complexity in optimal detector design with GG noise model, we apply expectation maximization (EM) algorithm to decompose the resultant channel distribution in terms of weighted sum of Gaussian density functions. By having multiple antennas at the receiver, we also exploit spatial diversity to improve error performance at the receiver. In this context, we compute decision boundary for detecting the binary phase shift keying (BPSK) modulated signal. In addition, we also discuss variation in decision boundary under various signal to noise ratio (SNR) levels observed at receiver front end. Finally, we compare the detector performance under new decision boundary with traditional detectors and validate the approach by showing improvement in symbol error rate performance.**

***Index Terms*—Underwater Acoustic Communication, Generalized Gaussian Statistics, Expectation Maximization Algorithm, Detector Design**

I. Introduction

Advancements in underwater communication will enable practical and efficient realization of several applications under sea water. These applications include collection of oceanographic data, military surveillance, disaster prevention etc [1]. The challenges in design of suitable communication system are attributed to some of the unique characteristics of underwater medium. Firstly, due to high attenuation of electromagnetic waves in water medium, underwater communication is generally enabled by acoustic waves [2]. However, low propagation speed of acoustic waves results in large multi-path delay spread and hence there is severe inter symbol interference at the receiver [3] [4]. Further, additive noise in the underwater acoustic (UWA) channel is significantly different from wireless channel. The UWA communication in oceanic medium is affected by prevailing noise sources like, surface waves, thermal noise, turbulence etc. [5] and spontaneous sources like marine life, shipping traffic, underwater explosives, offshore exploration etc. [6]. Due to dominance of different noise sources in various acoustic spectrum bands, the additive noise follows a decaying power spectral density [4]. This makes standard Gaussian distribution to be unsuitable for its statistical characterization. For example, in spectrum range of 1Hz to 100Hz there is dominance of seismic noise, while in range of 10Hz to 100KHz there is dominance of noise due to merchant ships [4]. Since most communication systems are designed assuming additive white Gaussian noise (AWGN) model, in this paper we investigate a new receiver design technique to improve the communication system performance in UWA environment.

The noise source in UWA communication can be characterized by several non Gaussian models, like Middleton model, Gaussian Gaussian model, Generalized Gaussian (GG) model etc. each having its own limitations. Middleton models are generally used to model electromagnetic interference (EMI) and dont have irreducible form [7]. Similarly, Gaussian Gaussian models are not able to capture the shape and tail of actual noise distribution [8]. In this work, we choose GG distribution to model UWA channel noise source as by adjustment of distribution parameters, the model can be easily adapted to super Gaussian and sub Gaussian densities [9].

The UWA channel noise is modeled by GG distribution in [10] to derive an analytical expression for probability of error performance. The authors have considered BPSK, QPSK, and M-ary PAM modulation schemes and analyzed the receiver performance under various Kurtosis values. In this paper, we consider mixture of GG distribution as channel noise in UWA communication is generated by several prevailing and spontaneous noise sources. We also exploit spatial diversity to improve the probability of error performance. The spatial diversity is achieved by considering multiple antennas at the receiver. Here we assume that antennas are placed sufficienty apart so that observed channel noise at corresponding antennas is statistically independent.

Considering simplicity in design, a linear detector which is optimal for noise with Gaussian distribution can be used. However, its performance is expected to degrade as GG statistics move away from Gaussian statistics. Similarly, a sign correlator based receiver can be used; but it performs better only if supplied with odd copies of same transmitted signal [8]. Sub-optimal linear detectors based on optimal nonlinear function can be also used, but it performs poorly at high SNR values [7]. In this work, we simplify the detector design by application of expectation maximization (EM) algorithm to decompose the channel noise density function in terms of weighted sum Gaussian density function. In this context, we compute decision boundary for detecting BPSK modulated signal. Assuming receiver is supplied with two copies of transmitted signal, the decision boundary is in two dimensional space. Compared with traditional detectors, our simulations indicate that the proposed detector has superior symbol error rate performance.

II. Non Gaussian UWA Channel Model

The Generalized Gaussian Distribution (GGD) [9], apart from mean and variance is characterized by another parameter called shape parameter which relates to its Kurtosis. The probability distribution function of a stochastic process N with GG statistics is represented as:

$$f_{N-GG}(n) = \frac{\beta}{2\alpha\Gamma(1/\beta)} exp\{-(|n-\mu_n|/\alpha)^{\beta}\} \quad (1)$$

where, μ_n is the mean, $\Gamma(\cdot)$ is the standard Gamma function, and α, β are the scale and shape parameters, respectively. The corresponding variance σ_n^2 and kurtosis $Kurt[N]$ of N are defined as,

$$\sigma_n^2 = \frac{\alpha^2\Gamma(3/\beta)}{\Gamma(\frac{1}{\beta})} \qquad kurt[N] = \frac{\Gamma(1/\beta)\Gamma(5/\beta)}{\Gamma(3/\beta)^2}.$$

By selecting proper shape parameter (or Kurtosis), the GG distribution can be used to statistically model UWA acoustic noise in various scenarios. For example, low kurtosis values can be used for modeling noise due to propeller cavitation and high kurtosis values can be used to model noise due to snapping shrimps, breaking waves etc. Similarly, kurtosis value of 2.5 and 3.5 can be used to model ship transit noise and noise due to surface wave agitation respectively [11]. In addition, it can be noted that for $\beta = 2$ ($Kurt[N] = 3$), the distribution expression in equation (1) becomes standard normal distribution with mean μ_n and variance $\sigma_n^2 = \frac{\alpha^2}{2}$.

III. UWA Communication System Model

For simplicity in analysis, we assume signaling via binary phase shift keying modulation technique. The generalization to other modulation techniques can be obtained by straight forward modification in the analysis. The effect of multipath fading is not considered in this analysis and will be covered in our future work. We further assume there is one transmit antenna and N receive antennas for receiving multiple copies of transmitted signal. The received signal at any time instant t and at k^{th} receive antenna is given by,

$$r_k(t) = s(t) + n_k(t), \qquad k = 1, \cdots N \quad (2)$$

where, $s(t) \in \{\pm B\}$ and $n_k(t)$ is the noise sample observed at k^{th} receive antenna. Since n_k is assumed to be modeled as mixture of GG distribution, the resultant noise distribution is highly complex and mathematically intractable. So we first decompose the distribution off additive noise into weighted sum of Gaussian distribution with appropriate mean and variances. This is achieved by expectation maximization (EM) algorithm which consists of alternate iteration between two steps: expectation step and maximization step. Thus, the distribution of noise observed at k^{th} receive antenna can be expressed as,

$$p(n_k) = \sum_{m=0}^{\infty} \alpha_m g(n_k; \mu_m, \sigma_m^2) \quad (3)$$

where α_m is the weight, μ_m is the mean, σ_m^2 is the variance of m^{th} Gaussian distribution component $g(\cdot;\cdot,\cdot)$. With the assumption that noise samples observed at receive antennas are independent of each other, the joint distribution of noise vector $\mathbf{n} = [n_1, \cdots n_N]$ can be expressed as,

$$p(\mathbf{n}) = \prod_{k=1}^{N} \sum_{m=0}^{\infty} \alpha_m g(n_k; \mu_m, \sigma_m^2). \quad (4)$$

IV. Detector Design

In this section we discuss the optimal and proposed sub-optimal detector design for the given received signal vector $\mathbf{r} = [r_1, \cdots r_n]$.

A. Optimum Detector

Let us consider a binary hypothesis testing criteria in which hypothesis H_0 represents transmitted symbol $-B$ and hypothesis H_1 represent transmitted symbol B. Further, denoting conditional distribution of received sample r_k assuming hypothesis H_0 as $p(r_k/H_0)$ and hypothesis H_1 as $p(r_k/H_1)$; the optimum detector to compute test statistics is expressed as,

$$\Gamma(\mathbf{r}) = \frac{\prod_{k=1}^{N} p(r_k|H_1)}{\prod_{k=1}^{N} p(r_k|H_0)} \gtrless_{H_0}^{H_1} 1 \quad (5)$$

Here we have assumed that transmitted symbols are equiprobable. Substituting equation (4) in equation (5), the log likelihood ratio test is expressed as,

$$ln(\Gamma(\mathbf{r})) = \sum_{k=1}^{N} ln \left(\frac{\sum_{m=0}^{\infty} \frac{a_m}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{(r_k - B - \mu_m)^2}{2\sigma_m^2}}}{\sum_{m=0}^{\infty} \frac{a_m}{\sqrt{2\pi\sigma_m^2}} e^{-\frac{(r_k + B - \mu_m)^2}{2\sigma_m^2}}} \right) \gtrless_{H_0}^{H_1} 0 \quad (6)$$

Since optimum detector is very complex to be implemented, in next sub-section we discuss sub-optimal detector design.

B. Sub-optimal Detector Design

As discussed in section III, the additive noise in UWA channel can be decomposed into infinite number of Gaussian components. Since weights of large number of Gaussian components will be close to zero, we can approximate the channel distribution by finite number of Gaussian components. For simplicity in analysis, we further assume that noise distribution can be well approximated by two Gaussian components, i.e.,

$$p(n_k) \approx \alpha_0 g(n_k; \mu_0, \sigma_0^2) + \alpha_1 g(n_k; \mu_1, \sigma_1^2). \quad (7)$$

An illustration of such simplification is shown in Figure 1. Here we assume that UWA noise is due to two noise

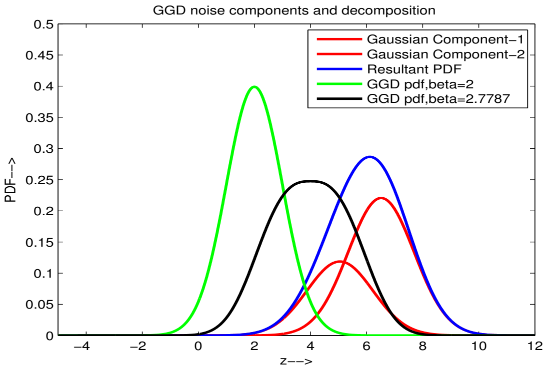

Fig. 1. Approximation of Generalized Gaussian Distribution by two Gaussian Components

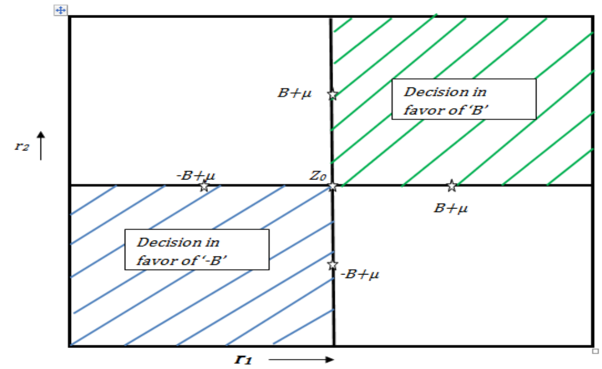

Fig. 3. Two dimensional decision region for two copies of received signal

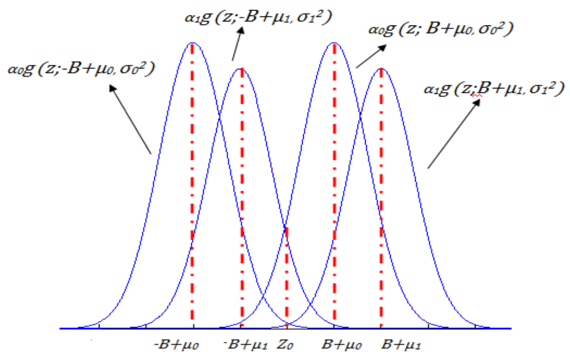

Fig. 2. Conditional distribution of received signal under Hypothesis H_0 and H_1

sources, each of which is modeled as GG distribution with mean $\mu gg_0 = 2; \mu gg_1 = 4$, variance $\sigma gg_0^2 = 1; \sigma gg_0^2 = 2$, and shape parameters $\beta_0 = 2; \beta_1 = 2.77$. The resultant noise distribution is shown in Figure 1. After applying EM algorithm, the resultant noise distribution can be approximated by two Gaussian components with following parameters: $\alpha_0 = 0.4142$, $\alpha_1 = 0.5788$; $\mu_0 = 4.8562$, $\mu_1 = 6.8283$; $\sigma_0^2 = 1.9075$, $\sigma_1^2 = 2.1844$.

Next, if we consider that only one copy r_1 of transmitted signal is present then the conditional distribution of received signal under different hypothesis can be expressed as,

$$\begin{aligned} P(\mathbf{r}|H_1) &= \alpha_0 g(n_1; +B+\mu_0, \sigma_0^2) + \alpha_1 g(n_1; +B+\mu_1, \sigma_1^2) \\ P(\mathbf{r}|H_0) &= \alpha_0 g(n_1; -B+\mu_0, \sigma_0^2) + \alpha_1 g(n_1; -B+\mu_1, \sigma_1^2) \end{aligned} \tag{8}$$

Figure 2 shows the conditional distribution of received signal under assumption of different hypotheses. The decision boundary Z_0 is the threshold point which decides whether decision should be in favor of $+B$ or $-B$ and is computed by equating log likelihood functions, i.e.,

$$l_0(r_1|H_0) + l_1(r_1|H_0) = l_0(r_1|H_1) + l_1(r_1|H_1). \tag{9}$$

The log likelihood functions l_m are defined as,

$$\begin{aligned} l_m(r_k|H_0) &= ln(\alpha_m/\sigma_m) - \frac{[r_k-\mu_m+B]^2}{2\sigma_m^2} \\ l_m(r_k|H_1) &= ln(\alpha_m/\sigma_m) - \frac{[r_k-\mu_m-B]^2}{2\sigma_m^2} \end{aligned} \tag{10}$$

where, subscript 'm' represents the m^{th} Gaussian component and subscript 'k' represents copy of transmitted signal received at k^{th} antenna. Further, our analysis assume that mean $\mu_0 < \mu_1$. On solving equation (9), the decision boundary Z_0 is expressed as,

$$Z_0 = \frac{\mu_0/\sigma_0^2 + \mu_1/\sigma_1^2}{1/\sigma_0^2 + 1/\sigma_1^2}. \tag{11}$$

Next, consider the scenario when receiver is with two copies of transmitted signal $\mathbf{r} = [r_1, r_2]$. Since we have assumed that the copies of received signal are independent of each other, there is a two dimensional decision region. In addition, the distribution of received samples are centered at $(\pm B - \mu_0, \pm B - \mu_0)$, $(\pm B - \mu_1, \pm B - \mu_0)$, $(\pm B - \mu_0, \pm B - \mu_1)$ and $(\pm B - \mu_1, \pm B - \mu_1)$ respectively. Thus, decision region can be divided in four quadrants as shown Figure 3.

It can be observed from the Figure 3 that in first and third quadrant both r_1 and r_2 give decision in favor of +B and -B, respectively. So there is no conflict in decision from two copies of received signal. However, for second and fourth quadrant there is conflict in the decision and hence the decision boundary needs to be reanalyzed. For second quadrant, the decision boundary can be computed by equating the corresponding log likelihood functions,

$$l_0(r_1|H_0) + l_1(r_1|H_0) = l_0(r_2|H_1) + l_1(r_2|H_1) \tag{12}$$

After simplification we get,

$$\begin{aligned} &r_1^2\left(\tfrac{1}{\sigma_0^2}+\tfrac{1}{\sigma_1^2}\right) - 2r_1\left(\tfrac{-B+\mu_0}{\sigma_0^2}+\tfrac{-B-\mu_1}{\sigma_1^2}\right) + \tfrac{(-B+\mu_0)^2}{\sigma_0^2} + \tfrac{(-B+\mu_1)^2}{\sigma_1^2} \\ &= r_2^2\left(\tfrac{1}{\sigma_0^2}+\tfrac{1}{\sigma_1^2}\right) - 2r_2\left(\tfrac{B+\mu_0}{\sigma_0^2}+\tfrac{B-\mu_1}{\sigma_1^2}\right) + \tfrac{(B+\mu_0)^2}{\sigma_0^2} + \tfrac{(B+\mu_1)^2}{\sigma_1^2} \end{aligned} \tag{13}$$

The decision boundary for fourth quadrant is symmetric to second quadrant and considering the noise distribution presented in Figure 1, the complete decision region is shown in Figure 4.

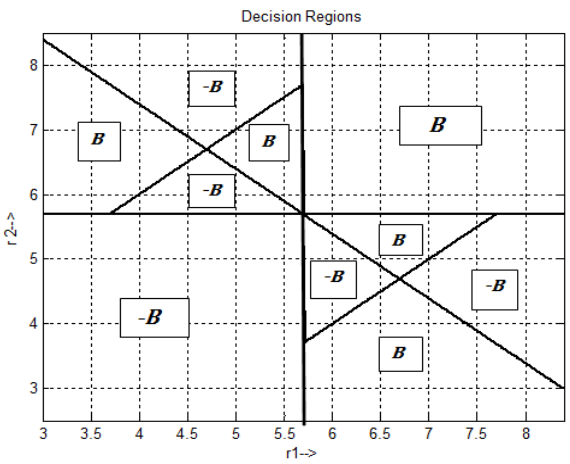

Fig. 4. Decision boundary with two receive antennas

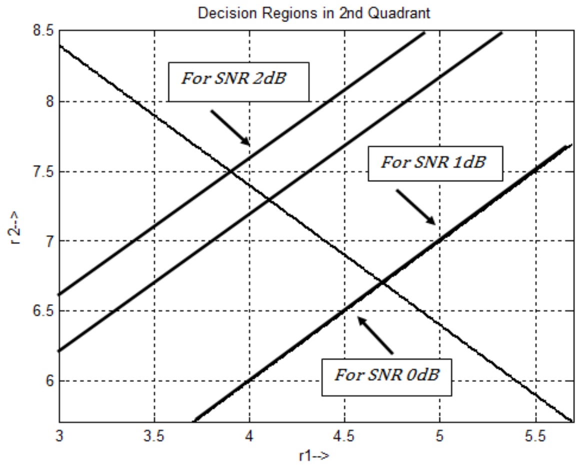

Fig. 5. Variation in decision boundary with SNR

V. Discussion on Decision Boundary

The decision region in second and fourth quadrant of figure 4 can be divided in two halves: linear region: if $r_1 > r_2$ and decision from r_1 is $-B$, r_2 is $+B$, the final decision is for B, and vice-versa; Nonlinear region: if $r_1 > r_2$ and decision from r_1 is $-B$, r_2 is $+B$, the final decision is for $-B$, and vice-versa. For this typical characteristic, once again consider equation (13) defined to compute decision region in second quadrant. For a given values of observed received signal, i.e., $\mathbf{r} = [r_1 r_2]$, at high SNR $\frac{(-B+\mu_0)^2}{\sigma_0^2}$, $\frac{(-B+\mu_1)^2}{\sigma_1^2}$ are the dominant terms on left side and $\frac{(B+\mu_0)^2}{\sigma_0^2}$, $\frac{(B+\mu_1)^2}{\sigma_1^2}$ are the dominant terms on right side of the equation. Thus, at high SNR equation (13) can be approximated as,

$$\begin{aligned} &\frac{(-B+\mu_0)^2}{\sigma_0^2} + \frac{(-B+\mu_1)^2}{\sigma_1^2} - 2r_1\left(\frac{-B+\mu_0}{\sigma_0^2} + \frac{-B-\mu_1}{\sigma_1^2}\right) \\ &= \frac{(B+\mu_0)^2}{\sigma_0^2} + \frac{(B+\mu_1)^2}{\sigma_1^2} - 2r_2\left(\frac{B+\mu_0}{\sigma_0^2} + \frac{B-\mu_1}{\sigma_1^2}\right) \end{aligned} \quad (14)$$

which follows a linear boundary region. Thus, for given value of r_1 and r_2, with the increasing value of SNR, there is increase in linear boundary region as shown in Figure 5.

VI. Simulation Results

Firstly, we discuss simulation result which validates our approach to decompose the UWA channel noise into Gaussian components by EM algorithm. Here we assume BPSK signaling and UWA noise modeled by mixture of two GG distribution with parameters: mean $\mu gg_0 = 2; \mu gg_1 = 4$, variance $\sigma gg_0^2 = 1; \sigma gg_0^2 = 2$, and shape parameters $\beta_0 = 2; \beta_1 = 2.77$. The approach for GG noise generation is similar to [9] with different shape parameters. Figure 6 shows the comparison of symbol error rate performance under following detection schemes with the assumption that receiver has only one copy of transmitted signal:

- *Gaussian detector with average threshold*: In this detection mechanism, receiver assumes additive channel noise to be Gaussian distributed with average mean $\mu = (\mu gg_0 + \mu gg_1)/2$ and variance $\sigma^2 = (\sigma gg_0^2 + \sigma gg_1^2)/2$. Thus, the decision boundary is simply given by threshold $(\mu gg_0 + \mu gg_1)/2$.
- *Gaussian detector with likelihood based threshold*: In this detection mechanism, receiver assumes additive channel noise to be mixture of two Gaussian distribution with mean $\mu_0 = 2; \mu_1 = 4$ and variance $\sigma_0^2 = 1; \sigma_0^2 = 2$. The decision boundary in this case can be computed using equation 11.
- *Detector with EM decomposition*: In this detection mechanism, receiver first decomposes the resultant distribution function formed by two GG components into weighted sum of Gaussian densities. The two significant Gaussian destinies have weights $\alpha_0 = 0.4142; \alpha_1 = 0.5788$, mean $\mu_0 = 4.8562, \mu_1 = 6.8283$ and variance $\sigma_0^2 = 1.9075; \sigma_1^2 = 2.1844$. The decision boundary after this is computed by equation 11.

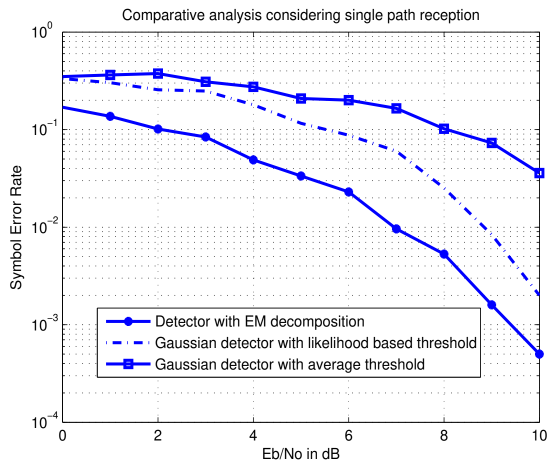

Fig. 6. Detector performance comparison with single antenna reception

It can be observed that *Gaussian detector with average threshold* is the simplest detector, but it has very poor symbol error

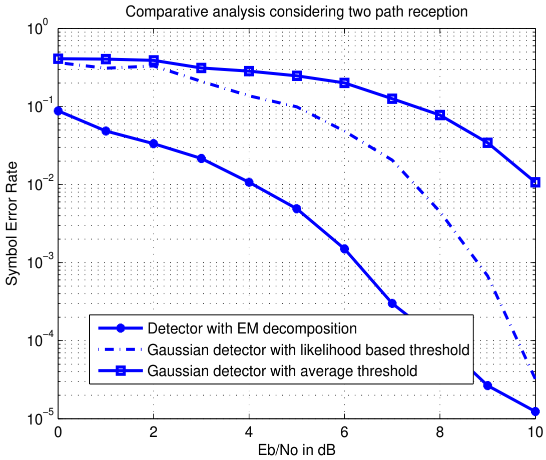

Fig. 7. Detector performance comparison with multiple antenna reception

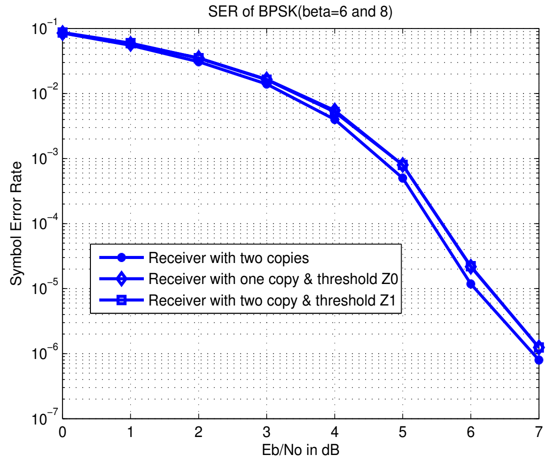

Fig. 8. Performance improvement via spatial diversity

rate performance. The performance is improved in *Gaussian detector with likelihood based threshold* where GG distribution is considered same as Gaussian distribution with same mean and variance parameters. The proposed *Detector with EM decomposition* shows superior performance compared to other two detectors as the resulting additive noise distribution of UWA communication is suitably approximated by mixture of Gaussian densities. Similar, observations can be inferred from Figure 7 where receiver is provided with two copies of transmitted signal. Here, threshold for decision boundary is computed by equation (13) and decision region is followed from Figure 4. Once again it can be verified that *Detector with EM decomposition* following decision region of Figure 4 give the superior performance compared to other two detectors.

Another observation can be made from Figures 6 and 7 is regarding convergence of symbol error rate curves for *Gaussian detector with likelihood based threshold* and *Detector with EM decomposition* at high SNR. This observation is in accordance with our discussion in section V, where for high SNR the decision region in quadrant 2 and 4 follow linear boundary. Thus, at high SNR, performance improvement obtained by EM decomposition becomes marginal and both *Gaussian detector with likelihood based threshold* and *Detector with EM decomposition* give similar performance. Finally, we verify the performance improvement obtained by exploiting spatial diversity at the receiver. Figure 8 shows symbol error rate performance for UWA noise modeled by mixture of two GG distribution with parameters: mean $\mu gg_0 = 2; \mu gg_1 = 4$, variance $\sigma gg_0^2 = 1; \sigma gg_0^2 = 2$, and shape parameters $\beta_0 = 6; \beta_1 = 8$. It can be observed that by having two copies of transmitted signal, there is improvement in symbol error rate performance.

VII. Conclusion

In this work we have computed close form expression of decision boundary for detecting binary phase shift keying (BPSK) modulated signals in underwater acoustic (UWA) channel. The additive noise is modeled as mixture of GG densities and further approximated by weighted sum of Gaussian densities using Expectation Maximization (EM) algorithm. In order to improve the error performance, spatial diversity is exploited by having multiple antennas at receiver. The simulation results validate the approach by indicating improvement in BER performance when compared with traditional detectors.

References

[1] J. Heidemann, W. Ye, J. Wills, A. Syed, and Y. Li, "Research challenges and applications for underwater sensor networking," in *Wireless Communications and Networking Conference, 2006. WCNC 2006. IEEE*, vol. 1, April 2006, pp. 228–235.

[2] M. Stojanovic, "Underwater acoustic communications: Design considerations on the physical layer," in *Wireless on Demand Network Systems and Services, 2008. WONS 2008. Fifth Annual Conference on*, Jan 2008, pp. 1–10.

[3] A. Singer, J. Nelson, and S. Kozat, "Signal processing for underwater acoustic communications," *Communications Magazine, IEEE*, vol. 47, no. 1, pp. 90–96, January 2009.

[4] M. Stojanovic and J. Preisig, "Underwater acoustic communication channels: Propagation models and statistical characterization," *Communications Magazine, IEEE*, vol. 47, no. 1, pp. 84–89, January 2009.

[5] B. Mazhl, M. Wahlberg, P. Madsen, A. Heerfordt, and A. Lund, "The monopulsed nature of sperm whale clicks," *The Journal of the Acoustical Society of America*, vol. 114, no. 2, pp. 1143–54, Aug 2003.

[6] J. Goold and P. Fish, "Broadband spectra of seismic survey airgun emissions, with reference to dolphin auditory thresholds," *The Journal of the Acoustical Society of America*, vol. 103, no. 4, pp. 2177–84, 1998.

[7] D. Middleton, "Non-gaussian noise models in signal processing for telecommunications: new methods an results for class a and class b noise models," *Information Theory, IEEE Transactions on*, vol. 45, no. 4, pp. 1129–1149, May 1999.

[8] D. Stein, "Detection of random signals in gaussian mixture noise," *Information Theory, IEEE Transactions on*, vol. 41, no. 6, pp. 1788–1801, Nov 1995.

[9] M. Novey, T. Adali, and A. Roy, "A complex generalized gaussian distribution; characterization, generation, and estimation," *Signal Processing, IEEE Transactions on*, vol. 58, no. 3, pp. 1427–1433, March 2010.

[10] S. Banerjee and M. Agrawal, "Underwater acoustic noise with generalized gaussian statistics: Effects on error performance," in *OCEANS - Bergen, 2013 MTS/IEEE*, June 2013, pp. 1–8.

[11] M. Bouvet and S. Schwartz, "Under water noise: Statistical modeling, detection and normalization," *Journal of Acoustic Society America*, vol. 83, no. 3, pp. 1023–1033, 1988.